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THE
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AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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THE
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VOLUME XXXIV JULY 1911 NUMBER 1

SOME INFRA-RED SPECTRA

By H. M. RANDALL

In 1895 Percival Lewis[1] at Johns Hopkins measured the wave-lengths of thirteen infra-red spectral lines whose wave-lengths lay between *Ca* 7664 and *Th* 11,512 Å.U., with an actual accuracy of about one Å.U.; an accuracy hitherto unattained and not equaled in the subsequent important researches of Lehmann,[2] Bergmann,[3] and Moll.[4] In two very noteworthy contributions in 1908 and 1909 Paschen[5] very materially extended the work of Lewis, attaining an accuracy in case of sources of constant intensity of a few tenths Å.U. and a maximum wave-length of approximately 28,000 Å.U. This latter limit was again extended the following year to 50,000 Å.U. by the same observer. Many of the lines found will serve as standards of wave-length in the infra-red, on account of their intensity and sharpness and the accuracy with which they were measured; while the results in general have furnished much needed data for the establishment of series laws and especially the combination principle of Ritz.

The work described in the present paper was done at Tübingen and deals with the spectra of 11 elements in the region 7500 to

---

[1] *Astrophysical Journal*, **2**, 1 and 106, 1895.

[2] *Annalen der Physik*, **5**, 633, 1895; **8**, 643; **9**, 246 and 1330, 1902.

[3] Dissertation, Jena, 1907.

[4] *Archives néerlandaises des sciences exactes et naturelles* (2), **13**, 100, 1908.

[5] *Annalen der Physik*, **27**, 537; **29**, 625; **33**, 717, 1910.

30,000 Å.U. Fig. 1 shows the general scheme of the arrangement. An enlarged image of the arc and carbons was thrown on the screen *K* by the quartz lens *Q*, the radiation from the carbons being intercepted by the screen, while that from the arc itself passed through the aperture, and was converged upon the slit *S* by means of the silvered mirrors *X* and *Y*. This slit being at the focus of the silvered mirror *M*, the beam came to the grating *G* as a parallel beam and the portion diffracted to *M* was brought to a focus at the thermopile *P*, with the aid of the plane mirror *T*. This approximately autocollimating arrangement is quite similar to that previously devised by Kayser and Runge,[1] and by it the advantages of an increased sharpness of definition due to smallness of the angle of incidence at the reflecting surface *M* is obtained. The spectrometer embodying this arrangement had been previously designed by Professor Paschen and constructed in the laboratory shops.

Fig. 2 shows a cross-section of this spectrometer somewhat more in detail. The slits, concave mirror, grating, and thermopile are all contained within a thick-walled chamber of brass, cast in a single piece with its two portions in the form of hollow cylinders. The portion containing the grating has its axis vertical and its upper end open, while the portion containing the mirror *M* has its axis horizontal and is open back of the mirror. The diameter of this horizontal portion is somewhat less than the height of the other portion. Besides the openings just mentioned there are openings before the slit and thermopile. The slit *S* is covered by a fluorite plate, and all other openings are furnished with suitable covering plates of brass with ground joints, and as the axis of the grating table passes through the bottom of the vertical chamber air tight,

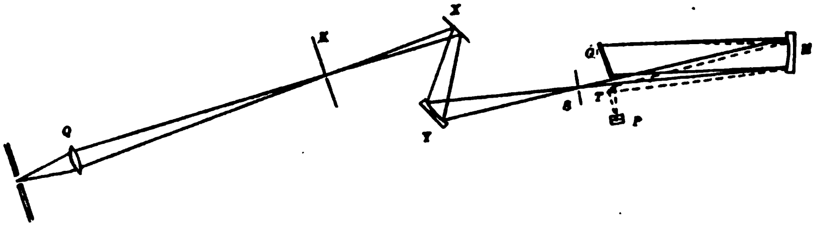

FIG. 1

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[1] Kayser's *Handbuch der Spectroscopie*, 1, 635.

it is possible to use the spectrometer as a vacuum spectrometer. The graduated circle and grating table are permanently clamped together on the same axis and are turned from beneath the brass case. The circle was made by Fuess and is graduated to read directly to seconds. The entire spectrometer is held at a convenient height above the table upon suitable feet. This spectrometer, designed for special work, is much simpler in construction than are the best spectrometers which may be purchased, as these are usually designed to serve many purposes. It has been possible, therefore, to build it remarkably free from mechanical defects. In fact no variation of the zero point or spectrometer constant was noticed during the entire work, and the reading upon a line was the same whether the line was brought to the bolometer from the left or from the right. In addition to this great freedom from mechanical defects, the thick walls, with all the openings closed after the necessary adjustments had been made, prevented errors arising from rapid changes in temperature. A thermometer with the bulb within the chamber followed such slow changes as occurred with sufficient accuracy. This comparative freedom from error arising from mechanical faults or temperature effects has possibly somewhat increased the accuracy attainable, and, as a result of the

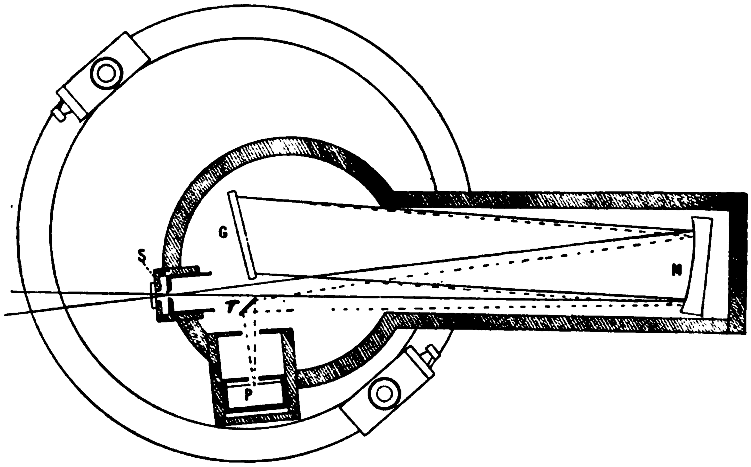

FIG. 2

arrangement chosen (Fig. 1), the range has been extended to somewhat more than 30,000 Å.U.

The grating was a Rowland grating having 14,450 lines per inch and a ruled surface of 48 mm×35 mm. The accurately ground silvered mirror *M* was furnished by Zeiss and had an aperture of 10 cm and focal length of 35 cm. The slits were of equal width, 0.25 mm. This corresponded to about 1.'25 and the thermopile therefore would cover in the first order a spectral region of about 12.5 Å.U. at 7000 Å.U., 10.5 Å.U. at 20,000 Å.U., and 6.6 Å.U. at 30,000 Å.U.

The thermopile used has very recently been fully described[1] and it will be sufficient here to say that in type it was a Rubens linear thermopile expressly adapted to the present purpose. The principal changes were a reduction of the heat capacity and conductivity of the junctions by rolling the wires to a thickness of 0.002 mm and the adoption of a method of mounting which gave a thermopile remarkably free from irregular temperature effects. As a consequence of this reduction in heat capacity the thermopile practically acquired its final temperature within two seconds after being exposed to radiation. The copper leads came out of the brass case through air-tight insulators. The resistance of the pile was approximately 12 ohms. The galvanometer was of the Paschen type.[2] It had a sensibility of about 5×10^{-11} when the period was 7.5 sec. and the resistance 16 ohms.[3] The galvanometer was adjusted to give readings proportional to the current up to deflections of 800 mm, was very steady, and, after the first adjustment, required no further attention during the year it was in use. Since the thermopile acquired its maximum temperature in approximately 2 seconds, and the half-period of the galvanometer was between 3 and 4 seconds, the first throw of the galvanometer was quite accurately proportional to the energy incident upon the pile. The copper cable, connecting the pile and galvanometer, was carefully insulated and protected against moisture and changes in temperature.

---

[1] F. Paschen, *Annalen der Physik*, **33**, 736, 1910.

[2] F. Paschen, *Zeitschrift für Instrumentenkunde*, **13**, 17, 1893.

[3] Scale 2.8 meters distant.

ADJUSTMENT AND CONSTANT OF SPECTROMETER

By the usual methods the spectrometer was placed in exact adjustment. The small mirror T and the thermopile were then adjusted until the image of the slit S exactly coincided with the thermopile slit and the image of a strip of foil crossing the slit S at its middle point exactly fell on the line marking the middle of the thermopile slit.

The equation $\lambda = C \sin\theta$ expresses the relation between the wave-length to be measured and the angle between the diffracted and undeviated ray. C depends upon the grating constant and the angle between the two telescopes and was determined experimentally by measuring θ for known values of λ. For this purpose the accurately determined infra-red lines[1] *Hg* 10140.58±0.2 Å.U. and *He* 10830.42±0.15 Å.U. were employed. For the first of these lines a Heraeus quartz-glass mercury lamp with a storage battery current was used and for the second a helium tube driven by a high-potential battery was employed. With each source the intensity of the radiation was so constant that the positions of the diffracted images could be very accurately located by means of energy-curves;[2] the accuracy with such sharp lines being about 1″. In every case the positions of the images right and left were determined and half the angle between these positions taken as the value of θ. The position of the undeviated image was the mean of the readings of the right-and-left images. The values of C obtained at various temperatures were reduced to the equivalent values at

TABLE I

Date	Temp.	λ	Angle	Constant	Constant at 17° C.
Oct. 22.......	15°.0	10140.58	16° 45′ 11″.1	35180.1	35181.4
Nov. 1.......	15.0	10140.58	16 45 9.9	0.8	2.0
Nov. 15......	16.0	10830.42	17 55 47.3	0.4	1.0
Nov. 15......	15.7	10830.42	17 55 47.3	0.4	1.2
Dec. 11	16.6	10140.58	16 45 8.5	1.5	1.7
Dec. 13	16.6	10140.58	16 45 8.5	1.5	1.7
					Mean 35181.5

---

[1] F. Paschen, *Annalen der Physik*, **27**, 555, 558, 1908.

[2] *Ibid.*, p. 548.

17° C. by using 0.000018 as the temperature-coefficient of the grating. The results of this calibration are found in Table I.

This mean value, 35181.5 Å.U., has been used throughout the work.

METHOD OF MEASUREMENT

The carbons used in the lamp were the "A" grade furnished by Siemens Bros., and were especially free from impurities. In fact no trouble from this source was experienced. The upper negative carbon was about 1.5 cm in diameter and the lower positive carbon 2 cm; this latter was bored a convenient depth, the hole about 4 mm in diameter being filled with the material[1] to be examined: the metals themselves in some cases, and the fused chlorides in the others. The storage battery operated the lamp, the voltage being 64 volts and the current generally 15 to 20 amperes. An assistant, by more or less constant adjustment of the quartz lens, kept the image of the arc upon the screen *L* in such a manner that the images of the carbons fell above and below the opening through which therefore only the radiation of the arc itself went. By means of the slow-motion screw the entire spectrum, between 7500 and 30,000 Å.U. of the substance under examination, was made to pass the bolometer slit, and the spectrometer readings were taken when the galvanometer deflections indicated the presence of lines. Generally two such examinations of each spectrum were made, and it is unlikely that any lines having an intensity of 10 mm have been overlooked. The varying intensity of the arc making energy-curves unsatisfactory, the position of a line was determined by bringing the line upon the bolometer slit by turning the slow-motion screw slowly and at as uniform a rate as possible, stopping the instant the galvanometer showed its maximum deflection. The spectrometer reading was taken and the grating was then further turned in the same direction until the line had passed off the slit. The screw was then turned in the reverse direction at as nearly the same rate as before as was possible, the line being brought to the slit from the opposite side. The spectrometer reading for the maximum deflection was again noticed. Except for accidental variations these two readings were equidistant from the true maximum for sharp lines or for lines equally broadened on both sides. The average of a sufficient number, generally ten to twenty, of such pairs of readings therefore located accurately the position of the line. This method was found to yield the same results as the method of setting directly on the apex of the energy-curves and for the writer it proved easier and more rapid. By either method it is possible to determine the position of a sharp line whose intensity is varying more or less rapidly with an accuracy of about 5″.

---

[1] All the substances used were furnished by Kahlbaum with the exception of the copper, where a sample of electrolytic copper was used.

In the tables which follow, the wave-lengths given are the wave-lengths in air according to the Rowland scale, while the frequencies are the reciprocals of the wave-lengths reduced to corresponding values *in vacuo*. The tables contain all the measurements which have been made and in every case great care has been taken that the lists of lines given contain no higher orders of lines of shorter wave-lengths. The intensities of the lines are but roughly estimated by means of the galvanometer deflections which occurred when a moderate amount of material was in the arc, the current being 15 amperes for silver and copper and 20 for the remaining materials. When the lines were sufficiently intense and sharp to permit measurements in higher orders, the determinations in the different orders have been indicated. No mean value[1] is thought to be in error by an amount greater than the quantity preceded by the plus and minus sign; however, should any of the lines be broadened on one side only, the measurements may be in error by greater amounts. As an inspection of the results will show, these possible errors do not involve simply the accidental errors of observation. They embrace in fact all instrumental errors as well, and are arrived at largely by considering the accuracy with which the stronger visible lines can be measured in the second- and third-order spectra.

With the exception of a number of lines in barium and a single line in strontium, all the infra-red lines of the first six elements have found places in the regular series or in Ritz combination series. Immediately following the tables of results, those series are given which contain the new infra-red lines as members and which at the same time furnish the terms from which the combination series are obtained. Only so many terms of these series are given as are necessary for this purpose. The wave-lengths are in general those of Kayser and Runge and Saunders, while the limits are those of Ritz.

---

[1] In a few instances the values of the wave-lengths here given vary slightly from those given in the preliminary paper (*Annalen der Physik*, 33, 739, 1910); this is due in general to the fact that subsequent measurements have been incorporated with the earlier ones.

COPPER

It was found that a bead of copper about 5 mm in diameter in a shallow cavity of the positive carbon gave the steadiest and most intense radiation, 15 amperes being the most suitable current. The entire spectrum in the region 7000 to 30,000 Å.U. was twice carefully examined.

Intensity	130	6	250	15	6	60
Order..	I	II	I	II	III	I
λ......	7935.2		8093.4		8093.3	16009.3
	5.0		4.2	8093.4	3.4	7.4
	5.1	7934.1	3.9	3.1	3.6	8.8
	4.6	3.8				8.5
Mean λ	7935.0	7934.0±0.5	8093.8	8093.3	8093.4±0.5	16008.5±1.0
ν......		12600.5			12352.2	6245.00

Intensity.........	12*	10*	6
λ................	16654.9	18194.8	18229.7
	2.0	4.5	9.5
			9.3
Mean............	16653.4±2.5	18194.7±3.0	18229.5±0.5
ν................	6003.16	5494.63	5484.14

7934.0	$2p_2-2.5s$			Subordinate series II
8093.4	$2p_1-2.5s$			Subordinate series II
16008.5	$2.5s-3p_1$	16004.9 Comp.	$3p_1=12924.8$	Combinations according to Ritz
16653.4	$3p_1-4d'$	16650.8 Comp.		
18194.7	$3d'-4\Delta p$	18199.8 Comp.	$4\Delta p=6879.1$†	Bergmann series
18229.5	$3d-4\Delta p$	18221.3 Comp.		

* These lines were found only when currents greater than 20 amperes were used.

† The value of $4\Delta p$ obtained from the line 2369.97 is used as this line is more accurately measured than any of the other lines from which it may be obtained.

The following lines in the visible spectrum are combination lines.

4015.8	$2p_2-4\Delta p$*	$4\Delta p$ 6876.9
4056.8	$2p_1-4\Delta p$*	$4\Delta p$ 6880.0
3652.56	$2p_2-5\Delta p$	$5\Delta p$ 4401.4
3512.19	$2p_1-6\Delta p$	$6\Delta p$ 3059.1

*Ritz, *Physikalische Zeitschrift*, 9, 527, 1908.

The very strong pair, 5782.30 and 5700.39, may be represented as $x-2p_i$ which makes $x=49061.2$. The following lines are combinations in which this term appears.

2766.50	$x-3p_1$	$3p_1 = 12924.8$
2768.94	$x-3p_2$	$3p_2 = 12956.6$
2369.97	$x-4\Delta p$	$4\Delta p = 6879.1$
2238.52	$x-5\Delta p$	$5\Delta p = 4402.4$
2724.04	$x-3d$	Comp. = 2724.35

The principal series lies in the ultra-violet and the terms $M=3$ may be computed from the values of $3p_1$ and $3p_2$ obtained above.

	$M=2$	$M=3$	
λ	3247.65	2024.42 Comp.	Observed by Kayser and Runge 2025.1±0.2
ν	30782.9	49381.4	
Mp_1	31523.3	12924.8	
λ	3274.06	2025.73 Comp.	
ν	30534.5	49349.6	
Mp_2	31771.8	12956.6	

SUBORDINATE SERIES II

$2p_2=31771.8$

M	1.5	2.5	3.5
λ	3274.06	7934.0	4480.59
ν	30534.5	12600.5	22312.3
Ms	62306.3	19171.3	9459.5
$2p_1=31523.3$			
λ	3247.65	8093.4	4531.04
ν	30782.9	12352.2	22063.9
Ms	62306.2	19171.1	9459.4

SUBORDINATE SERIES I

$2p_1=31523.3$

M	3	4	5
λ	5218.45	4062.94	3688.6
ν	19157.6	24605.8	27103.0
Md	12365.7	6917.5	4420.3
λ	5220.25	4063.50	
ν	19151.0	24602.6	
Md'	12372.3	6920.7	
$2p_2=31771.8$			
λ	5153.33	4022.83	3654.6
ν	19399.7	24851.2	27355.1
Md'	12372.1	6920.6	4416.7

SILVER

The metallic silver was fed into the arc until the hole in the lower carbon, about 5 cm deep, was filled. With a current of 15 amperes the arc burned satisfactorily until the silver was nearly used up. The spectrum was twice examined to 30,000 Å.U.

Int....	200	30	10	250	40	10	10*
Order..	I	II	III	I	II	III	
	7688.9	7687.6	7688.1	8273.9		8274.2	
	8.8	8.2	8.6	4.2		4.0	
	9.0	8.5	7.9	4.3	8274.2	4.0	12551.0
Mean....	7688.9	7688.1	7688.2±0.5	8274.1	8274.2	8274.1±0.5	12551.0±5.0
ν........			13003.4			12082.6	7965.33

Int..........	60	20	15*	15
	16819.0	17416.6		18381.3
	9.5	5.1	18307.1	2.1
	8.9	5.3	8.7	2.6
Mean........	16819.1±0.8	17415.7±1.0	18307.9±3.0	18382.0±1.0
ν............	5944.09	5740.40	5460.66	5438.65

Previous determinations—Lewis:† 7688.4, 8274.02

* Found only when the current was over 20 amperes.

† *Astrophysical Journal*, 2, 1 and 106, 1895.

7688.2..................	$2p_2-2.5s$		Subordinate series II
8274.1..................	$2p_1-2.5s$		Subordinate series II
12551.0..................	$3d'-5\Delta p$	$5\Delta p=4384.7$	Bergmann series
18307.9..................	$3d'-4\Delta p$	$4\Delta p=6889.3$	Bergmann series
18382.0..................	$3d\ -4\Delta p$	$4\Delta p=6891.0$	Bergmann series
16819.1................	$3p_1-2.5s$	16819.8 Computed	Ritz combination
17415.7................	$3p_2-2.5s$	17414.2 Computed	Ritz combination

The values of $3p_1=12594.1$ and $3p_2=12797.0$ are obtained from the following combinations of Ritz:

5276.4....................	$2p_2-3p_1$	$3p_1=12593.8$
5545.86..................	$2p_1-3p_1$	$=12594.2$
5333.5....................	$2p_2-3p_2$	$3p_2=12797.0$

The principal series lies in the ultra-violet, the terms $M=3$ having been computed as in the case of copper:

	$M=2$	$M=3$
λ...........	3280.8 Observed	2061.3 Computed
ν..........	30471.7	48498.3
Mp_1........	30620.7	12594.1
λ...........	3383.0	2070.0
ν..........	29551.3	48295.4
Mp_2........	31541.1	12797.0

SUBORDINATE SERIES II

$2p_1=30620.7$ [Ritz]*			
M............	1.5	2.5	3.5
λ.............	3280.80	8274.1	4668.70
ν.............	30471.7	12082.6	21413.3
Ms...........	61092.4	18538.1	9207.4
$2p_2=31541.1$			
λ.............	3383.00	7688.2	4476.29
ν.............	29551.3	13003.4	22333.8
Ms...........	61092.4	18537.7	9207.3

* *Annalen der Physik*, 12, 300, 1903.

SUBORDINATE SERIES I

$2p_1=30620.7$			
M............	3	4	5
λ.............	5465.66	4212.1	3810.6
ν.............	18291.0	23734.5	26235.2
Md..........	12329.7	6886.2	4385.5
λ.............	5471.72		
ν.............	18270.8		
Md'..........	12349.9		
$2p_2=31541.1$			
λ.............	5209.25	4055.44	3681.8
ν.............	19191.3	24651.3	27152.9
Md'..........	12349.8	6889.8	4388.2

The values of $4d^i$ and $5d^i$ are, within the limits of observational error, practically equal to $4\Delta p$ and $5\Delta p$; so that it is possible that the lines 4212.1, 4055.44, 3810.6, and 3681.8 correspond to the combination $2p_i - M\Delta p$ instead of belonging to the subordinate series I as Kayser and Runge write them.

With the four following substances, caesium, rubidium, strontium, and barium, the fused chloride, moisture free, was used in the arc. The current was in general 20 amperes. The spectra of all the materials were twice examined to 30,000 Å.U., except rubidium. Here only those lines were measured which had been found by Moll or whose positions were foretold by the Ritz combination principle.

CAESIUM

Int.....	130	20	250	10	250	30
Order...	I	I	I	I	I	I
	8021.5	8017.7	8083.3	8080.7	8524.2	8522.6
	19.4	19.1	3.6	0.9	1.7	2.6
	21.0	19.4	2.4	0.9	4.5	2.6
		19.4			4.0	1.5
		19.0			4.4	2.9
Mean...	8020.6	8018.9±2.0	8083.1	8080.9±2.0	8523.8	8522.4±1.5
ν.......		12467.0		12371.5		11730.6

Int.....	230	20	270	40	350	40
Order...	I	I	I	I	I	I
	8761.5	8762.0	8947.0	8945.0	9173.3	9172.1
	1.0	3.0	4.7	4.5	3.5	4.2
	2.6	2.6	3.3	4.8	3.4	4.6
			5.3	5.1	2.5	3.7
					4.9	
	8761.7	8762.5	8945.1	8944.9	9173.5	9173.9
Mean...		8762.1±1.5		8945.0±1.5		9173.7±1.0
ν.......		11409.7		11176.6		10897.8

Int......	170	10	200	5	200	5
Order...	I	I	I	III	I	III
	9210.5		10026.0			
	08.9		5.4		10124.1	
	08.7		5.6	10025.5	3.8	10124.0
	10.2	9209.3				
	10.0	10.3				
	9209.7	9209.8				
Mean.		9209.7±1.0	10025.7	10025.5±0.5	10124.0	10124.0±0.5
ν.....		10855.2		9971.78		9874.86

Int......	80	12	100	20
Order...	I	I	I	II
	13591.7	13761.0	14698.1	14695.9
	0.5	1.4	7.0	6.2
	0.0	1.7	7.1	7.0
	1.3			
	0.1			
Mean...	13590.7±1.5	13761.4±1.5	14697.4	14696.4±1.5
ν.......	7356.06	7264.75		6802.51

Int..........	6	6
λ............	29318.5	30103.1
	7.8	4.2
	7.5	4.0
	6.0	
Mean........	29317.4±2.0	30103.8±3.0
ν............	3410.04	3320.92

The spectrum of caesium shows relatively many strong lines, but most of these are quite broad, only two, 10025.7 and 10124.0, being sufficiently sharp to give definite maxima in the higher orders. To increase the sharpness of the lines, measurements were made with but small quantities of the salt in the arc, and the results show that certain lines, notably 8083.1 and 8523.8, give smaller readings, while others are apparently unchanged. As many of the caesium lines are broadened toward the red it is probable that the smaller values are the more correct; they give in general better agreements when they enter into Ritz combinations.[1] An attempt to sharpen the lines by reduced pressure about the arc only resulted in so decreasing the intensity of the lines that no measurements could be made.

8522.4	$1.5s-2p_1$	Principal series
8945.0	$1.5s-2p_2$	Principal series
14696.4	$2p_1-2.5s$	Subordinate series II
13590.7	$2p_2-2.5s$	Subordinate series II
9173.7	$2p_1-4d$	Subordinate series I
9209.7	$2p_1-4d'$	Subordinate series I
8762.1	$2p_2-4d'$	Subordinate series I
30103.8	$2p_2-3d'$	Subordinate series I
10025.5	$3d'-4\Delta p$	Bergmann series
10124.0	$3d-4\Delta p$	Bergmann series
8018.9	$3d'-5\Delta p$	Bergmann series
8080.9	$3d-5\Delta p$	Bergmann series

29317.4	$2.5s-3p_1$	29292.8 Comp.	Ritz combination
13761.4	$3p_2-3d'$	13757.4 Comp.	Ritz combination

Previous determinations—
Moll: 0.803, 0.855, 0.895, 0.920, 1.01, 1.12, 1.37, 1.48, 3.00 in μ.
Lehmann: 8019.62, 8082.02, 8527.72, 8766.10, 8949.92, 9171.38, 9211.86 in μμ.
Bergmann: 1002.8, 1012.7, 1359.7, 1377.1, 1476.6 in μμ.

PRINCIPAL SERIES

$1.5s=31397.8$

M	2	3	4
λ	8522.4	4555.44	3876.73
ν	11730.6	21945.8	25787.7
Mp_1	19667.2	9452.0	5610.1
λ	8945.0	4593.34	3888.83
ν	11176.6	21764.6	25707.5
Mp_2	20221.2	9633.2	5690.3

SUBORDINATE SERIES II

$2p_1=19667.2$

M	1.5	2.5	3.5
λ	8522.4	14696.4	7944.7
ν	11730.6	6802.51	12583.5
Ms	31397.8	12864.7	7083.7

$2p_2=20221.2$

λ	8945.0	13590.7	7609.7
ν	11176.6	7356.06	13137.5
Ms	31397.8	12865.1	7083.7

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[1] Paschen, *Annalen der Physik*, 33, 731, 1910.

SUBORDINATE SERIES I

	$2p_1=19667.2$		
M	3	4	5
λ	34892.5*	9173.7	6973.1
ν	2865.2	10897.8	14336.9
Md	16802.0	8769.4	5330.3
λ	36127.7*	9209.7	6983.8
ν	2767.3	10855.2	14315.0
Md'	16899.9	8812.0	5352.2
	$2p_2=20221.2$		
λ	30103.8	8762.1	6723.7
ν	3320.9	11409.7	14868.8
Md'	16900.3	8811.5	5352.4

* Paschen.

BERGMANN SERIES $3d^i-M\Delta p$

	$3d'=16900.1$			
M	4	5	6	7
λ	10025.5	8018.9	7228.8	6826.9
ν	9971.8	12467.0	13829.7	14643.9
$M\Delta p$	6928.3	4433.1	3070.4	2256.2
	$3d=16802.0$			
λ	10124.0	8080.9	7280.5	6872.6
ν	9874.7	12371.5	13731.6	14546.5
$M\Delta p$	6927.3	4430.5	3070.4	2255.5

RUBIDIUM

Int.	160	5	5	100
λ	10080.1	12924.4	12987.2	13236.8
	3.5	5.5	8.3	6.8
	1.7	2.3	4.5	8.0
	2.4			6.6
				7.1 (30)*
Mean	10081.9±1.5	12924.1±3.0	12986.6±3.0	13237.0±1.0
ν	9916.11	7735.39	7698.17	7552.57
Int.	270	150	150	100
λ	13444.8	13668.4	14754.0	15289.6
	3.9	7.8	4.5	91.0
	2.9	6.9	3.5	90.3
	3.5	6.5(15)*	3.8	90.6(15)*
	3.4		4.4(10)*	
	4.8(30)*			
Mean	13443.9±1.5	13667.4±1.5	14754.0±1.0	15290.4±1.5
ν	7436.33	7314.70	6776.03	6538.34
Int.	35	12	8	8
λ	22533.0	22936.6	27319.9	27910.3
	3.1	7.6	18.8	9.3
	2.7	6.3	20.8	9.7
	3.3	6.4		
Mean	22533.0±0.8	22936.7±1.0	27319.8±2.0	27909.8±2.0
ν	4436.75	4358.67	3659.37	3582.01

* None of the rubidium lines shows decrease in value when intensity is lowered to amount indicated by the number in brackets.

Previous determinations—
Moll: 1.01, 1.35, 1.51, 2.28, 2.80 in μ.
Bergmann: 1006.9, 1322.3, 1344.2, 1366.6, 1483.0, 1541.0 in $\mu\mu$.

13667.4	$2p_1-2.5s$		Subordinate series II
13237.0	$2p_2-2.5s$		Subordinate series II
15290.4	$2p_1-3d$		Subordinate series I
14754.0	$2p_2-3d'$		Subordinate series I
13443.9	$3d-4\Delta p$		Bergmann series
10081.9	$3d-5\Delta p$		Bergmann series
22533.0	$3d-3p_1$	22533.2 Computed	Ritz combination
22936.7	$3d-3p_2$	22934.5 Computed	Ritz combination
27319.8	$3p_1-2.5s$	27313.3 Computed	Ritz combination
27909.8	$3p_2-2.5s$	27904.9 Computed	Ritz combination
12924.1	$4p_1-2.5s$	12920.8 Computed	Ritz combination
12986.6	$4p_2-2.5s$	12979.5 Computed	Ritz combination

PRINCIPAL SERIES

Limit $1.5s=33694.0$

M	2	3	4
λ	7800.2	4201.98	3587.23
ν	12816.7	23791.7	27868.9
Mp_1	20877.3	9902.3	5825.1
λ	7947.6	4215.72	3591.74
ν	12579.0	23714.1	27833.9
Mp_2	21115.0	9979.9	5860.1

SUBORDINATE SERIES II

$2p_1=20877.3$

M	1.5	2.5	3.5
λ	7800.2	13667.4	7408.5
ν	12816.7	7314.70	13494.3
Ms	33694.0	13562.6	7383.0

$2p_2=21115.0$

λ	7947.6	13237.0	7280.3
ν	12579.0	7552.5	13731.9
Ms	33694.0	13562.5	7383.1

SUBORDINATE SERIES I

M	3	4	5
λ	15290.4	7757.9	6298.7
ν	6538.34	12886.6	15872.0
Md	14338.96	7990.7	5005.3
		7759.5	
		12883.9	
Md'		7993.4	
λ	14754.0	7619.2	6206.7
ν	6776.03	13121.1	16107.2
Md'	14339.02	7993.9	5007.8

BERGMANN SERIES

$3d = 14339.0$

	4	5	6
λ	13443.9	10081.9	8872.
ν	7436.33	9916.11	11268.4
$M\Delta p$	6902.7	4422.9	3070.6

STRONTIUM

Int.	100	200		200	
Order	I	I	II	I	II
λ	10038.6	10329.9	10328.6	10916.4	10915.2
	8.6	7.9	8.0	7.5	4.5
	7.7	8.3		5.2	5.2
				6.1	
Mean	10038.3±1.0	10328.7	10328.3±0.8	10916.3	10915.0±0.8
ν	9959.16		9679.32		9159.19

Int.	50	10	6	6
λ	11243.4	20262.5	26023.8	26914.5
	4.2	3.0	4.6	5.0
	2.3	3.1	5.0	5.7
	1.1			6.3
Mean	11242.7±2.0	20262.9±1.0	26024.5±1.0	26915.4±1.5
ν	8892.30	4933.81	3841.50	3714.34

Int.	6	6	5
λ	27356.2	29226.4	30109.5
	6.4	5.1	11.2
	5.9	6.2	11.4
Mean	27356.2±1.5	29225.9±1.5	30110.7±3.0
ν	3654.50	3420.72	3320.2

Previous determinations—Lewis: 10326.8, 10915.7.

THE FIRST TERM OF THE SUBORDINATE SERIES I OF TRIPLETS

	$2p_1=31026.1$ Group I	$\Delta\nu$	$2p_2=31420.5$ Group II	$\Delta\nu$	$2p_3=31607.7$ Line III	Mean
	[30667]*		27356.2		26024.5	
	3260.0	394.5	3654.5	187.0	3841.5	
$3d''$	27766.1		27766.0		27766.2	27766.1
	30110.7		26915.4			
	3320.2	394.1	3714.3			
$3d'$	27705.9		27706.2			27706.0
	29225.9					
	3420.7					
$3d$	27605.4					27605.4

* Estimated.

The differences between the terms $3d^i$, 100.6 and 60.1, and the close agreement between the value of $3d$ and the limit which

Fowler[1] gives for the less refrangible components of the narrow strontium triplets discovered by him, indicate that this series, as well as the corresponding Saunders series of calcium,[2] is of the Bergmann type and that it may be represented by the expression $3d^i - M\Delta p$.

The following arrangements of lines, corresponding to similar arrangements in calcium,[3] seem possible.

20262.9 $1.5s - 3p_1$ As $1.5s = 16887.1$ $3p_1 = 11953.3$ Principal series of triplets

With this value of $3p_1$ the term 12012.1 which appears in the following combinations may be considered to be $3p_2$.

		$3d$	$3d'$	$3d''$
$3p_1$......	Observed	6386.74	6345.9*	
	Computed	6387.2	6346.4	
$3p_2$......			6370.2	6345.9*
			6370.2	6345.9
$3p_3$......	[12057.6?]			[6364.3]?

* Used twice, as also is done in calcium.

5257.12	$2p_1 - 3p_2$	5257.8 Computed
10038.3	$2p_1 - 3d'$	Subordinate series I of doublets
10328.3	$2p_1 - 3d$	
10915.0	$2p_2 - 3d'$	

BARIUM

Int..........	100	100	20	70
λ............	9530.1	9613.6	9714.1	9833.4
	23.9	10.5	13.3	0.8
	27.5	7.8	12.7	0.8
	27.5	10.9		
Mean........	9527.3 ± 3.5	9610.7 ± 3.0	9713.4 ± 1.5	9831.7 ± 1.5
ν............	10493.3	10402.2	10292.3	10168.5
Int..........	25	60	5	60
λ............	10002.9	10036.0*		
	1.0	4.7		
	2.5	2.5		10233.3
	2.0	7.2	10189.4	4.3
		7.5	8.9	4.0
Mean........	10002.1 ± 1.0	10035.6 ± 3.0	10189.1 ± 2.0†	10233.8 ± 1.0
ν............	9995.3	9961.8	9811.8	9768.95

* 10038.3, 29225.9 } not strontium, as the strongest strontium lines are lacking.

† In the preliminary paper a misprint made this line 10199.1.

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[1] *Astrophysical Journal*, **21**, 82, 1905.

[2] *Annalen der Physik*, **29**, 657, 1909.

[3] *Op. cit.*, p. 658.

Int.	5	60	70	10
λ	10272.2 3.6	10475.3 10474.7 3.2	10652.4†	23254.8 4.7
Mean	10272.9±2.0	10474.4†		23254.8±2.0
ν	9731.7	9544.5	9385.0	4299.0

Int.	10
λ	29224.0* 2.7 3.6
Mean	29223.4±2.0
ν	3421.0

* 10038.3, 29225.9 } not strontium, as the strongest strontium lines are lacking.

† But once found; possibly *Pb*.

8210.73*	$2p_1-3d'$	Subordinate series I of doublets	As $2p_2=51616.4$	$3d'=62101.7$
8654.33*	$2p_1-3d$		$2p_1=49925.8$	$3d=61477.6$
9527.3	$2p_2-3d'$			$3d'=62109.7$
10233.8	$1.5s-3d'$		10230.8 Computed	$1.5s=71877.5$
9610.7	$1.5s-3d$		9612.8 Computed	
29223.4	$2p_1-3d'$	Subord. series I of triplets	As $2p_1=28472.0$	$3d'=\begin{cases}25051.0\\25051.4\end{cases}$
23254.8	$2p_2-3d'$		$2p_2=29350.4$	

* Hermann, Dissertation, Tübingen, 1907.

The above arrangements are somewhat doubtful.

TIN

Intensity	Wave-Length	Error	Frequency
12	8554.7	1.5	11686.3
8	9023.2	1.5	11079.8
7	9414.9	2.0	10618.6
6	9619.4	2.0	10392.8
6	9746.0	1.0	10257.9
6	9808.7	2.0	10192.3
10	9852.5	1.0	10147.0
10	10458.6	1.5	9558.95
12	10808.8	1.0	9249.24
40	10896.0	1.5	9175.24
70	11194.0	1.0	8930.96
120	11279.2	1.0	8863.52
10	11339.4	1.0	8816.47
60	11457.3	1.0	8725.70
65	11618.0	1.0	8605.04
20	11672.6	1.0	8564.80
90	11741.9		
8*	11740.4*	0.5	8515.28
40	11827.2	1.0	8452.80
40	11853.3	2.0	8434.19
170	11935.3		
10*	11934.0*	1.0	8377.18
50	12982.9	1.0	7700.36
25	13022.0	1.0	7677.25

* Second order.

LEAD

Intensity	Wave-Length	Error	Frequency
100	10292.9		
10*	10291.3*	0.5	9714.30
150	10501.3		
15*	10500.0*	0.5	9521.22
60	10650.8	2.0	9386.40
12	10888.6	2.0	9181.46
30	10971.5	1.0	9112.14
40	12563.8	1.0	7957.21
40	13101.9	1.0	7630.45
30	14744.4	1.0	6780.46
30	15315.6	1.0	6527.54

* Second order.

ARSENIC†

Intensity	Wave-Length	Error	Frequency
10	8823.3	1.0	11330.6
15	8871.2	1.0	11269.4
15	8936.9	1.5	11186.5
100	9599.3	1.0	11414.6
140	9627.7	1.0	10383.9
80	9834.1	1.0	10166.0
150	9923.2	1.0	10074.7
100	10024.4	2.0	9972.98
60	10453.8	1.5	9563.32
25	10614.6	1.5	9418.48

Arsenic was examined to but 16,000 Å.U.

ANTIMONY

Intensity	Wave-Length	Error	Frequency
20	9519.9	2.0	10501.5
15	9950.5	2.0	10047.2
35	10079.9	1.5	9918.07
40	10262.9	1.5	9741.25
50	10587.2	1.0	9442.87
130	10678.6		
10*	10678.0*	0.5	9362.60
50	10742.9	1.0	9305.96
50	10840.6	1.0	9222.10
30	10880.3	2.0	9188.46
20	11013.4	1.5	9077.37
15	11082.7	1.5	9020.62
15	11109.7	2.0	8998.72
10	11190.3	2.0	8933.90
45	11268.5	1.0	8871.94
40	11864.3	1.0	8426.36
15	12118.9	1.5	8249.32

* Second order.

BISMUTH

Intensity	Wave-Length	Error	Frequency
7	7841.1	1.5	12749.9
10	8628.5	2.0	11586.4
10	8761.8	1.5	11410.2
10	8910.0	1.5	11220.3
15	9059.5	2.0	11035.2
40	9344.1		
5*	9343.6*	0.5	10699.7
300	9657.9		
20*	9657.7*	0.5	10351.7
20	9828.8	1.0	10171.5
20	10106.1	1.0	9892.38
15	10301.7	2.0	9704.50
8	10540.2	2.0	9484.98
15	11073.2	1.0	9028.38
5	11555.5	2.0	8651.58
160	11711.9		
10*	11711.1*	1.0	8536.66
13	11994.5	2.0	8334.94
40	12166.5	1.0	8217.05
30	12690.5	1.0	7877.82
25	14331.5	1.5	6975.78
7	22554.2	1.0	4432.57

*Second order.

In the preceding tables the average values only of the measured wave-lengths are given. Each line was measured at least three times, the individual determinations being on different days. With these elements but a single examination of the spectrum between 7000 and 30,000 Å.U. was made in each case. The arcs burned so quietly that there was not much possibility of overlooking lines whose intensity was 10 mm.

This investigation was conducted in the Physical Laboratory of the University of Tübingen, and the writer wishes to express his appreciation of the many courtesies extended to him by its director, Professor F. Paschen, and also his great indebtedness for the many valuable suggestions so kindly offered.

PHYSICAL LABORATORY
UNIVERSITY OF MICHIGAN
January 1911

THE EFFECT OF PRESSURE ON THE ARC SPECTRUM OF VANADIUM

BY R. ROSSI

This research was carried out in the same manner as a previous one on the effect of pressure on the arc spectrum of titanium;[1] the region studied being again from λ 4000 to λ 4600.

The same apparatus was used, the arc being struck between a carbon pole and a graphite tube filled with vanadium carbide. The vanadium arc under pressure does not burn as well as the titanium, so that at high pressures very long exposures of the photographic plates were needed, with the result that the plates were often fogged owing to the continuous spectrum due to the carbon poles. Photographs at 100 atmospheres were especially hard to obtain, the exposure sometimes amounting to one hour. This accounts for the lack of readings at that pressure, as will be noticed in the table given below. Two sets of photographs were chosen at 15, 30, 50, and 100 atmospheres, and these were employed in all subsequent measurements. (See Plates I and II.)

All lines were found to be broadened and displaced toward the less refrangible end of the spectrum by pressure. Several lines were found to be reversed; they are marked *R* and *r* in the last column of Table I according to their large or small tendency to reverse under pressure. The wave-lengths are taken from Hasselberg's tables.[2] The displacements were measured by means of a Kayser measuring machine, 12 readings being taken for each line on each plate.

It will be noticed that the displacements do not vary much from line to line, with the result that no grouping of lines of equal shift was possible.

The mean displacement of all vanadium lines studied at each pressure is given in Table II, which also confirms once more the approximate linear relation between pressure and displacement. The mean displacement per atmosphere of all vanadium lines is found to be 0.00266 Ångström unit.

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[1] *Proc. Roy. Soc.*, A, 83, 414, 1910.

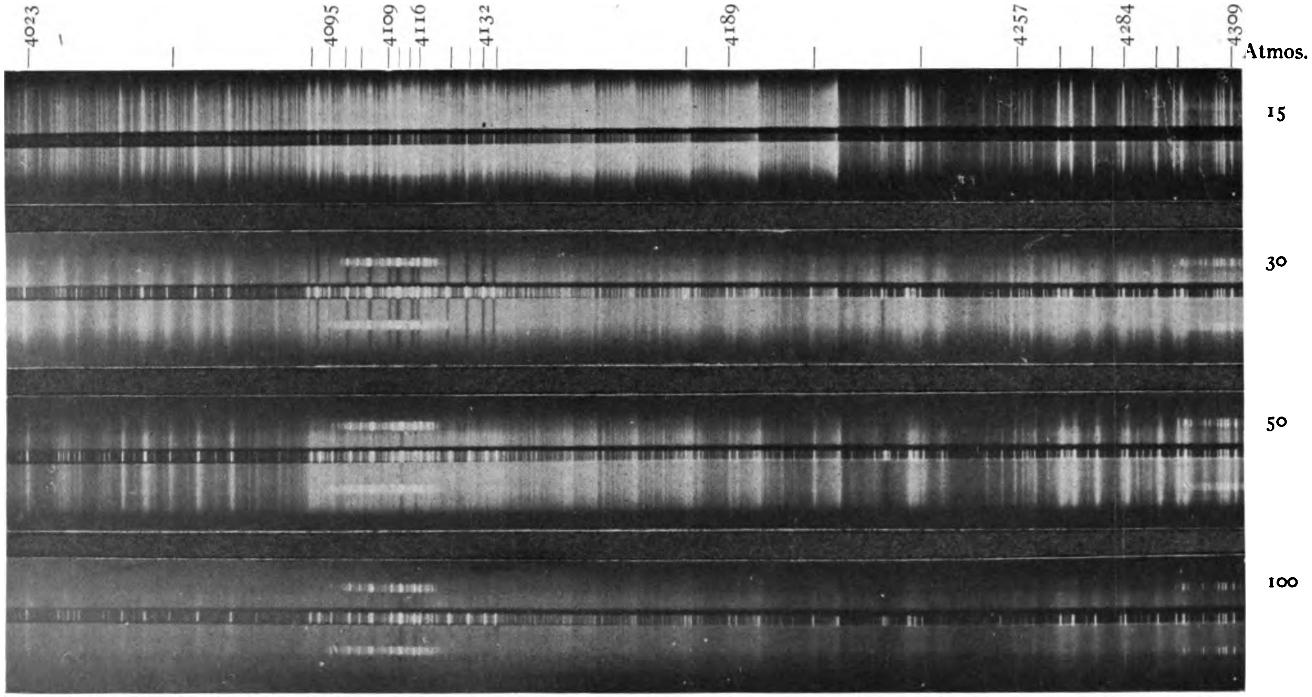
[2] *Astrophysical Journal*, 11, 67, 1900.

TABLE I

Atmospheres	15	30	50	100	
Wave-Length	**Displacements in thousandths of an Å.U.**				**Tendency to Reverse**
4023.50	47	68	...	(199)	...
57.21	58	112	175	309	...
90.70	71	93	174	...	*R*
95.64	61	91	149	...	*r*
99.93	49	80	134	...	*R*
4102.32	47	89	...	...	*r*
09.94	52	83	131	...	*R*
11.92	43	92	108	...	*R*
15.32	43	91	128	...	*R*
16.64	42	89	(121)	...	*R*
23.65	50	92	147	259	*R*
28.25	46	94	150	270	*R*
32.13	49	93	142	243	*R*
34.61	44	92	141	274	*R*
79.53	..	(82)	...	...	*r*
89.99	..	99	147	...	*r*
4209.98	46	95	144	...	*r*
35.90	43	80	122	...	..
57.53	41	104	149	290	..
68.78	47	89	(133)	...	*r*
77.12	47	89	156	246	*r*
84.19	53	90	164	...	*r*
91.97	46	92	164	261	*r*
96.28	46	93	138	...	*r*
4309.95	38	85	129	...	..
30.18	42	78	(120)	(259)	*r*
32.98	41	72	113	(227)	*r*
41.15	52	77	134	...	*r*
53.02	40	75	130	...	*r*
79.38	46	92	122	266	*R*
84.87	46	93	124	233	*R*
90.13	43	86	119	234	*R*
95.40	47	78	(131)	268	*R*
4400.74	47	82	133	208	*R*
12.30	43	79	139	...	*r*
16.63	43	85	148	...	*r*
20.08	40	87	128	...	*r*
21.73	46	85	(128)	...	*r*
26.17	44	84	...	...	*r*
28.68	46	86	146	...	*r*
29.95	47	87	146	...	*r*
36.31	49	80	139	...	*r*
38.02	43	80	142	...	*r*
41.88	50	81	121	...	*r*
44.40	49	83	139	...	*r*
52.19	50	79	132	...	*r*
62.56	51	82	138	...	*r*
68.19	42	91	116	...	*r*
69.88	50	96	144	...	*r*
80.20	39	103	156	190	..
4502.12	38	91	106	160	..

Figures in parentheses denote doubtful readings.

PLATE I



Effect of Pressure on the Arc Spectrum of Vanadium

Central strips at atmospheric pressure; outer strips at pressures indicated on margin
Wave-lengths are given for every fourth line of Table I

TABLE I (*Continued*)

Atmospheres....	15	30	50	100	
Wave-Lengths	Displacements in thousandths of an Å.U.				Tendency to Reverse
4514.36.....	47	109	179	...	..
24.38.....	41	87	142	251	r
45.57.....	48	81	137	234	r
49.81.....	42	86	145	274	r
60.90.....	43	83	134	230	r
71.96.....	44	80	130	243	r
77.36.....	44	72	125	...	r
80.57.....	42	68	123	...	r
83.96.....	43	81	115	...	r
91.39.....	46	86	128	246	..
94.27.....	41	68	114	...	r

TABLE II

Pressure in Atmospheres (Excess above One Atmosphere)	15	30	50	100
Mean displacement in Å.U.............	0.042	0.082	0.132	0.245
Mean displacement per atmosphere in Å.U.	0.0028	0.0027	0.0026	0.0024

The relative intensities of the vanadium lines are very little affected by pressure. A few lines show very slight changes in intensity, but these changes are not always the same on all photographs. Only the two lines λ 4095.5 and λ 4102.3 show an appreciable decrease in intensity with increase of pressure; on some plates at 100 atmospheres they are quite obliterated.

It has often been claimed on theoretical grounds that there should exist a connection between the pressure-displacement and the Zeeman effect. A. S. King[1] recently collected a number of data for lines on which both phenomena have been studied and came to the conclusion that there seems to be a correspondence between the magnitudes of the two effects for the majority of the lines considered.

The magnetic separation of the vanadium lines has been studied in a very exhaustive paper by Purvis,[2] and his results are compared with the pressure-displacements in Table III. They are

[1] *Astrophysical Journal*, 31, 433, 1910.

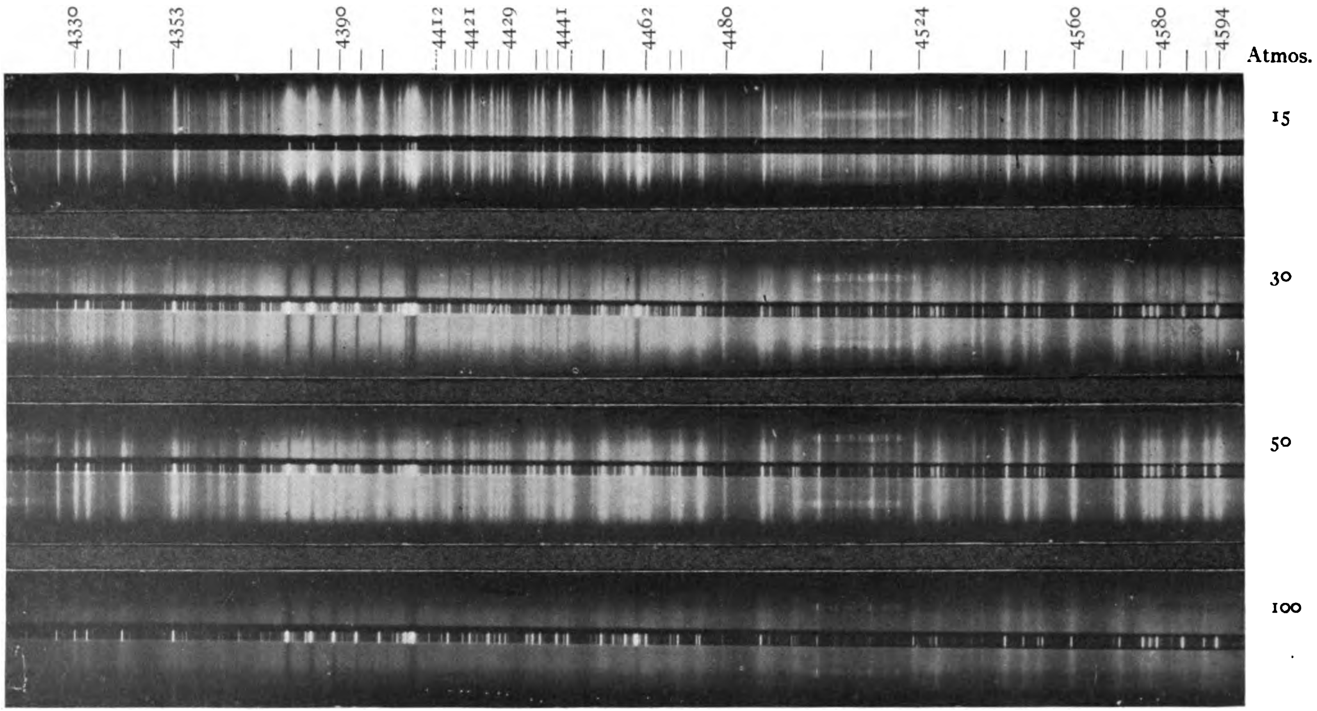
[2] *Cambridge Phil. Soc., Trans.*, 20, 193.

TABLE III

Wave-Length	Mean Pressure-Displacement per Atmosphere in 10^{-5} Å.U.	Magnetic Separation in 10^{-3} Å.U.	*R*
	TRIPLETS		
4023.50.........	247	498	2.02
90.70.........	377	524	1.39
95.64.........	337	517	1.53
4102.32.........	305	378	1.24
15.32.........	283	762	2.69
28.25.........	297	710	2.39
32.13.........	290	728	2.51
34.61.........	287	728	2.53
4268.78.........	293	634	2.16
77.12.........	292	564	1.93
84.19.........	327	270	.82
91.97.........	302	572	1.89
96.28.........	300	572	1.90
4332.98.........	257	636	2.47
41.15.........	293	634	2.16
53.02.........	260	698	2.68
79.38.........	282	642	2.27
84.87.........	275	634	2.30
90.13.........	262	530	2.02
4426.17.........	285	1057	3.70
28.68.........	297	1010	3.70
52.19.........	283	600	2.12
62.56.........	297	578	1.94
4469.88.........	313	570	1.82
4545.57.........	275	500	1.88
60.90.........	267	510	1.91
71.96.........	265	502	1.85
77.36.........	260	380	1.46
80.57.........	253	566	2.23
94.27.........	263	650	2.47
	QUADRUPLETS		
4209.98.........	307	590*	1.92
4395.40.........	275	410*	1.49
4416.63.........	290	622 1651	2.14 5.69
4421.73.........	283	260 1290	.92 4.55
4436.31.........	293	236 1074	.80 3.67
4438.02.........	280	156 957	.56 3.42
4441.88.........	280	248 951	.88 3.39
4444.40.........	297	448 1231	1.51 4.14

* Inner doublet of quadruplet.

PLATE II



Effect of Pressure on the Arc Spectrum of Vanadium

Central strips at atmospheric pressure; outer strips at pressures indicated on margin
Wave-lengths are given for every fourth line of Table I

TABLE III (*Continued*)

Wave-Length	Mean Pressure-Displacement per Atmosphere in 10^{3} Å.U.	Magnetic Separation in 10^{3} Å.U.	*R*
	QUINTUPLETS		
4116.64.........	273	801 1690	2.93 6.19
4400.74.........	265	610 1114	2.30 4.20
	SEXTUPLETS		
4109.94.........	297	348 422 1242	1.17 1.45 4.18
4123.65.........	297	350 428 1246	1.17 1.44 4.19

arranged according to the type of magnetic separation (triple, quadruple, etc.). The separations are for a field of 39,980 C.G.S. units, perpendicular to the lines of force. The column headed *R* gives the ratio of the magnetic separation to the pressure-displacement. No grouping of lines giving the same value of *R* seems possible.

It is known that the magnetic separation increases with the square of the wave-length, but an exact relation between the pressure-displacement and wave-length has not yet been found. The displacement seems to be roughly proportional to the square or a higher power of the wave-length,[1] so that the values given in Table III may not be comparable. Comparisons were therefore also made between the magnetic separation and the pressure-displacements divided by the wave-length, its square, and its cube; thus assuming the displacements to vary respectively from the cube to the fifth power of the wave-length; but even then no grouping of lines seemed possible.

In conclusion I wish to express my thanks to Professor Rutherford for placing the necessary apparatus at my disposal, and to Professor Schuster for the interest he has taken in this work.

Manchester University
December 1910

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[1] Duffield, *Phil. Trans. Roy. Soc.*, A, **209**, 205, 1908.

PHOTOGRAPHIC DETERMINATIONS OF STELLAR PARALLAX MADE WITH THE YERKES REFRACTOR. VII

By FRANK SCHLESINGER

SUMMARY AND DISCUSSION

The results are summarized in Table IV. The magnitudes and the assignment of spectra are for the most part copied from the recent compilation of parallaxes by Kapteyn and Weersma in *Groningen Publications*, No. 24. The third column from the last informs us in what directions the displacements were measured.

The list contains three double stars of which both components were measured. The difference for each pair of parallaxes, and the square root of the sum of the squares of their probable errors, are as follows:

	Difference	$\sqrt{\epsilon_1^2+\epsilon_2^2}$
Groombridge 34	0.″013	±0.″022
Fedorenko 1457–58 (R.A.)	.023	.038
" " (decl.)	.033	.048
P.M. 2164 (R.A.)	.001	.007
" " (decl.)	.028	.022

A more severe test as to the presence of systematic error is afforded by comparing the results from the declination displacements with those from the right ascensions:

	Difference R.A. *minus* Decl.	Probable Error
Fedorenko 1457 (preceding star)	+0.″027	±0.″044
" *1458* (following ")	+ .037	.043
Lalande 23917	+ .060	.048
Fedorenko 2544	+ .028	.031
P.M. 2164 (preceding star)	− .005	.013
" " (following ")	− .034	.020
Groombridge 3689	− .062	.023
Krüger 60	+ .019	.013
Lalande 46650	+ .066	.036

Five of these differences exceed the probable error and four are less than it.

The complete observing list for this work contained a number of stars of the helium type. The intention was to ascertain by observation whether these stars are at the great distance that their proper motions would imply. The results for four of these stars appear in the present table: *c Persei*, π^4 *Orionis*, ψ *Orionis*, and *S Monocerotis*. Three of these parallaxes come out negative and the fourth is only slightly positive; so far then as they go, they confirm the inference drawn from their proper motions. If we take the mean of these four parallaxes, weighting them in accordance with their probable errors, we obtain $-0''.006 \pm 0''.005$, indicating that these fourth magnitude stars are nearly or quite as distant as the ninth magnitude stars in the same regions of the sky.

With the exception of these helium stars none of the deduced parallaxes come out negative. This is also true of the separate results (for the two co-ordinates, and for the two components of double stars) with the exception of the value from the declinations of *Lalande 23917;* and in this case the negative value is much less than its probable error.

In connection with the detailed results for each star, I have given the determinations by other observers, in order that the reader may see how the present values agree with theirs. A careful study of the recent and extensive series of parallax determinations, from the point of view of systematic error, is well worth while; but it would be out of place here.

The average number of plates of each region is thirteen. This is also about the number than can be secured during a clear night of average length and under average atmospheric conditions. The average probable error of one of our parallaxes is $\pm 0''.013$. We may therefore state this conclusion, which will be important in planning similar work in the future: *With a telescope of this size and character, the number of stellar parallaxes that can be determined per annum with an average probable error of* $\pm 0''.013$ *will (in the long run) be about equal to the number of clear nights available for this work.*

This statement does not neglect the fact that in most situations there is more cloudy weather in winter than in summer. This will be compensated for by the fact that a winter night is *for this purpose* at least three times as valuable as a summer night, not only on account of its greater length, but also because it is possible to secure plates for which the parallax factors are on the average considerably greater, always assuming that the exposures are to be made when the regions are near the meridian.

TABLE IV

SUMMARY OF RESULTS

Star	R.A. 1900	Decl. 1900	*B.D.* Number	Magnitude and Spectrum	Proper Motion	Relative Parallax	Probable Error	Co-ord.	No. of Plates	Probable Error of One Plate
Groombridge 34	0^h 13^m	+43° 27′	+43°44	8.2, Ma	2″.85	+″.256	±″.019	long.	12	±0″.040
Groombridge 34 (Companion)				10, –		+.269	.011	long.	10	.019
					Mean	**+.266**	**.01**			
μ Cassiopeiae	1 2	+54 26	+54°223	5.3, G5	3.75	**+.105**	**.010**	long.	9	.019
Lalande **5761**	3 3	+25 58	+25°495	8.0, F2	0.90	**+.010**	**.020**	long.	10	.046
Lalande **7443**	3 56	+35 3	+34°796	8.6, G2	2.19	**+.039**	**.013**	long.	11	.025
c Persei	4 1	+47 27	+47°939	4.0, B3	0.05	**+.008**	**.009**	long.	8	.016
π⁴ Orionis	4 46	+ 5 26	+ 5°745	3.8, B3	0.01	**–.012**	**.007**	long.	11	.014
ψ Orionis	5 22	+ 3 1	+ 2°962	4.7, B2	0.01	**–.019**	**.015**	long.	14	.029
Weisse I,5^h **592**	5 26	– 3 42	– 3°1123	8.9, Ma	2.23	**+.189**	**.010**	long.	18	.027
S Monocerotis	6 35	+ 9 59	+10°1220	4.7, Oe5	0.01	**–.007**	**.012**	long.	8	.022
Lalande **13427**	6 54	+48 32	+48°1469	8.2, –	0.71	**+.049**	**.011**	long.	10	.022
Lalande **15290**	7 47	+30 55	+31°1684	8.2, G0	1.97	**+.072**	**.019**	long.	14	.042
Fedorenko **1457**	9 8	+53 7	+53°1320	7.9, Ma	1.69	+.172	.025	R.A.	12	.044
Fedorenko **1457**						+.145	.036	decl.	12	.027
Fedorenko **1458**			+53°1321	7.9, Ma		+.149	.028	R.A.	12	.050
Fedorenko **1458**						+.112	.032	decl.	12	.025
					Mean	**+.148**	**.015**			
Lalande **23917**	12 45	+ 1 45	+ 2°2585	8.3, –	0.68	+.035	.020	R.A.	13	.028
						–.025	.044	decl.	13	.029
					Mean	**+.025**	**.018**			

Berlin A 4999	13ʰ	40ᵐ	+18°	24′	+18°2776	9.6, G5	1″.92	**+″.077**	**±″.014**	long.	11	±0″.020
Lalande 25372	13	41	+15	26	+15°2620	8.7, K5	2.33	**+.152**	**.007**	long.	11	.012
Fedorenko 2544	14	52	+54	4	+54°1716	7.9, G	1.08	+.056	.019	R.A.	12	.027
								+.028	.025	decl.	12	.022
							Mean	**+.046**	**.015**			
Weisse I, 17ʰ.322	17	21	+ 2	14	+ 2°3312	7.8, Ma	1.36	**+.14**	**.020**	long.	16	.032
P.M. 2164 (prec.)	18	42	+59	29	+59°1915	8.8, K	2.28	+.281	.004	R.A.	23	.014
P.M. 2164 (prec.)								+.286	.012	decl.	22	.013
P.M. 2164 (fol.)						9.3, —		+.280	.006	R.A.	23	.019
P.M. 2164 (fol.)								+.314	.019	decl.	22	.020
							Mean	**+.282**	**.003**			
Lamont 18180	18	53	+ 5	48	+ 5°3993	9.4, Ma	1.24	**+.06**	**.012**	long.	21	.027
Lamont 18816	19	2	+ 7	29	+ 7°3967	9.3, —	0.80	**+.036**	**.007**	long.	11	.010
31 *b Aquilae*	19	20	+11	44	+11°3833	5.2, G5	0.96	**+.101**	**.022**	long.	10	.021
Lalande 39866	20	35	+ 4	37	+ 4°4510	8.4, K	0.86	**+.067**	**.022**	long.	14	.033
Groombridge 3689	22	3	+52	39	+52°3112	8.1, —	0.61	+.018	.014	R.A.	16	.031
								+.080	.018	decl.	16	.021
							Mean	**+.041**	**.020**			
Krüger 60	22	24	+57	12	+56°2783	9.2, K5	0.92	+.257	.007	R.A.	19	.018
								+.238	.011	decl.	19	.014
							Mean	**+.252**	**.006**			
Lalande 46650	23	44	+ 1	52	+ 1°4774	8.9, Ma	1.40	+.161	.014	R.A.	13	.021
								+.095	.033	decl.	12	.022
							Mean	**+.151**	**.013**			

If the observer should decide to determine his parallaxes with smaller accidental errors than the above, a greater number of plates would of course be required, and the total number of determinations would be cut down. But in the writer's opinion, it is not especially desirable to strive for any considerable reduction in the accidental errors at the present time. It may possibly be that modern methods are capable of yielding parallaxes that are free from *systematic* errors of the order of 0."01, but this remains to be proven.

TABLE V

TABULATION OF FINAL RESIDUALS

No. of Measures	No. of Exposures	Quality of Images	No. of Residuals*	Sum of Residuals
1	1	Good	2	±0.033
1	1	Fair	5	.074
1	1	Poor	6	.149
1	2	Good	5	.032
1	2	Fair	12	.184
1	2	Poor	7	.096
1	3	Good	90	.924
1	3	Fair	141	1.717
1	3	Poor	58	0.806
2	1	Good	0	.....
2	1	Fair	7	.063
2	1	Poor	0	.....
2	2	Good	10	.151
2	2	Fair	13	.140
2	2	Poor	3	.014
2	3	Good	82	.717
2	3	Fair	35	.402
2	3	Poor	21	.448

* In those cases in which displacements in two directions were measured, both residuals have been tabulated; and likewise for measurements of both components of a double star.

In Table V the residuals for all the plates have been grouped according to the number of exposures, the quality of the images, and whether the plate has been measured by one or both of the observers. The sums in the last column are of course taken without regard to sign. Most of the plates have three exposures, 11 per cent having two, and only 5 per cent having one exposure. This table is therefore not suited to answer the question as to what relative weights should be assigned to plates containing less than three exposures, and for the same reason this question (since it applies to few plates) is not of great importance. Better information on this point can be obtained from a study of the separate residuals for each *exposure*. Such a study shows that a plate that contains one exposure is entitled to about half the weight of a plate that contains three; and that two exposures are but slightly inferior to three. But these ratios differ greatly for different regions: where the image of the parallax star is much larger than the others, one exposure is nearly as good as the full number. On the other hand, in those cases for which the rotating disk was used to reduce the light of the parallax star, or for which the latter was, to begin with, about equal to the comparison stars in brightness, then the weights are more nearly proportionate to the number of exposures.

Let us next inquire what improvement is brought about by measuring the plates in duplicate. The average of the 289 residuals referring to plates that contain three images and that were measured once is ±0.0119. The average of the 138 residuals referring to plates that contain three images and that were measured twice is ±0.0114. These figures indicate that the second measurement has increased the weight by only 9 per cent. Even this small increase has probably been exaggerated, as the quality of the plates that were measured twice averages better than those that were measured once. A somewhat greater improvement is indicated by the small number of residuals that refer to plates containing less than the full number of exposures. For practical purposes we may then conclude that *the measurement of the plates in duplicate adds only 10 per cent to their weights.*

It was principally for this reason that such duplicate measurements were discontinued after March, 1905. It is true that the two measurements serve to check each other and call immediate attention to discrepancies; but the same advantage is retained by the separate reduction of the three exposures that appear on each plate.

Finally, and most important of all, Table V can tell us how to weight plates of different quality. Taking the averages of the plates that contain three exposures (whether they were measured twice or not) we obtain

	Average Residual	Relative Weight
For the 172 "good" plates........	±0.0095	1.00
" " 176 "fair" "	.0121	0.62
" " 79 "poor" "	.0159	0.36

Collecting these various results we have Table VI. These relative weights are not very different from those actually employed (Table III, p. 170, March); the latter were, in fact, derived in much the same way, but from preliminary and less numerous residuals. The chief difference between the two tables may be stated thus: the former (Table III) slightly underrates good plates.

TABLE VI

RELATIVE WEIGHTS

	Two Measurements			One Measurement		
	Good	Fair	Poor	Good	Fair	Poor
One exposure	0.50	0.31	0.18	0.45	0.28	0.16
Two exposures.....	0.80	0.50	0.29	0.72	0.45	0.26
Three exposures.....	1.00	0.62	0.36	0.90	0.56	0.32

Of the plates here discussed, twenty-one were secured with the telescope east of the pier, and (as a natural consequence) at hour-angles that differ considerably from the others for the same regions. The residuals for these plates are collected in Table VII. The last column gives the relative hour-angle, obtained by subtracting the weighted mean hour-angle for the other plates of that region from the hour-angle for the plate in question. The first three residuals in this table result from least-squares solutions that include the plates themselves. As there is only one of these plates for each region, new solutions without these plates are hardly worth while. For the five other regions, solutions were made from which these plates were excluded, and the resulting residuals are tabulated. The parallax factors in the fourth column refer either to right ascension or declination.

In some cases, notably *Weisse* 17^h322, these residuals show a decided systematic tendency; while in others, notably *P.M. 2164*, they seem to be accidental in character. We may at least conclude that reversing this telescope is not entirely unattended with danger, so far as the relative positions of the parallax stars are concerned; in the writer's opinion, the observations should be strictly confined to one or the other position of the telescope.

TABLE VII

RESIDUALS FOR PLATES SECURED WITH THE TELESCOPE EAST OF PIER

STAR	PLATE	WEIGHT	PARALLAX FACTOR	RESIDUALS			RELATIVE HOUR-ANGLE
				Long.	R.A.	Decl.	
Lal. 5761	203	0.8	−0.97	+0″.07	......	......	$2^{h}6$ West
Lal. 7443	218	0.8	− .95	+ .05	......	......	1.9 West
W. $5^{h}592$	131	0.2	+ .82	− .02	......	......	1.4 West
W. $17^{h}322$	68	0.7	− .85	+ .20	......	......	2.0 West
	86	0.4	−1.01	+ .12	......	......	2.5 "
	93	0.5	−1.00	+ .06	......	......	2.4 "
P.M. 2164........ (preceding)........	84	1.0	− .74	......	+0″.01	−0″.03	2.4 West
	87	1.0	− .93	......	− .02	− .01	2.3 "
	90	1.0	− .97	......	+ .02	+ .02	2.5 "
	94	0.8	− .98	......	+ .01	.00	2.1 "
	106	0.8	−1.00	......	− .01	.00	1.3 "
	150	0.4	− .87	......	− .03	+ .02	2.8 "
P.M. 2164 (following)............	84	1.0	− .74	......	− .01	+ .01	2.4 West
	87	1.0	− .93	......	+ .01	.00	2.3 "
	90	1.0	− .97	......	+ .01	− .01	2.5 "
	94	0.8	− .98	......	− .02	− .01	2.1 "
	106	0.8	−1.00	......	− .03	− .03	1.3 "
	150	0.4	− .87	......	− .17	− .11	2.8 "
Lam. 18180.......	57	0.2	− .42	+ .11	......	......	0.9 West
	76	0.8	− .62	+ .04	......	......	1.8 "
	107	0.9	− .98	+ .02	......	......	0.4 "
	111	0.9	−1.00	+ .09	......	......	2.7 "
	136	0.8	− .95	+ .09	......	......	1.8 "
Gr. 3689..........	170A	0.7	− .92	......	+ .09	......	2.0 West
	181	0.7	− .89	......	+ .05	......	2.6 "
Krüger 60.........	171A	1.0	− .92	......	+ .01	+ .03	2.2 West
	182	0.6	− .90	......	− .02	+ .06	2.8 "

It should be remarked that our final parallaxes are practically independent of the effect that we are discussing. Solutions were made from which the plates taken in the unusual position of the telescope were excluded, and preference was given to these latter results in the two cases where the parallaxes differed from each other by more than a few thousandths of a second. The results for these five stars are summarized below.

Star	Solution from All the Plates	Solution without "Telescope East" Plates	Adopted Parallax
Weisse I, 17ʰ322	+″.113	+″.150	+″.140
P.M. 2164	+.282	+.276	+.282
Lamont 18180	+.051	+.065	+.060
Groombridge 3689	+.041	+.046	+.041
Krüger 60	+.252	+.259	+.252

Whether this effect is directly due to the reversal of the objective, or whether it is an hour-angle error, the present data cannot decide. This is a matter of little importance in the present connection, but it would be of general interest to make a special investigation as to this pont. For this purpose, plates in the two positions would best be secured in rapid succession on the same night.

Returning now to Table IV, let us consider the last column, which contains the probable error of one plate, or more properly that which corresponds to unit weight. These were computed from the full material without rejecting a single plate, although in one or two instances the omission of one of the plates would have reduced the probable error for that region by as much as 50 per cent. The diversity in these probable errors for different stars is not altogether accidental: it depends, for one thing, upon the brightness of the parallax star. The truth of this statement is well brought out in Table VIII, where the stars are arranged in the order of the "relative diameter of the parallax star." This quantity was computed by subtracting from the diameter of the parallax star the sum of the products formed by multiplying the diameter of each comparison star by its dependence. The data necessary to this computation are given above in connection with the comparison stars for each region. Where a plus sign is affixed to this relative diameter, the parallax star is brighter than the mean of the comparison stars. Table VIII includes (in italics) those stars for which the rotating disk was used; but, for reasons that will appear presently, it does not include probable errors in declination.

The table shows a progressive decrease in accuracy for the brighter parallax stars. The mean of all twenty-eight probable errors is ±0″.0260. The mean of the first fourteen (for each of which the parallax star is about as bright as the comparison stars) is only ±0″.0184; while for the last fourteen (for which the parallax star is considerably brighter than the comparison stars) the mean is ±0″.0336. Other things being equal, the latter group would require three times as many plates to produce parallaxes of the same degree of accuracy as for the former group.

TABLE VIII

Relation between the Brightness of the Parallax Star and the Probable Error in Right Ascension or in Longitude

Star	Relative Diameter of the Parallax Star	Probable Error	Star	Relative Diameter of the Parallax Star	Probable Error
Groombridge 34 (comp.)	−0.26	±″.019	*Lamont 18180*	+0.45	±″.027
P.M. 2164 (fol.)	− .23	19	*Fedorenko 2544*	+ .50	27
S Monocerotis	− *.12*	*22*	*Lalande 46650*	+ .53	21
Krüger 60	− .11	18	*Weisse* 17^h *322*	+ .64	32
31 b Aquilae	− *.09*	*21*	*Groombridge 3689*	+ .67	31
ψ Orionis	− *.08*	*29*	*Lalande 5761*	+ .72	46
μ Cassiopeiae	+ *.03*	*19*	*Lalande 13427*	+ .74	22
c Persei	+ *.05*	*16*	*Weisse I*, 5^h*952*	+ .74	27
π^4 *Orionis*	+ *.06*	*14*	*Lalande 23917*	+ .78	28
Lalande 7443	+ .17	25	*Lalande 39866*	+ .79	33
Berlin A. 4999	+ .19	20	*Groombridge 34*	+ .79	40
P.M. 2164 (prec.)	+ .23	14	*Lalande 15290*	+ .91	42
Lamont 18816	+ .35	10	*Fedorenko 1458*	+ .92	50
Lalande 25372	+ .37	12	*Fedorenko 1457*	+ .95	44

If we take the mean of the probable errors that relate to the six stars for which the rotating disk was employed to reduce the brightness of the parallax star, we obtain ±0″.0202. It appears then that *measurements upon images whose diameters have been reduced by the use of the disk are practically as accurate as those made upon correspondingly faint stars.*

In the present work I have used the disk only for those cases for which a reduction in the brightness of the parallax star was necessary in order to make its image measurable. But in the light of present experience I should, for similar work in the future, use the disk in almost every case; whenever, in fact, the parallax star exceeds the comparison stars in brightness by more than half a magnitude. Under these circumstances the probable error of one good plate secured with this telescope will be about ±0″.02, and thirteen plates of average quality should yield a value of the parallax with a probable error of ±0″.01.

It is of interest to ascertain to what is due the progressive increase of probable errors with brightness. The results for the declination measures enable us to answer this question with considerable certainty. In Table IX the probable errors for such measures are contrasted with those for right ascensions of the same stars. We see that so long as the parallax star does not differ much in brightness from the comparison stars, the measurements in the two co-ordinates are about equally accurate. But when the parallax star is considerably brighter (as is the case for each of the last six regions in Table IX), the probable error in declination is considerably smaller than that in right ascension, the two means being ±0.″0243 and ±0.″0335. The progressive increase noted in Table VIII can therefore be due only in small part to greater uncertainty in bisecting larger images, since this uncertainty applies equally to both co-ordinates. The true explanation is doubtless to be found in the guiding error, which depends upon the brightness of the image, and which will be greater in right ascension than in declination, since in the former direction the observer must contend with irregularities in the driving train in addition to atmospheric fluctuations. We have here another indication of the importance of guiding error in work of this character and the value of the rotating disk in keeping down to a minimum both accidental and systematic errors.

TABLE IX

PROBABLE ERRORS IN DECLINATION COMPARED WITH THOSE IN RIGHT ASCENSION

STAR	RELATIVE DIAMETER OF THE PARALLAX STAR	PROBABLE ERRORS	
		Right Ascension	Declination
P.M. 2164 (fol.)	−0.23	±0.″019	±0.″020
Krüger 60	− .11	18	14
P.M. 2164 (prec.)	+ .23	14	13
Fedorenko 2544	+ .50	27	22
Lalande 46650	+ .53	21	22
Groombridge 3689	+ .67	31	21
Lalande 23917	+ .78	28	29
Fedorenko 1458	+ .92	50	25
Fedorenko 1457	+ .95	44	27

ALLEGHENY OBSERVATORY
April 1911

THE EFFECT OF PRESSURE UPON ELECTRIC FURNACE SPECTRA[1]

BY ARTHUR S. KING

The effect of high pressure around the light-source in displacing the spectrum lines given by the arc and spark has been studied to such an extent that the general phenomena are well known. Measurements of the shift of lines for arc spectra through a considerable range of pressures and for a number of elements have been published by Humphreys[2] and by Duffield.[3] The effect of pressure upon electric furnace spectra is a new field, involving some phenomena peculiar to the apparatus.

The electric resistance furnace in the Pasadena laboratory was described by the writer[4] at the time of its construction and has since been used extensively in the study of the effect of varying temperatures upon spectra. The furnace chamber was built for high pressures, having been tested hydraulically to 5000 lbs. per square inch. A series of experiments has been made recently to observe the effect of pressure on the furnace spectrum, and although the work has been entirely in the nature of a preliminary survey, the results show clearly what is to be expected from the apparatus in this line of research.

EXPERIMENTAL METHOD

1. *The furnace.*—The furnace was used for this investigation with only slight changes. The tubes of Acheson graphite used at first were 30.5 cm long, 12.5 mm inside diameter and 18.5 mm outside diameter. In later experiments resistance tubes of the same length and inside diameter but of 19.5 mm outside diameter

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 53.

[2] *Astrophysical Journal*, **26**, 18, 1907.

[3] *Philosophical Transactions of the Royal Society of London*, A, **208**, 111, 1908; **209**, 205, 1908; **211**, 33, 1911.

[4] *Contributions from the Mount Wilson Solar Observatory*, No. 28; *Astrophysical Journal*, **28**, 300, 1908.

were used. A length of 38 mm at each end of the tube was clamped in the vertical graphite blocks which connected the ends of the tube with the water-cooled copper pipes carrying the current. The heated portion of the tube between the contact blocks was then 22.9 cm long. This portion was protected first by a graphite tube of 32 mm inside diameter to provide a clear space immediately about the heated tube, then by carborundum powder heaped around the outer tube to serve as a heat jacket.

The pressure-chamber was modified only by the use of conical glass windows 21.5 mm thick in the water-cooled window-holders, and by simple connections for the gas tank and gauges to one of the holes in the head of the furnace chamber, while a valve with connection for rubber tube was screwed into the other hole. Two gauges were used on the apparatus, one hydraulic, the other a low-reading gas pressure gauge. After allowing for the difference of one atmosphere in their scales, the gauges always agreed in their readings within 5 pounds.

When ready for use, a vacuum pump was connected to the valve last mentioned, and the furnace with connecting tubes was pumped out to less than 1 cm of mercury. A potential of 15 volts A.C. gave an initial current of about 900 amperes with the thinner tubes described above, and 1000 to 1100 amperes with the thicker ones. The current would quickly fall 50 amperes or more as the tube heated, then very slowly if there was no opportunity for oxidation. A photograph taken under these conditions gave the first part of the comparison for a pressure exposure. On the completion of this preliminary exposure, the outlet valve was closed, the vacuum pump disconnected, and the chamber filled with gas to the desired pressure. The furnace is almost perfectly air tight, so that the pressure of the gas would rise rapidly from contact with the heated parts within. To prevent this, the outlet valve was either frequently opened or set so as to give a constant leak sufficient to counteract the increase of pressure within. At the conclusion of the pressure exposure, the valve was opened and the gas reduced to atmospheric pressure, after which the vacuum pump was again connected and the furnace pumped out for the second portion of the comparison photograph.

2. *The optical system.*—The photographs were made with the 30-ft. (9.1 m) vertical Littrow spectrograph. The grating used was ruled by Michelson, and has a ruled surface 19 cm wide, with 500 lines per mm. The second-order spectrum was used, giving a scale of about 0.95 Å per mm. The light from the furnace was focused on the slit by means of a simple lens 10 cm in diameter and of 60 cm focal length. The position of this lens was so adjusted that the grating was centrally illuminated by a beam of light about twice the diameter of the 20 cm objective of the spectrograph. The method of making comparison exposures was to leave all parts of the optical system absolutely untouched between exposures, except for a moving of the sliding occulting plate above the slit, which is supported independently of the latter. The first comparison exposure in vacuum was made with the plate so adjusted as to give two narrow strips of spectrum. The pressure exposure was then placed between these. For the second exposure in vacuum the plate was moved back to its first position with its two openings adjusted to give longer strips of spectrum. The comparison spectrum was thus made up of the superposition of exposures taken before and after the pressure photograph, with the lines due to the second exposure usually extending a little beyond the superposed portion. The slightest instrumental disturbance was then shown by the imperfect coincidence of the comparison lines.

The furnace with the optical system as described gives a minimum risk of instrumental disturbance, since there need not be the least movement of anything which can affect the direction of the beam of light. The furnace supplies its own comparison spectrum, and this being taken in vacuum, the pressure-effect is that due to the total pressure, not to the excess of the furnace pressure above one atmosphere.

THE GAS USED, AND SPECIAL PHENOMENA

For these preliminary observations, commercial carbon dioxide in cylinders was employed, being the most easily obtained of any gas which would serve. Compressed air is out of the question for furnace work, as the large supply of oxygen would cause the furnace tube to be quickly consumed. Considerable oxidation has resulted from the use of carbon dioxide, as the gas dissociates from contact with the hot tube and the surrounding jacket, which is raised to red heat for a considerable distance from the tube itself. The free oxygen given off causes the tube to wear thin near its ends, where the gas comes most freely in contact with the graphite. More serious than the wearing of the tube is the formation of a white smoke, probably an oxide, which becomes rather thick at high pressure and weakens the light by collecting in the end of the furnace tube as well as in the steel chamber, where it deposits on the cool metal parts as a white powder. This probably results from the contact of carbon dioxide with the heated carborundum, which is the most efficient jacketing material thus far found for the regular use of the furnace. The formation of white vapor increases with the pressure and sets a limit to the effective use of carbon dioxide. Some other material for heat insulation, combined perhaps with the use of compressed hydrogen, may give an improvement in this direction.

Two refraction effects are noticeable. One is a trembling of the image of the interior of the tube as projected on the slit of the spectrograph. This increases with the pressure, but is not disturbing at moderate pressures. The other is an interesting bending of the beam of light within the furnace owing to the fact that the gas in the lower half of the chamber is cooler than that above, there being a length of about 20 cm of compressed gas between the end of the tube and the window with the arrangement thus far used. The image on the slit was moved about 3 mm when the chamber was filled to 9 atmospheres and in the direction to be expected according to this explanation. The effect decreases after the furnace has been heated some time and the gas has become of more uniform temperature throughout. As the image on the slit is over 1 cm in diameter, there is no difficulty in adjusting at the beginning so as to allow for this movement. The effect cannot influence the pressure results, as it amounts only to taking the light from a slightly different part of the cross-section of the tube. There is no appreciable change either in intensity or wave-length of the lines given by different portions of the cross-section, as the stigmatic action of the Littrow spectrograph readily shows.

The temperature of the furnace was regulated by the impressed voltage to give a large number of lines without strong continuous spectrum, the effect of a very high temperature being to widen the lines and the reversals and eventually to give all lines in absorption when the continuous spectrum became strong enough.

Temperature measurements were made during the production of both the pressure and the comparison spectra, a Wanner pyrometer being sighted on the interior of the furnace tube through the window opposite to that directed at the spectrograph.

Pyrometer readings are made to best advantage when a graphite plug is placed midway in the tube, which gives a close approach to black body conditions; but this is impracticable when observations of the spectrum are being made. The pyrometer recorded temperatures ranging from 2100° to 2300° C. under the regular conditions of operation. There was always a close agreement between the temperature readings with and without pressure for the same current through the tube, so long as the light from the furnace was not weakened by smoke during the operation under pressure.

RESULTS

1. *Nature of pressure-effect for the furnace.*—Pressure in the furnace as high as 9 atmospheres gives a clear displacement of all lines toward the red, the magnitude of the displacement varying greatly for different lines. The general effect is a widening combined with displacement, the degree of widening also being very different for different lines and having no apparent relation to the amount of displacement. It has been noted in previous publications that the furnace, as regularly used in vacuum, does not give strong widening except for a few lines, generally those appearing in low-temperature flames, for which the amount of vapor present seems to govern the width. With pressure, however, the furnace not only gives this class of lines enormously widened, usually beyond the possibility of close measurement, but the widening extends to a second class of lines, requiring higher temperature than the first mentioned though appearing at the lower furnace temperatures (usually under 2000° C.). Such lines are always strong with the furnace in vacuum,

though not greatly widened. They appear in the high-temperature flames and in the outer vapors of the arc. The effect of pressure upon these lines is to give very strong widening, usually combined with reversal, since the lines are also given by the cooler vapor near the ends of the furnace tube. The reversals are quite or very nearly symmetrical, indicating that the displacing effect of pressure upon the cooler vapor is the same as upon the hotter. These widened lines in the spectra thus far examined never show the largest shifts. As a rule the displacement is of medium amount, but sometimes is very small.

Lines which appear at the higher furnace temperatures show various degrees of widening, though never so large as the lines just discussed, and a very great variety of displacements. The largest shifts occur for such lines. As a rule they appear unreversed and moderately widened as compared with their condition with the furnace in vacuum. Sometimes the effect of pressure is almost or entirely to blot out a line. In most cases I think this is due to a rather strong widening of a weak line, since generally a very hazy trace of the line can be seen in the pressure photograph. No clear case has been observed of a line being rendered visible by pressure when the photograph without pressure was taken at about the same temperature and normally exposed. The effect of pressure appears to be confined to a widening and displacing of such lines as are brought out by the temperature employed.

The quality of the furnace lines for measurement is in general good, perhaps better than is usually given by the arc under pressure. The reversals are well defined and generally narrow. If the exposure is not greatly prolonged, the conditions of the furnace can be controlled more closely than probably is possible for any other source.

2. *Comparison with pressure results for the arc.*—The portion of the iron spectrum from λ 4060 to λ 4460 photographed with the furnace under pressure has been studied by Duffield[1] with the pressure arc, and a comparison with his results has led to two interesting conclusions. The measurements of furnace pressure plates showed that the displacements for the majority of the lines

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[1] *Philosophical Transactions of the Royal Society of London*, A, **208**, 111, 1908.

fall into three groups for which the ratio of the shifts is 1:2:4. The same relation, and in general for the same lines, was found by Duffield for the arc displacements. The good quality of the furnace lines for measurement gives this grouping very distinctly. Without any decided exception thus far observed, the relative displacements for furnace lines follow closely those for the arc. This is for two sources which have a considerable difference in temperature and which are shown by other effects to have the luminous vapor in a very different condition as regards response to the displacing influence of pressure.

The second important result in the comparison with the arc shows that *for the same pressure the furnace displacements are much larger than those of the arc.* This was established by a series of plates taken at 8 and 9 atmospheres total pressure, the magnitude of the shifts being found to run about 100 per cent larger than those given by Duffield for the same lines at 11 atmospheres and to agree closely with the arc displacements for 21 atmospheres. The superposition of the comparison spectra taken before and after the pressure exposure showed freedom from instrumental disturbance, and the relation was further shown decisively by the grouping of displacements in the 1:2:4 ratio mentioned in the preceding paragraph. An instrumental displacement of all lines would be added to the true shifts, and with the values observed for 9 atmospheres the 1:2:4 ratio would not hold even approximately.

The later measurements by Humphreys[1] of arc displacements for iron do not go lower than 42 atmospheres, but the shifts recorded are found to be in general rather less than twice those given by the furnace at 9 atmospheres, so that there is a fair agreement.

Table I gives the furnace displacements in thousandths of an Ångström unit measured on the best plate (F 140) taken at 9 atmospheres total pressure, the comparison spectrum being with the furnace in vacuum. The measurements of this plate were fully checked by four other good plates taken under the same pressure conditions. In the third and fourth columns are measurements for the same lines by Duffield at 21, and by Humphreys at 42 atmospheres. Two sets of measurements are given by Duffield, and

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[1] *Op. cit.*

TABLE I

PRESSURE DISPLACEMENTS

Iron

λ	Furnace 9 Atm.	Arc, 21 Atm. (Duffield)	Arc, 42 Atm. (Humphreys)	Remarks
4063.759	0.064	0.062	0.107	Reversed
4071.908	0.060	0.067	0.092	Reversed
4100.901	0.028	.....	.....	
4132.235	0.102?	0.064	0.105	Reversed. Measurement disturbed by close blend with *V* line
4134.840	n.m.	0.032	.....	Much weakened by pressure. Small displacement
4143.572	n.m.	0.060	.....	Faint. Medium displacement
4144.038	0.065	0.068	0.116	Reversed
4147.836	0.058	.....	.....	
4152.343	0.041	.....	.....	
4154.667	n.m.	0.054	.....	Faint. Medium displacement
4156.970	n.m.	0.054	.....	Faint. Medium displacement
4172.296	0.099	.....	.....	Much widened
4172.923	0.044	.....	0.140	
4174.095	0.030	.....	.....	
4175.082	0.034	.....	.....	
4177.698	0.037	.....	.....	
4181.919	0.085	0.043	.....	Faint
4185.058	n.m.	.....	0.040	Faint. Medium displacement
4187.204	0.111	0.128	.....	
4187.943	0.114	0.094	.....	
4191.595	0.119	0.271	.....	
4198.494	0.125	0.127	.....	
4199.267	0.026	0.043	0.073	
4200.148	0.039	.....	.....	
4202.198	0.062	0.044	0.071	Reversed
4206.862	0.033	.....	.....	
4210.494	0.113	0.142	.....	
4216.351	0.033	.....	.....	Reversed
4222.382	0.106	0.265	.....	
4232.887	0.027	.....	.....	
4233.772	0.101	0.107	0.240	
4236.112	0.119	0.125	0.274	
4250.287	0.126	0.083	.....	
4250.945	0.060	0.057	0.089	Reversed
4258.477	0.035	.....	.....	
4260.640	0.124	0.124	0.246	
4271.325	n.m.	0.075	.....	Weakened. Blend with wide reversal of 4271.934
4271.934	0.063	0.064	0.083	Reversed
4282.565	0.039	0.032	0.043	
4291.630	0.030	.....	.....	
4294.301	0.063	0.050	0.084	Reversed
4299.410	0.126	0.126	.....	
4306.078	0.075	.....	.....	
4308.081	0.055	0.063	0.090	Reversed
4315.262	0.042	0.041	0.036	
4325.939	0.064	0.062	0.097	Reversed
4337.216	0.062	0.069	0.090	
4347.403	0.035	.....	.....	

TABLE I—*Continued*

λ	Furnace 9 Atm.	Arc, 21 Atm. (Duffield)	Arc, 42 Atm. (Humphreys)	Remarks
4352.908....	0.049	0.048	0.052	
4376.107....	0.032	0.032	0.039	Reversed
4383.720....	0.068	0.070	0.125	Reversed
4389.413....	0.028	.....	.....	
4404.927....	0.071	0.064	0.110	Reversed
4415.293....	0.067	0.052	0.087	Reversed
4427.482....	0.037	0.028	0.055	Reversed
4435.321....	0.032	.....	.....	
4442.510....	0.097	0.101	0.190	
4443.365....	n.m.	0.047	0.060	Faint. Medium Displacement
4447.892....	0.107	0.121	0.180	
4459.301....	0.096	0.116	0.160	
4461.818....	0.049	0.045	0.060	Reversed
5143.111....	0.057	.....	.....	
5151.020....	0.067	.....	.....	
5152.087....	0.061	.....	.....	
5166.454....	0.033	.....	.....	
5167.678....	0.051	.....	.....	Reversed
5169.069....	0.051	.....	.....	
5171.778....	0.052	.....	0.075	
5195.113....	0.056	.....	0.080	
5216.437....	0.037	.....	.....	
5225.695....	0.021	.....	.....	
5227.362....	0.052	.....	.....	
5233.122....	0.160	.....	.....	Weak and diffuse
5247.229....	0.055	.....	.....	
5250.385....	0.024	.....	.....	
5250.817....	0.113	.....	.....	Faint
5255.121....	0.029	.....	.....	
5266.738....	0.169	.....	.....	Faint
5269.723....	0.056	.....	0.083	Reversed
5270.558....	0.064	.....	.....	
5283.802....	0.083	.....	.....	Faint
5324.373....	n.m.	.....	.....	Much weakened. Large displacement
5328.236....	0.062	.....	0.100	Reversed
5328.696....	n.m.	.....	.....	Blend with reversal of 5328.236
5333.089....	0.051	.....	.....	
5341.213....	0.057	.....	.....	
5371.734....	0.068	.....	0.095	Reversed
5397.344....	0.060	.....	0.080	Reversed
5405.989....	0.051	.....	0.100	Reversed
5429.911....	0.059	.....	0.085	Reversed
5434.740....	0.057	.....	0.120	Reversed
5447.130....	0.068	.....	0.095	Reversed
5455.834....	0.065	.....	0.105	Reversed
5497.735....	0.063	.....	0.110	
5501.683....	0.060	.....	0.095	
5507.000....	0.062	.....	0.120	

Chromium

λ	Furnace, 9 Atm.	Humphreys, 42 Atm.
4254.505............	0.037	0.056
4274.958............	0.056	0.076
4289.885............	0.048	0.087

Vanadium

λ	Furnace, 9 Atm.	λ	Furnace, 9 Atm.
4099.941....	0.046	4384.873	0.067
4105.318....	0.062	4390.149....	0.061
4109.95	0.042	4395.413....	0.065
4111.940....	0.067	4400.738....	0.044
4115.330....	0.056	4406.810....	0.067
4128.251....	0.042	4407.810....	0.064
4134.589....	0.025	4408.364....	0.059
4330.189....	0.037	4408.683....	0.063
4332.988....	0.046	4459.922....	0.061
4341.167....	0.048	4460.389....	0.064
4379.396....	0.066		

where these are very different for a certain line the value was used which most nearly accords with a linear relation between pressure and displacement.

The measurements for the region from λ 5143 to λ 5507 were made from a very good plate (F 141) taken under conditions as nearly identical as possible with those for the previous plate in the blue region. This is the only plate thus far taken in the green with the furnace under pressure, as it was highly satisfactory. The experiments have been concentrated on the blue region because of the great variety of displacements present there and the opportunity to compare with the arc results. There is every reason to believe that the displacements measured for the two parts of the spectrum correspond to the same furnace conditions. Values for the arc displacements of the green lines as measured by Humphreys for 42 atmospheres are given in a separate column.

The three strong blue lines of chromium were measured and the displacements are entered with the arc values of Humphreys, the ratio of furnace and arc shifts being about the same as for iron.

The iron filings used in the furnace proved to contain enough vanadium to give the stronger lines of this substance, which are accordingly listed, as measurements for this element are very scanty.

The remarks in the last column refer to the appearance of the furnace lines under pressure. The letters "n.m." denote that the line was not measurable in the pressure spectrum, but enough could be seen to give an estimate of the order of magnitude of the displacement, which is entered in the "remarks" column. The wave-lengths are on the Rowland scale.

The plates were measured with a small Gaertner comparator by Miss Griffin and myself. The quality of most of the lines was such that separate sets of measurements agreed to within 0.01 Å.

Nearly all of the furnace lines for iron between λ 4060 and λ 4460 fall into three groups, for which the mean displacements show a close approach to the 1:2:4 ratio, as shown by Table II.

TABLE II

GROUPING OF FURNACE DISPLACEMENTS

Group	No. Lines	Mean Displacement
I.	20	0.034
II.	15	0.064
III.	15	0.112
Intermediate.	2	0.049

A comparison, in Table III, of the mean displacements of those lines common to my tables and those of Duffield shows the close correspondence between the grouping for the furnace and the arc.

TABLE III

MEAN DISPLACEMENTS FOR FURNACE AND ARC

Group	No. Lines	Furnace	Arc
I.	5	0.035	0.035
II.	13	0.064	0.061
III.	14	0.113	0.138
Intermediate.	2	0.049	0.047

3. *Anomalous displacements under special conditions.*—The photographs thus far discussed were taken with the furnace in vacuum for the comparison spectrum and at a pressure which gave displacements of considerable size without introducing dis-

turbing conditions. It appeared possible to reproduce at will the shifts given under these conditions, only one plate, of poor quality, showing any appreciable discordance with the set thus taken.

A variation of conditions, however, has shown that the displacements for the furnace do not depend entirely upon the pressure, but may be influenced by the state of the vapor in the furnace tube in a way which at present is not clear.

The use of a pressure above 9 atmospheres or the presence of carbon dioxide at atmospheric pressure during the preliminary comparison exposure gives in either case a large increase in the white smoke previously referred to. Pressure photographs have then occasionally given displacements larger or smaller than those normally obtained. The most striking instance of this kind was a plate taken with carbon dioxide at atmospheric pressure during the comparison exposure, then with the pressure raised to 9 atmospheres above atmospheric pressure, instead of to 8 atmospheres as was usual for the best plates when the comparison was taken in vacuum. On this plate the shifts were almost twice as large as those given normally by the same difference in pressure with the comparison in vacuum, therefore about four times as large as for the arc at equal pressure. The plate was apparently free from instrumental disturbance and the ratio of 1:2:4 for different lines held as before. Several attempts were made to obtain these large displacements again, but the conditions could not be kept uniform for different runs as they can with the furnace in vacuum, and the smoke usually gave underexposure. One fairly good plate taken in this way gave shifts very close to those regularly obtained for 9 atmospheres with the comparison in vacuum. Another anomalous plate was taken for 11 atmospheres total pressure, the comparison being in vacuum. For this plate the shifts agreed closely with those of the arc for 11 atmospheres, being about half as large as the furnace usually gave for 9 atmospheres. An attempt to repeat this plate gave displacements of the magnitude to be expected for 11 atmospheres, being a little greater than normally obtained for 9 with the furnace.

4. *Influence of temperature and vapor-density.*—The effect of difference of temperature was tested by a plate for a total pressure

of 9 atmospheres with a temperature, as read by the Wanner pyrometer, about 300° higher than that regularly employed. The difference was probably considerably greater than this, as the increased amount of smoke in the furnace due to the high temperature tended to reduce the reading. In spite of weakening of the light from this cause, the exposure for a strong photograph was only one-third of that required for the lower temperature. The high temperature photograph gave displacements consistently 10 to 25 per cent smaller than those for the temperature regularly used.

A photograph taken under the usual conditions but with a very little iron in the tube gave displacements of about the usual magnitude, though the lines were faint and poor for measurement.

These results will be taken up later in the discussion.

5. *Evidence on the linear relation between displacement and pressure.*—The proportionality between pressure and displacement has not been fairly tested with the furnace, as a much greater range of pressures is required than has been used as yet. However, a comparison is given in Table IV for the displacements of a number of good lines measured for 6, 8, 9, and 11 atmospheres, the comparison spectra being always taken in vacuum. A regular increase is evident, indicating that a linear relation probably holds if the conditions of the furnace are properly controlled.

TABLE IV

CHANGE OF DISPLACEMENT WITH PRESSURE FOR SELECTED LINES OF IRON

λ	DISPLACEMENTS			
	6 Atm.	8 Atm.	9 Atm.	11 Atm.
4250.945.....	0.048	0.051	0.060	.....
4271.934.....	.038	.061	.063	0.082
4308.081.....	.040	.044	.055	.076
4325.939.....	.033	.057	.064	.070
4376.107.....	.018	.031	.032	.037
4383.720.....	.034	.051	.068	.071
4404.927.....	.042	.052	.071	.075
4415.293.....	.023	.058	.067	.064
4427.482.....	.023	.030	.037	.037

6. *Data concerning the relation between pressure-displacement and Zeeman separation.*—A comparison of the pressure-effect with the

magnetic separation for lines of the iron spectrum was given in a former paper,[1] using the pressure measurements of Humphreys and Duffield. The furnace material enables us to compare the average displacements and separations for two regions of the spectrum about 1000 Ångström units apart. The average displacement for the total number of lines in the two regions cannot be compared with fairness at present, as most of the weaker lines in the green were too faint for the measurement of their displacements and the largest pressure-effects usually occur among lines of this class. With the available material it seemed best to select lines showing a similar behavior in laboratory sources, as these probably arise from vibrating particles having at least some points of similarity. Such lines are those which appear at low temperatures of the furnace and reverse easily. Fifteen lines of this character are included in the region from λ 4050 to λ 4450 given in Table I, and 9 lines between λ 5050 and λ 5450, excluding λ 5434.740, which is not comparable, being unaffected by the magnetic field. Assuming that the pressure plates were taken under similar conditions, we see that the displacements of these lines show no tendency to change with the wave-length, the mean shift being 0.058 for the blue and 0.060 for the green lines. (The very close agreement is of course largely accidental.) The magnetic separations for these lines, however, show a decided increase with the wave-length, although the separations of the lines in each region are of the same order of magnitude. For a field-strength of 16,000 gausses, the mean separation of the 15 blue lines given as "reversed" in Table I is 0.330, while that for the 9 green lines is 0.520, an increase of 60 per cent for a difference of wave-length of about 1000 units. This affords further evidence against a close connection between the phenomena of pressure and of the magnetic field.

7. *The adaptability of the furnace to pressure investigations.*—While it is too early to state definitely the limitations of the furnace as a pressure source, a few points may be noted as to the relative usefulness of the furnace and arc in this line of work. In the total number of lines produced under pressure, there is little to choose

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 46; *Astrophysical Journal*, 31, 433, 1910.

between them. The tables for iron given by Duffield and in the present paper contain almost exactly the same number of lines for the same range of wave-length, though the lists are not identical owing to the difference in relative intensity of lines given by the furnace and arc. Nineteen lines measured by Duffield could not be measured on the furnace plates, while values for 18 furnace lines are given which do not appear in the arc tables. In the green region, measurements are given for almost double the number of lines measured by Humphreys in the arc, though probably the high pressure used by him increased the difficulty in obtaining the weaker lines.

As to quality, a line clearly and narrowly reversed given by any source usually admits of close measurement. I believe experience will prove that unreversed lines can be measured to better advantage on furnace plates, as there seems to be less tendency toward strong and unsymmetrical widening than is usually given by the arc.

The arc will have an advantage over the furnace in the production of the enhanced lines of some elements, such as titanium, in which most of the lines characteristic of the spark are given of moderate intensity by the arc.

For very high pressures the arc may be more efficient than the furnace, though this is by no means certain, since these preliminary experiments have suggested a large variety of ways in which present difficulties may be overcome. The use of a different compressed gas, different jacketing material, and probably shorter tubes should permit the use of much higher pressures. The relative displacements of lines have been given clearly by the pressures thus far employed.

The chief usefulness of the furnace for pressure work will probably occur when it is desirable to take advantage of the special conditions of radiation afforded by it. The present work has shown that its results may give large differences from those normally obtained with the arc. When an extended investigation has shown what conditions are responsible for these differences, the close control of the state of the radiating vapor which is possible with the furnace will probably open lines of work which cannot be carried out to advantage with any other source.

In astrophysical problems the temperatures involved are high, being approached best by the arc and spark; but in other respects the state of the luminous vapor resembles more nearly that of the furnace, so that certain phases of the pressure phenomena can be studied to best advantage by means of this source.

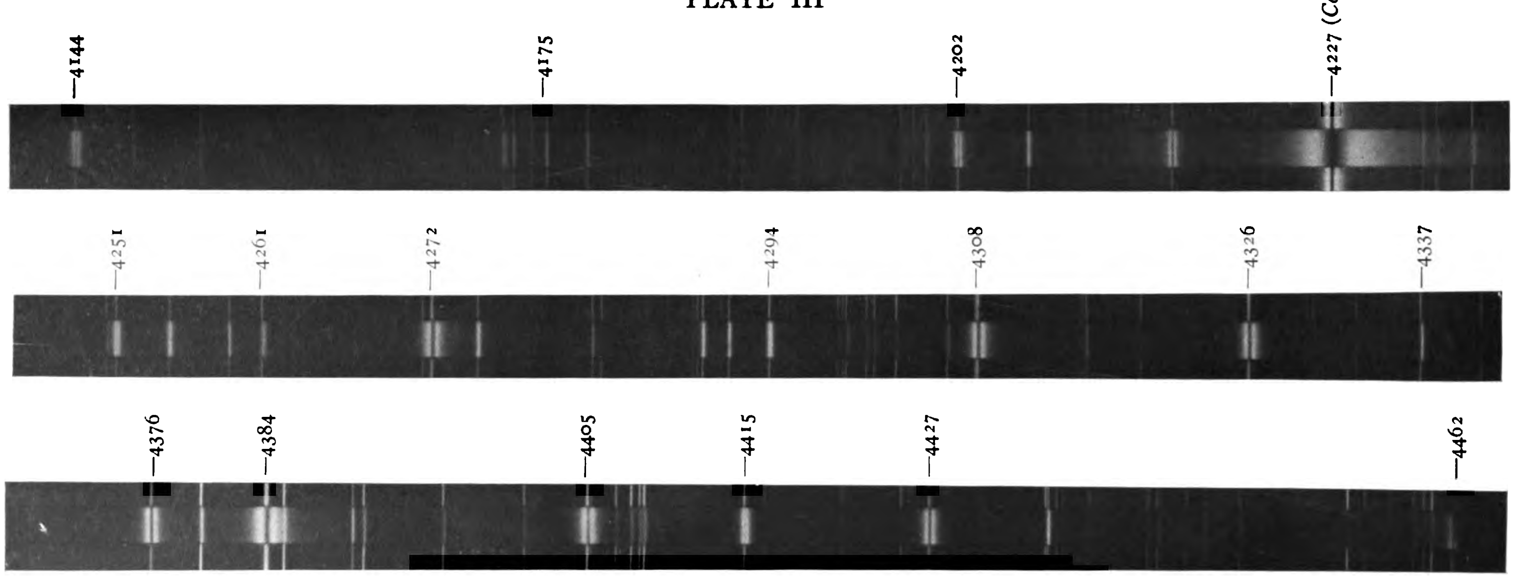
8. *Explanation of the plates.*—Plates III and IV present enlargements of the more interesting regions of the iron spectrum covered by the furnace plates. Certain groups of lines are favorable for the examination of the differences in relative magnitude of displacements. In Plate III, the 1:2:4 relation is represented by λ 4258.477, λ 4250.945 or λ 4271.934, and λ 4260.640 respectively. The group of reversed lines near λ 4383.720 shows this line with $\lambda\lambda$ 4404.927 and 4415.293 displaced approximately twice as much as $\lambda\lambda$ 4376.107 and 4427.482. An example of the very wide reversal of flame lines is given by λ 4227 of calcium, arising from impurities. The same degree of reversal was obtained for the three blue lines of chromium when a large amount of this metal was used in the furnace.

Plate IV shows a portion of the iron spectrum in the green. The spectrum given by the arc between iron poles is also reproduced for comparison. It is seen that the furnace under pressure gives in this region only lines which are strong and sharp in the arc. The other strong lines, with edges not so sharp, are given by the core of the arc, and appear in the furnace spectrum very faintly at the highest temperatures. The comparison spectrum for the pressure photograph shows a combination of effects owing to the fact that the second comparison exposure, taken with longer slit, ended by the tube burning through, giving during a few seconds a spectrum similar to that for the core of the arc. The result is that the regular furnace lines appear with fainter extensions, while those lines which are not usually given by the furnace show faintly and of uniform intensity throughout their length.

DISCUSSION

It has been the purpose of this paper to record the pressure-effects for the furnace as given by a definite set of experimental conditions, which as yet have been varied only to a limited degree.

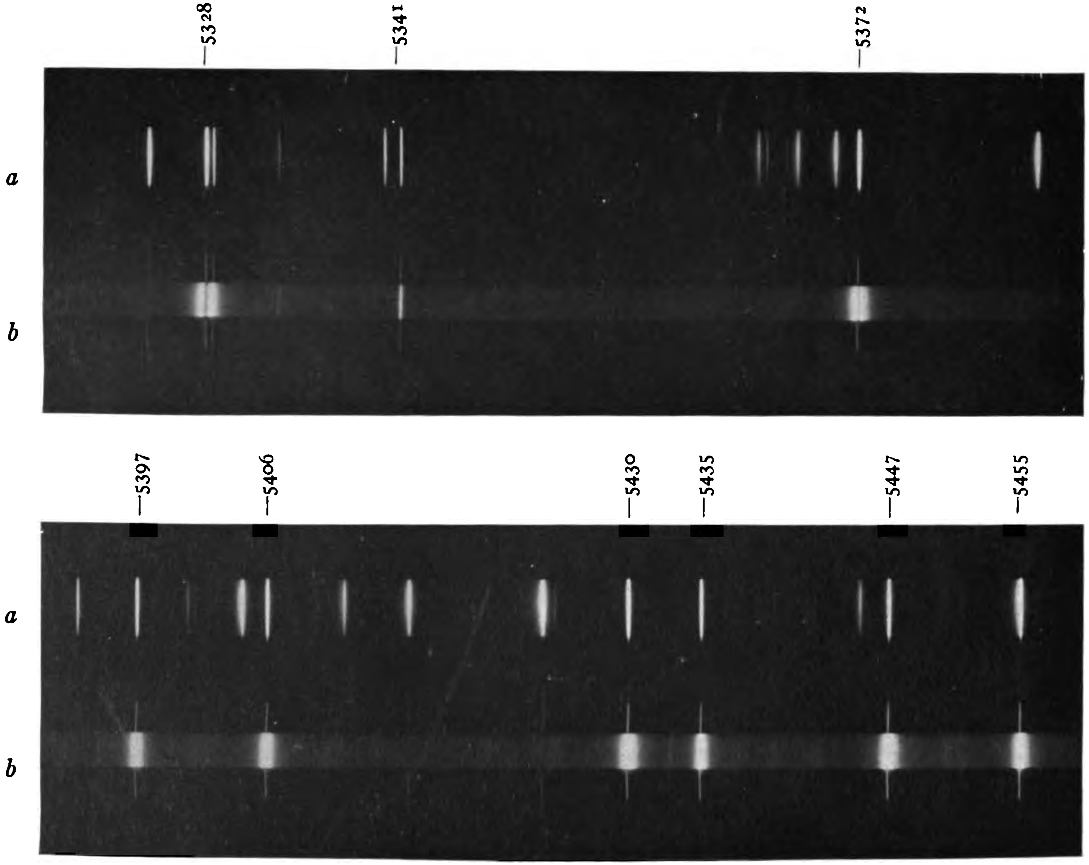
PLATE III



Effect of Pressure on the Electric Furnace Spectrum of Iron

Inner portion of each spectrum given by furnace at 9 atmospheres. Comparison spectrum given by furnace in vacuum
Scale: 1 mm = 0.5 Å

PLATE IV



a, Spectrum of iron arc

b, Effect of pressure on the electric furnace spectrum of iron
Inner portion of spectrum *b* given by furnace at 9 atmospheres
Comparison spectrum given by furnace in vacuum
Scale: 1 mm = 0.5 Å

At this time it would be merely speculative to discuss the results farther than to bring out a few leading points in the question as to why the furnace gives displacements consistently larger than those of the arc. The occasional departures from regularity in the furnace effects are to be regarded as a strengthening or weakening of a general displacing influence.

It may be noted that these results have been in a way predicted by observations on the arc under pressure. Workers with the arc have obtained many discordant results for pressures under 20 atmospheres, so that for this range it has been difficult to show a linear relation between shift and pressure. Duffield obtained a plate at 25 atmospheres which gave shifts twice as large as those which accorded with the linear relation. This is a difference of the same magnitude that the furnace at 9 atmospheres gives as compared with the arc at the same pressure. This and other departures from the linear relation observed by Duffield on certain plates were not to be explained by errors of measurement, and are considered by him as probably connected with the tendency to reverse, the reversals of lines being broader and stronger on the plates giving abnormal shifts.

The production of large displacements may arise from causes of two kinds, either or both of which may be active in the furnace. The luminous metallic vapor may be in such a condition as to weaken the resistance offered by the vibrating particle to a change in its period; or the condition of the compressed gas may enable it to exert a stronger displacing influence.

To consider first a possible weakening of the forces controlling the period, we find that the differences observed between the effects of the furnace and the arc indicate that a decrease in the internal energy of radiation, regulated in the case of the furnace by the temperature, may permit a large disturbance of the period by pressure; in other words, that lower temperature may favor large displacements. The result of the test as to the effect of increased furnace temperature showed that the difference produced was in this direction; and as there is probably an interval of at least 1200° between the lower furnace temperature and that of the arc, this might explain in large measure the differences obtained were it not that other lines of evidence indicate that temperature difference in itself cannot fully account for the results. In the first place, those furnace plates which gave differences larger or smaller than regularly obtained were made with the furnace at about the usual temperature. Other evidence is that the reversals for lines with the furnace under pressure are symmetrical, which should not be the case if the cooler vapor near the ends of the tube gives a line of different wave-length from the vapor in the middle. A difference in the amount of iron vapor present in the furnace tube also appeared to be without decided effect.

Evidence from arc investigations may be considered in this connection. The arc under pressure rarely burns steadily, the spectrum of different regions being taken in turn under ordinary conditions. Different investigators have used a variety of arrangements of the arc, including carbon and metallic terminals and the rotating type, and if either temperature or vapor-density exerts a controlling influence, it is doubtful if results for the arc under pressure would show even as much consistency as they do.

The furnace results give little ground to choose between the theories proposed by Humphreys,[1] Richardson,[2] and Larmor[3] to account for the pressure-effect, but since that of Larmor involves the dielectric constant of the luminous vapor, a possible influence upon this quantity may be considered. An increase in the dielectric constant should diminish the elasticity of the ether about each particle of matter, with a resulting tendency toward increase of vibration period. It would seem probable that the difference between the heated vapor in the furnace and the discharge conditions of the arc might give a change in the inductive capacity, but there is little evidence as to its direction or amount. However, Baedeker[4] measured the change of dielectric constant with temperature through various ranges, sometimes as high as 150° C., of several vapors, among them sulphur dioxide, ammonia, ethyl chloride, methylene chloride, and chloroform, and found in all cases a decrease in the dielectric constant with increase in temperature. If this relation is general for gases and holds at very high temperatures, the temperature difference between furnace and arc would change the dielectric constant in the right direction to give larger shifts for the furnace.

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[1] *Astrophysical Journal*, **23**, 233, 1906.

[2] *Philosophical Magazine* (6), **14**, 557, 1907.

[3] *Astrophysical Journal*, **26**, 120, 1907.

[4] *Zeitschrift für physikalische Chemie*, **36**, 305, 1901.

We may also consider whether the effect can be ascribed to the large dielectric constant of carbon dioxide as compared to compressed air. The dielectric constant of air at 20 atmospheres was measured by Occhialini[1] as 1.0101, that for carbon dioxide at 20 atmospheres by Linde[2] as 1.025. This difference in itself should give larger displacements for the furnace in carbon dioxide than for the arc in air, although, as has been remarked by Humphreys, a change directly proportional to the dielectric constant would be within the errors of measurement. Tests have been made of the displacements given by the spark in different atmospheres by Anderson[3] and by Hale and Kent.[4] The former found some evidence of an increased displacement in carbon dioxide as compared with hydrogen. The question was discussed at some length by Humphreys,[5] but a systematic investigation of the effect of different gases under pressure for the arc and spark has not yet been made. It is hoped to make such a comparison at an early date with the furnace. The irregular displacements occasionally obtained when white oxide was present in the furnace can perhaps be most easily explained through a change of the dielectric constant.

One feature in which the furnace conditions are different from those of the arc is that the effect of the compressed gas should be more steady and possibly stronger in the furnace. Here we have the carbon dioxide heated to about the same temperature as the radiating metallic vapor, and with considerable uniformity throughout several inches of the tube, with little circulation. With the arc, the heating of the gas is very much localized, with opportunity for constant movement of the gaseous particles in and out of the heated region. The fact that in the arc the metallic particles carry

[1] *Physikalische Zeitschrift*, 6, 669, 1905.

[2] *Annalen der Physik und Chemie*, 56, 546, 1895.

[3] *Astrophysical Journal*, 24, 221, 1906.

[4] *Publications of the Yerkes Observatory*, 3, Part II, 1907.

[5] *Astrophysical Journal*, 26, 18, 1907.

the greater part of the electric current makes it a question how nearly the thermal condition of the gas approaches that of the metallic vapor.

SUMMARY

The results obtained may be summarized as follows:

1. The furnace under pressure gives a spectrum containing practically all lines brought out by the same temperature with the furnace in vacuum.

2. The general effect of pressure is a widening of lines and displacement toward greater wave-length, the widening for low-temperature lines being very strong and usually accompanied by reversal.

3. The number of lines which are measurable is approximately the same as for the arc under pressure, with some differences due to the different relative intensity of lines in the two sources.

4. The displacements of nearly all iron lines from λ 4050 to λ 4450 fall into three groups, the ratio of the displacements being 1:2:4, which ratio was also found for these groups of lines in the arc.

5. So far as observed, the *relative* displacements of the furnace lines appear to follow closely those of the arc.

6. The *absolute* displacements of furnace lines at a pressure of 9 atmospheres are about twice those given by the arc at the same pressure and compare closely in magnitude with the arc displacements for 20 atmospheres.

7. Some furnace photographs have shown shifts larger or smaller than those usually given by the same pressure. These were apparently due to differences which evidently prevailed in the condition of the luminous vapor.

8. For iron lines of the same general character, produced at low furnace temperatures and easily reversed, the displacement shows no tendency to increase with the wave-length, the pressure-effect being in this respect very different from the effect of a magnetic field for the same lines.

I am indebted to Miss Griffin for much assistance in the measurement of the spectrograms.

Mount Wilson Solar Observatory
March 6, 1911

MOTION AND CONDITION OF CALCIUM VAPOR OVER SUN-SPOTS AND OTHER SPECIAL REGIONS.[1] I

BY CHARLES E. ST. JOHN

In a former paper, "The General Circulation of the Mean and High-Level Calcium Vapor in the Solar Atmosphere,"[2] the investigation was confined to those portions of the solar disk that were free from spots and flocculi. The points at which measurements were made were 1, 12, 25, 35, 50, and 70 mm from the limb and at the center, on an image 172 mm in diameter. The investigation was continued through 12 rotations of the sun, during which period the position angle of the solar axis described nearly a complete cycle; the points selected for measurement lay along a horizontal diameter of the image produced by the Snow coelostat, and as the plates were taken at irregular intervals each point corresponded to a random distribution of positions on the solar surface but at definite distances from the center of the disk. The combined results of the measurements showed a descending motion for the absorbing vapor of 1.14 km per second, and an ascending motion for the emitting vapor of 1.97 km per second over the surface of the sun. This result, depending on some 500 positions on the solar surface, represents a mean that is practically free from the effects of causes that are occasional and diverse in their action and may be assumed to show the condition of the calcium vapor over regions of the surface not subject to great or exceptional disturbances, such as spots, flocculi, and prominences in projection.

In the present paper are presented the results of an investigation in which the measurements of the wave-lengths of the absorption line K_3 and the emission line K_2 of calcium have been extended over the umbrae and penumbrae of spots, over the accompanying flocculi and circumfloccular regions, and over some other points where the calcium vapor showed exceptional phenomena. Measurements are given also for the intensity of the emission line in terms of the continuous spectrum and for the relative intensities of K_1, K_2, K_3, and for different points of the same line. The series of plates extends over nearly two years. The spectrographs employed are of the auto-collimating type and are described as Nos. 2 and 3 in *Contribution* No. 44.[1] In the 18-foot (5.5 m) instrument (No. 2) a 20-cm Michelson grating having 590 lines to the centimeter and a 10-cm Rowland grating having 568 lines to the centimeter were used, the scale in each case being approximately 1 mm = 1.0 Å. This instrument can be used as a section-spectrograph, and is an application of the principle of the *spectro-enregistreur des vitesses* suggested by M. Deslandres.[2] The arrangement permits one easily and rapidly to take exposures of successive points or successive exposures of the same point. The majority of plates were taken with this spectrograph, using a second slit 12 mm wide in order to include the solar standards, and 45 mm long, the maximum length of the second slit. Often successive exposures were taken over a spot, but more frequently the first slit was moved by steps of 2 mm each over the region under investigation. The other plates were taken with the 30-foot spectrograph (No. 3) used in connection with the 60-foot tower telescope in which the above-described Rowland grating was used, giving a scale of approximately 1 mm = 0.60 Å. These plates are designated by the letter "*c*."

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 54.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 48; *Astrophysical Journal*, 32, 36, 1910.

The measurements of the spectra were made upon a Gaertner comparator reading directly to 0.001 mm. The plates were measured red right and red left, and at least four settings were made for each position. The two iron lines λ 3930 and λ 3935 were employed as standards. The solar wave-lengths of these lines were determined at the center of the sun in terms of the International Standards as described in *Contribution* No. 48, p. 42.[3] The wave-lengths of these lines at the center of the sun were found to be λ 3930.306 and λ 3935.823, the arc values being λ 3930.301

[1] *Astrophysical Journal*, **31**, 143, 1910.

[2] *Comptes Rendus*, **114**, 277, 1892.

[3] *Astrophysical Journal*, **32**, 36, 1910.

and λ 3935.818 respectively. Their wave-lengths over the surface of the sun, away from the limb, have been assumed to remain constant except in the immediate neighborhood of sun-spots where the radial motions discovered by Evershed may affect them when the slit is not placed perpendicular to the radius drawn through the spot. To eliminate any such effect it was the practice to make settings on portions of the standard lines outside the penumbral regions which are the seat of the radial motions in the reversing layer. To fix the points of the line at which measurements were to be made, and to avoid the effect that might be produced by having other portions of the line in the field of view, the photographic plate was often covered by an adjustable slide in which there was a slit 1 mm wide and sufficiently long to include the standard lines. The platform of the comparator carried a millimeter scale at each end by means of which the plate could be moved in a direction perpendicular to the screw in order to bring any desired point of the line into the field of view.

Distortions of the H and K lines of calcium are not uncommon during periods of activity and violent changes in and around sun-spots, when eruptions often take place in the immediate vicinity. Displacements of the lines occur at such times, which indicate very high velocities in the line of sight, amounting to one hundred kilometers or more per second. The measurements that form the basis of the present article were made upon plates that did not show such extreme and unusual conditions, but were representative, rather, of the more common and steady states. The purpose has been to determine what is the usual rather than the unusual, to find what are the prevailing movements of the calcium vapor in and around sun-spots, if such movements do exist in any stable form. There is undoubtedly much that is billowy, tumultuous, and apparently irregular in the movements in the upper regions of the solar atmosphere, but it is a question whether underneath the adventitious and casual there may not exist some systematic effects that point to generally acting causes. The present investigation was undertaken mainly to obtain quantitative data bearing upon this question.

TABLE I

The K Line of Calcium over the Umbrae of Central Spots

Plate	Date	Spot	On C.M.	Wave-Length of K Line	Eccentric Distance	Character	Remarks
					mm		
126..........	1908 Dec. 7	6577	1908 Dec. 8.3	3933.680 (4)	37.5	Bright, single	2d apparition. Large, regular. Returned as 6592, 6609, 6627, 6647, and 6661
138..........	Dec. 9	6577	Dec. 8.3	.679 (2)	35.0	Bright, single	Large, regular
				3933.680			
215..........	1909 Jan. 18	6602	1909 Jan. 19.9	.660	26.0	Bright, single	2d apparition. Large, regular. Returned as 6618
219..........	Jan. 19	6602	Jan. 19.9	.658 .655 .652 .650 .655	6.3	Bright, single	On 2 successive days this spot showed from 6 consistent measurements an upward motion over the umbra
				3933.655			
228..........	Jan. 20	6602	Jan. 19.9	.691 .690 .681 .688	14.6	Bright, single	On this, the following day, the motion was strongly downward
				3933.688			
228..........	Jan. 20	6604	Jan. 22.3	.686 (2)	41.8	Bright, single	1st apparition. Large. Regular. Returned as 6621
230..........	Jan. 23	6604	Jan. 22.3	.682 (2)	33.0	Bright, single	Large, regular
				3933.684			

	1909		1909				
237..........	Jan. 28	6606	Jan. 28.9	3933.675 (3)	31.0	Bright, single	Only appearance. Large, irregular cluster. Active region
247..........	Jan. 29	6606	Jan. 28.9	.676 (4)	29.0	Bright, single	Irregular cluster
				3933.676			
247..........	Jan. 29	6608	Jan. 30.3	.678 (2)	37.4	Bright, single	2d apparition. Large, irregular spot about to break up
257..........	Jan. 30	6608	Jan. 30.3	.682 (3) .665 (5)	28.0	Bright, single	Line strongly curved over umbra showing a change in motion over different parts
263..........	Jan. 30	6608	Jan. 30.3	.683 (5) .667 (1)	28.0	Bright, single	The table gives the means for the 2 parts of the line
				3833.675			
258..........	Jan. 30	6609	Feb. 1.1	.677 (4)	44.5	Bright, single	Large spot. 4th apparition. Return of 6592, a return of 6577
263..........	Jan. 30	6609	Feb. 1.1	.682 (2)	44.5	Bright, single	Large, regular
				3933.679			
290..........	Feb. 24	6625	Feb. 26.5	.693 (4)	40.0	Bright, single	1st apparition. Large, regular. Returned as 6645
296..........	Feb. 25	6625	Feb. 26.5	.685 (6)	25.2	Bright, with dark center	λ for weak absorption line 3933.684 (4)
304..........	Feb. 26	6625	Feb. 26.5	.687 (3)	27.0	Bright, with dark center	λ for weak absorption line 3933.685 (3)
309..........	Feb. 27	6625	Feb. 26.5	.694 (2)	42.4	Bright, single	Regular
				3933.689			
304..........	Feb. 26	6627	Feb. 28.5	.679 (3)	54.5	Bright, with dark center	5th apparition. Return of 6609. λ for absorption line 3933.673 (3)
309..........	Feb. 27	6627	Feb. 28.5	.678 (4)	44.0	Bright, single	Spot shows indications of breaking up
				3933.678			

TABLE I—*Continued*

Plate	Date	Spot	On C.M.	Wave-Length of K Line	Eccentric Distance	Character	Remarks
	1909		1909		mm		
439..........	April 24	6660	April 24.6	3933.684 (2)	C'ntr'l	Bright, single	Large, regular spot. One apparition
563..........	July 23	6695	July 24.4	.684 (2)	11.0	Bright, single	Large, regular spot. Leader of stream
567..........	July 24	6695	July 24.4	.679 (4)	10.0	Bright, single	Regular
				3933.681			
617..........	Oct. 20	6746	Oct. 20.7	.687 (2)	15.0	Bright, single	Large spot. 2d apparition. Returned as 6765
621..........	Oct. 30	6753	Oct. 31.4	.679 (2)	18.2	Bright, single	3d apparition. Large, regular spot. Line weak
624..........	Oct. 31	6753	Oct. 31.4	.678 (4)	C'ntr'l	Bright, single	Line weak
				3933.678			
649*a*.........	Nov. 6	6756	Nov. 6.4	.682 (2)	19.0	Bright, single	Large, regular. 2d apparition. Leader of train. Returned as 6778
649*b*.........	Nov. 6	6756	Nov. 6.4	.686 (3)	19.0	Bright, single	Regular
				3933.685			
651..........	Nov. 15	6765	Nov. 17.0	.674 (3)	27.5	Bright, single	3d apparition. Return of 6746
652..........	Nov. 16	6765	Nov. 17.0	.671 (1)	17.4	Bright, single	Regular
653..........	Nov. 16	6765	Nov. 17.0	.685 (3)	17.4	Bright, single	Regular
660..........	Nov. 17	6765	Nov. 17.0	.697 (1)	18.3	Bright, single	Regular
				3933.682			
669..........	Nov. 27	6771	Nov. 26.7	.687 (3)	36.0	Bright, single	1st apparition. Large, regular. Returned as 6786
674..........	Nov. 27	6772	Nov. 26.7	.665 (4)	21.0	Broad, bright	4th apparition. One of changing group
679..........	Nov. 28	6772	Nov. 26.7	.667 (2)	22.0	Broad, bright	Strong, bright line
				3933.666			

680..........	1909 Nov. 28	6774	1909 Nov. 27.4	3933.670 (2)	27.0	Bright, single	One of developing group
714..........	Dec. 23	6786	Dec. 24.0	.696 (6)	29.5	Bright, single	Large, regular. Return of 6771. Line weak
714..........	Dec. 24	6787	Dec. 24.9	.690 (2)	29.0	Bright, single	Large, regular. Return of 6771
716..........	Dec. 24	6787	Dec. 24.9	.693 (4)	29.0	Bright, single	Large, regular
				3933.692			
714..........	Dec. 23	6788	Dec. 25.1	.688 (2)	44.0	Bright, single	Large, regular. Return of 6774
717..........	Dec. 24	6788	Dec. 25.1	.681 (4)	27.0	Bright, single	Large, regular. K line weak
				3933.683			
76*c*..........	1910 May 18	6847	1910 May 18.6	.688 (2)	20.0	Bright, single	Regular
76*c*..........	May 18	6849	May 19.7	.684 (2)	19.0	Bright, single	Large, regular. Leader of stream. Shows inflow of calcium vapor on plates taken near limb
79*c*..........	May 19	6849	May 19.7	.691 (3)	C'ntr'l	Bright, single	Large, regular
				3933.688			
195*c*..........	Aug. 9	6874	Aug. 9.5	.684 (3)	C'ntr'l	Bright, single	Large, regular. Shows inflow. Terrestrial magnetic disturbances. Returned as 6880
256*c*..........	Sept. 27	6886	Sept. 26.9	.687 (2)	C'ntr'l	Bright, single	Large, regular. 2d apparition
296*c*..........	Oct. 21	6897	Oct. 21.3	.683 (2)	25.0	Bright, single	Shows inflow on plates taken near limb. At first absorption line showed even with fine seeing. Later only bright line

VERTICAL MOTION OF CALCIUM VAPOR OVER THE UMBRAE OF SPOTS

The usual appearance of the K line of calcium over spot umbrae is that of a relatively narrow but actually broad bright line, its width being approximately 0.3 Å. See Plate V (*a*). Compared with the line over the surrounding faculae it is broader than K_3 and narrower than K_2. Occasionally there is superposed upon it a narrow absorption line. This is often due doubtless to photospheric light that reaches the slit when the seeing is poor. The question whether it is ever a real spot phenomenon will be discussed later in the paper. The measurements were confined to the plates obtained when the spots were near the central meridian. The average diameter of the solar image was 170 mm. On an image of this diameter the vertical component of the motion of the rising or falling vapor at a point 35 mm from the center of the disk is about 10 per cent less than at the center. Few of the spots upon which measurements were made were so far from the center. Not all the plates were taken for this investigation, and the most available have been selected from those already on hand. Table I gives the results of the measurements. The Greenwich spot numbers are in the third column; the wave-length of the bright K line of calcium is given in the fifth column in International Units; the figures in parentheses following the wave-lengths show the number of separate measurements upon which the result depends; in the main these refer to separate exposures on the same plate. The distances of the spots in millimeters from the center of the image are given in the sixth column. In the last column under "remarks," the history and description of the spot are outlined, and the wave-lengths of the absorption line K_3 are included. The spots are arranged serially in the order of their appearance. The wave-length of the K line of calcium in the arc in International Units, to which the wave-lengths in the spots may be referred for the purpose of determining the motion of the calcium vapor, is λ 3933.667, as determined in *Contributions* No. 44, p. 10.[1]

Of the twenty-five spots studied, one, 6577, was observed at three apparitions; and four, 6746, 6771, 6772, and 6774, were observed at two apparitions. Twenty-two spots showed at all

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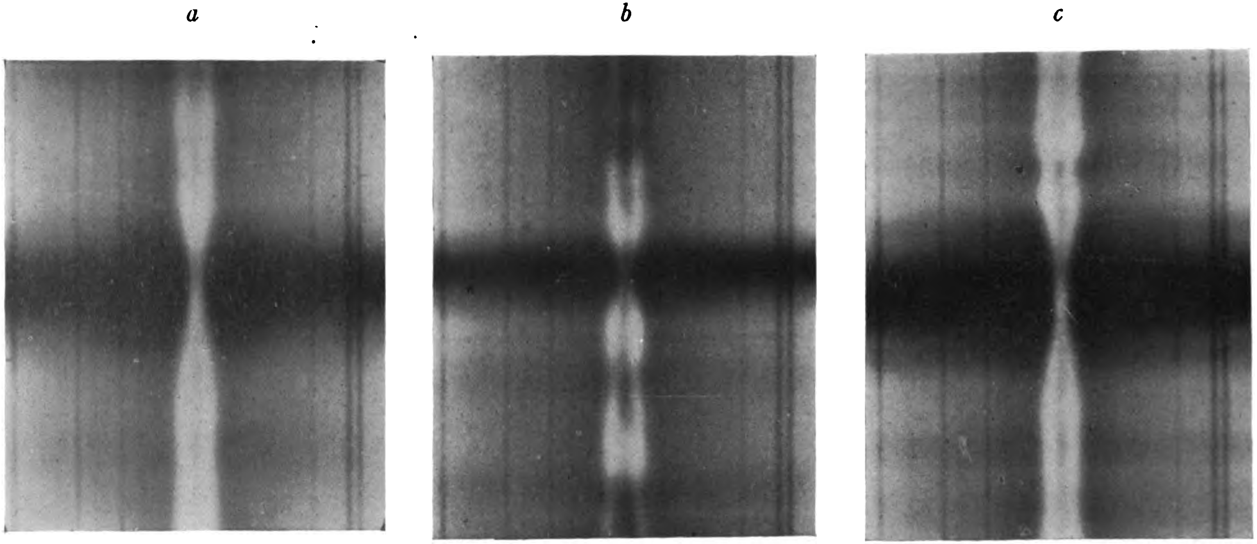
[1] *Astrophysical Journal*, 31, 152, 1910.

PLATE V



THE K LINE OF CALCIUM OVER SUN-SPOT NO. 6874

Enlargement: 6 times from original negatives with the 30-foot spectrograph and tower telescope

a, Normal reversal over umbra. Spot central. 1910, August 9.

b, Displacements due to inflow of penumbral vapor. Slit parallel to solar radius through spot, 17 mm from east limb. 1910, August 5.

c, Displacements due to rotation of penumbral vapor. Slit perpendicular to solar radius through spot, 23 mm from west limb. 1910, August 12.

times a downward movement of the vapor producing the bright K line of calcium. The velocity of descent varied for individual spots from 0.68 km per second in the case of 6606 to 2.2 km per second in the case of 6786, the general mean being 1.30 km per second. Spot 6602 gave some interesting results. On the first two of the three days on which observations were obtained the calcium vapor was rising with a velocity of 0.9 km per second. On the following day it was descending with a velocity of 1.6 km per second. This sudden change in the direction of the movement of the vapor was so surprising that the plates were remeasured. The results were similar, as indicated by the individual measurements given in full in the table. Spot No. 6608 was a large spot approaching dissolution. The bright calcium line over the umbra was strongly curved. The portion displaced to the violet was the brightest and the measurement indicated a possible upward velocity of the vapor of 0.15 km per second. The measurement of the portion of the line displaced to the red indicated a downward velocity of 1.22 km per second. The result given in the table for this spot is the mean, and indicates on the whole a considerably greater velocity downward than upward. The interesting fact, however, is that two parts of the umbra show these different conditions. The measurements in the case of spot 6772 point to a practically stationary state of the vapor producing the bright line, namely, an upward motion of less than 0.1 km per second. On its return as 6787 the emitting vapor was descending with a velocity of 1.9 km per second. From the large percentage (88 per cent) of the spots showing only downward movement of the emitting vapor and from the varying results for the other spots, one can conclude that in the umbrae of spots the calcium vapor is in general descending, but that under certain conditions upward motions do occur.

VERTICAL MOTION OF CALCIUM VAPOR OVER THE PENUMBRAE OF SPOTS

The usual appearance of the K line of calcium in the penumbra of a spot is very similar to that over the surrounding flocculus, consisting of the two bright components of K_2 and the central dark line K_3, except that the width of the K_2 line decreases rapidly as it nears the umbra and the absorption line grows less distinct and finally disappears over the umbra. These two portions of the line lie, of course, upon the broad shading of K_1, which is not considered in the wave-length measurements, as it is not possible to fix its position with sufficient accuracy. Both K_2 and K_3 have been measured; the center of the bright line was obtained by adding to the wave-length of the violet component the difference of wave-length between the absorption line and the red component.[1] Upon plates taken as these were, near the center of the solar image, the line usually runs straight over flocculus, penumbra, and umbra. No correction for distance from the center has been made to the wave-lengths in Table I, but in Table II, containing the results of the measurements over the penumbrae of the spots, the data for the umbrae of the individual spots are given with such corrections, which in no case exceed 0.002 Å.

TABLE II

THE K_2 AND K_3 LINES OF CALCIUM OVER THE PENUMBRAE OF CENTRAL SPOTS

NUMBER	NORTH		UMBRA	SOUTH		REMARKS
	K_2	K_3		K_2	K_3	
6577	3933.665(5)	3933.681(5)	3933.681	3933.680(5)	3933.692(5)	2d apparition. Returned as 6592
6602	.633(2)	.661(2)	.655	.643(2)	.676(2)	For 2 days
	.658(2)	.685(2)	.688	.662(2)	.690(2)	On the following day
6604	.668(1)	.685(1)	.686	.691(1)	.686(1)	1st apparition
6606	.677(2)	.671(2)	.677	.679(1)	.668(1)	Only appearance
6608	.661(2)	.671(2)	.676	.667(2)	.678(2)	2d apparition, about to break up
6609	.657(5)	.670(5)	.681	.679(3)	.686(3)	Return of 6592
6625	.670(2)	.684(2)	.690	.666(2)	.684(2)	1st apparition. Returned as 6645
6627	.681(2)	.692(2)	.680	.659(2)	.680(2)	Return of 6609
6660	.684(4)	.685(4)	.684	.692(3)	.684(4)	Only appearance
6695	.657(1)	.674(1)	.681	.672(1)	.687(1)	Leader of stream
6746	.684(1)	.684(1)	.687	.670(1)	.686(1)	2d apparition
6753	.651(4)	.674(4)	.678	.659(4)	.674(4)	3d apparition
6756	.674(5)	.689(5)	.685	.667(5)	.680(5)	2d apparition
6765	.653(4)	.678(4)	.681	.661(4)	.675(4)	Return of 6746
6771	.658(3)	.688(4)	.688	.632(3)	.659(3)	1st apparition
6772	.653(2)	.678(2)	.666	.654(3)	.675(3)	1st apparition
6774	.661(1)	.672(1)	.670	.648(1)	.684(1)	Developing spot
6786	.680(2)	.689(2)	.697	.670(2)	.680(2)	Return of 6771
6787	.667(4)	.694(4)	.693	.659(4)	.679(4)	Return of 6771
6788	.662(4)	.673(4)	.684	.668(4)	.678(4)	Return of 6774
6847	.665(2)	.694(2)	.688	.672(2)	.683(2)	1st apparition
6849	.685(1)	.689(1)	.688	.672(1)	.681(1)	Shows inflow
6874	.681(1)	.681(1)	.684	.666(1)	.661(1)	Shows inflow and vortex
6886	.655(1)	.653(1)	.687	.666(1)	.687(1)	
6897	.674(1)	.678(1)	.683	.684(1)	.696(1)	Shows inflow
	3933.665(63)	3933.682(63)	3933.682	3933.665(57)	3933.679(57)	

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[1] Adams, *Contributions from the Mount Wilson Solar Observatory*, No. 6, 3, 1905; *Astrophysical Journal*, 23, 47, 1906.

The headings "north" and "south" are not to be taken too literally, as they refer only to opposite sides of the spots in a very general north-and-south direction. The separation into these two groups was made in order that the results of the two independent sets of measurements might serve as checks upon each other. The mean of the two surprisingly concordant results for the emitting vapor is 3933.665. The measurements are difficult to execute, but considerable reliance may be placed on the final mean, depending as it does upon 120 measurements. The wave-length in the arc being 3933.667 Å, it appears that in the penumbrae of spots the calcium vapor producing the emission line is on the average neither ascending nor descending, as the mean would imply an upward velocity of only 150 m per second. The variation in the wave-length of the K_2 line in the penumbrae of different spots is greater than the errors of measurement in some cases, and should not be left out of consideration in the discussion of the individual spots. The high-level absorbing vapor is descending with a velocity of approximately 1 km per second, if its wave-length be referred to 3933.667. The measurements carried out on the absorption line K_3 reported in *Contribution* No. 48[1] point to a slightly shorter wave-length at the limb of the sun, where on the whole the effect of line-of-sight motion was probably eliminated; and, moreover, at the elevation at which the absorption line is produced, the pressure is probably less than one atmosphere, which is the value to which the arc wave-length is referred.

VERTICAL MOTION OF CALCIUM VAPOR OVER FLOCCULI

In Table III are assembled the results of the measurements of the wave-lengths of the K_2 and K_3 lines of calcium over the surrounding flocculi and outside the penumbral region of the spots. Points were taken on flocculi north, south, east, and west, and at different distances from the spots in order that the mean might fairly represent the average condition over the flocculi.

The division of the table into two parts was, as in the preceding one, for the purpose of using the two sets of measurements as checks upon each other. The K_2 line is very difficult to measure, because its components over the flocculi are very broad and each component is bounded differently on its two edges—on one edge by the K_1 shading and on the other by the K_3 line, and these boundaries are of different intensities. Moreover, the intensity of the K_2 component falls off more gradually toward the K_1 shading than toward the K_3 line. The plates have been measured red right and red left, with from four to ten settings on each component in each position. It was found of advantage to use the adjustable slide over the plate to fix the portion of the line under measurement, and to eliminate as far as possible effects that might arise from the rapidly varying width of the bright component. In general these components appear to the eye to be of nearly equal widths over the flocculi.

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[1] *Astrophysical Journal*, 32, 36, 1910.

TABLE III

THE K_2 AND K_3 LINES OF CALCIUM OVER CENTRAL FLOCCULI

NUMBER	NORTH AND EAST		SOUTH AND WEST	
	K_2	K_3	K_2	K_3
6577.........	3933.677(2)	3933.679(2)	3933.680(2)	3933.696(2)
6602.........	.643(2)	.667(2)	.657(2)	.671(2)
6604.........	.648(3)	.682(2)	.673(2)	.676(1)
6606.........	.677(2)	.670(2)	.678(1)	.674(1)
6608.........	.656(2)	.665(2)	.678(2)	.682(2)
6609.........	.650(3)	.666(3)	.670(6)	.677(5)
6625.........	.675(3)	.685(3)	.663(3)	.683(3)
6627.........	.659(3)	.673(3)	.667(3)	.680(3)
6660.........	.683(2)	.689(2)	.689(2)	.688(2)
6695.........	.680(7)	.681(5)	.676(7)	.690(6)
6746.........	.670(4)	.685(4)	.661(4)	.677(4)
6753.........	.658(10)	.673(8)	.663(3)	.680(2)
6756.........	.657(4)	.688(4)	.664(6)	.680(6)
6765.........	.654(2)	.685(2)	.666(6)	.678(6)
6771.........	.660(2)	.678(2)	.644(2)	.659(2)
6772.........	.657(3)	.682(3)	.653(7)	.665(7)
6774.........	.633(2)	.660(2)	.646(2)	.679(2)
6786.........	.666(3)	.682(3)	.658(3)	.679(3)
6787.........	.651(5)	.684(5)	.651(5)	.678(5)
6788.........	.659(6)	.675(6)	.671(8)	.689(8)
6847.........	.684(2)	.695(2)	.672(2)	.684(2)
6849.........	.681(3)	.691(3)	.667(3)	.666(3)
6874.........	.677(2)	.676(2)	.673(2)	.667(2)
6880.........	.666(3)	.676(3)	.675(2)	.680(2)
6886.........	.663(3)	.679(3)	.666(2)	.683(2)
6897.........	.672(2)	.685(2)	.656(3)	.686(3)
	3933.663(85)	3933.679(80)	3933.665(90)	3933.679(86)

The marked dissymmetry that is evident over the central portion of the disk outside of the flocculi shows but slightly, partly because the components are so broad, but mainly because it does not exist to the same degree. The mean wave-length for the two closely agreeing sets depending upon 175 measurements is 3933.664 Å against 3933.667 Å in the arc and 3933.641 Å at the center of the disk.[1] This indicates a small ascending movement of 0.23 km per second for the emitting vapor, against an upward velocity of 1.97 km per second for this vapor over the general solar surface. There are some indications in the measurements at different distances from the spot that the upward movement increases with increasing distance, but the number of measurements is not sufficient to fix this point. This result for the bright calcium over the flocculi is not in agreement with the general assumption that over the flocculi this vapor is rising rapidly from the interior of the sun or from a hotter lower stratum. The present investigation has been confined to the usual conditions in the flocculi and has not dealt with the exceptional eruptive phenomena. No measurements are known to the writer supporting the assumption of rapid upward motion, which seems to rest upon what appeared to be a very natural explanation of spectroheliographic data.

VERTICAL MOTION OF CALCIUM OVER CIRCUMFACULAR REGIONS

In the previous paper, "The General Circulation of Calcium Vapor," the measurements of the K_3 line were limited to regions free from spots and faculae. In the present investigation such measurements have been extended to the facular and circumfacular regions in order to detect any local circulation that might have its seat there, and to determine whether the facular and circumfacular regions are in any way complementary to each other. The measurements are given in Table IV and refer to the faculae surrounding the spots investigated, which were always near the center of the disk.

The mean of all the measurements is 3933.678, but omitting spots 6771, 6772, 6774, which appear to form a group by themselves, the mean is 3933.680. The mean of 166 measurements of the wave-lengths of K_3 over faculae is 3933.679. In the paper "The General Circulation of the Calcium Vapor" already referred to, the wave-length of the K_3 line at the center of the sun was found to be 3933.680, and in the arc 3933.667. The agreement between the means of the three solar determinations is so close that it seems evident that the conditions obtaining in the case of the upper calcium layer of the solar atmosphere are the same over the general surface, the faculae, and the region immediately surrounding the faculae; and that a local system of circulation between the facular and circumfacular regions is not the usual condition in the high-level vapor. Twenty-six circumfacular regions were investigated, twenty-three of which show a descending motion of the absorbing vapor at every point at which measurements were made. These points have been taken on all sides and at varying distances from the edges of the brilliant flocculi. In nine cases out of ten, therefore, the measurements indicate a descent of the high-level calcium vapor in the regions surrounding the faculae. In a tenth of the cases, namely in the circumfacular regions of spots 6771, 6772, and 6774, the mean of the measurements is 3933.665 Å. Compared with wave-length 3933.667 in the arc this would indicate a possible upward motion of 150 m per second; but the outstanding differ-

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 48, 21, 1910; *Astrophysical Journal*, **32**, 56, 1910.

TABLE IV

THE K_3 LINE OVER CIRCUMFACULAR REGIONS

Spot Number	Wave-Length	Spot Number	Wave-Length
6577	3933.681(1)	6765	3933.672(6)
6602	.676(5)	6771	.668(6)
6604	.684(4)	6772	.666(5)
6606	.683(4)	6774	.661(5)
6608	.687(4)	6786	.682(8)
6609	.686(9)	6787	.677(3)
6625	.684(6)	6788	.677(5)
6627	.678(7)	6847	.675(3)
6660	.685(3)	6849	.680(3)
6695	.673(10)	6874	.687(6)
6746	.681(4)	6880	.679(4)
6753	.676(5)	6886	.672(3)
6756	.681(5)	6897	.677(1)

Mean . . . . 3933.678(134)

ence of 0.002 Å is too small to predicate motion upon, and moreover the low pressure at this high level might account for the slight difference, as the shift per atmosphere is only 0.002 Å. Reference was made above to the possibility that these three spots form a special group. They were on the disk at the same time; two of them, 6771 and 6772, crossed the central meridian at the same hour; spot 6772 crossed 0.7 day later. It is of interest in this connection to compare the entire set of measurements at their following apparitions as 6786, 6787, and 6788, and with the mean of the whole series of spots. These data are assembled in Table V.

TABLE V

COMPARISON OF SPECIAL SPOTS WITH WHOLE SERIES

Spot No.	Circumfaculae K_3	Faculae K_3	Penumbrae K_3	Umbrae Bright Line
6771.........	3933.668	3933.668	3933.673	3933.688
6772.........	.666	.670	.676	.666
6774.........	.661	.670	.678	.670
6786.........	.682	.685	.681	.697
6787.........	.677	.687	.681	.693
6788.........	.677	.676	.682	.684
Whole series..	.680	.679	.680	.682

It will be noticed that in the case of these three spots at their first apparition the state of the absorbing vapor was exceptional over the faculae as well as over the circumfacular regions, in neither instance showing any indications of a circulation between the two regions; and that in the case of two spots the calcium vapor was not descending over the umbrae, while on their second apparition all three were in agreement with the majority of the spots observed in showing a descent of the calcium.

The results of this investigation of twenty-six circumfacular regions appear to differ from those obtained by M. Deslandres as reported in the *Annales de l'Observatoire de Meudon*, Tome IV, pp. 75–79. Measurements were carried out at Meudon over the faculae and their immediate surroundings upon plates taken with the *spectro-enregistreur des vitesses*.

> Les mesures ont porté seulement sur la raie centrale K_3, qui est nette et bien délimitée sur le disque entier; on a laissé de côté, au moins provisoirement,

> la raie K_2, dont les composantes ne se prêtent pas toujours à un pointé précis. Les vitesses sont seulement relatives, car on a adopté pour déplacement zéro la moyenne des déplacements de tous les points de l'image. Or sur la facule, dans la grande majorité des cas, ... les vitesses sont descendantes, et tout autour ascendantes. Ce fait expérimental a été vérifié sur une dizaine des facules.

And again, page 93:

> Cette conclusion, avant d'être acceptée comme absolument définitive, devra être appuyée sur des mesures plus nombreuses de vitesse radiale, faites non seulement au centre du Soleil, où les vitesses verticales seules interviennent, mais à une certaine distance de ce centre; ce qui permettrait de reconnaître les vitesses parallèles à la surface. Dès, à présent, on peut affirmer qu'elle est extrêmement probable.

The apparent disagreement in the results of the two investigations may possibly be referred to the different types of faculae studied in the two cases. In the present work all the faculae are large and in the immediate neighborhood of spots. The case shown in the Meudon *Annales* is that of a small facula unconnected with any spot. Then, too, the Meudon measurements are relative, the zero of displacement being the mean of all the displacements. In the present investigation the second slit was wide enough to admit the two iron standards, one on each side of K_3, from which the absolute wave-length of the displaced K_3 line was determined and through which the measurements were referred to the wave-length of the K line in the arc. From the adoption of the arbitrary zero in the former case it may be that the rise of the absorbing vapor in the circumfacular regions is only apparent, and that it is really descending in both regions, which would bring the results into nearer agreement. In view of the bearing of the results upon hypotheses relative to systems of circulation in the solar atmosphere the investigations at Mount Wilson will be extended to include large and small faculae and their immediate surroundings in cases where there are no associated spots.

RADIAL AND ROTATIONAL MOTION OF THE CALCIUM VAPOR OVER THE PENUMBRAE OF SPOTS

Radial movement of the reversing layer in sun-spots, showing an outflow with a velocity of one or two kilometers per second, was announced by Evershed in 1909.[1] In a subsequent paper, under the heading "Radial Motion in the Higher Chromosphere,"[2] he writes as follows:

> I have said that the spectrum-lines are universally found to be displaced toward the red on the side of the penumbrae nearest the limb, and to the violet on the opposite side, thus indicating an outward movement. There are, however, two notable exceptions among the host of absorption lines in the solar spectrum. The central absorption lines, known as H_3 and K_3 (possibly also the underlying emission-lines H_2 and K_2) when clearly defined in the penumbrae, are displaced in exactly the opposite sense in every case I have examined, that is, towards the violet on the limb side of the spot, thus indicating an indraught of the calcium vapour of the higher chromosphere. . . . . It would seem natural to suppose that the calcium vapour is drawn downwards as well as inwards, and I cannot at present say whether this is so or not, although some of my measures indicate such a downdraught.

In the same paper Mr. Evershed considers the question of a spiral motion of the outflowing vapors of the reversing layer. He finds in the case of five spots a mean rotational movement of 0.35 km per second and a radial motion of about 2 km per second, by combining which he gets a spiral movement opposite in direction in the two hemispheres. In the present investigation of the movement of the chromospheric calcium vapor to which the origin of the lines K_2 and K_3 is referred, plates were taken in the case of four spots with the slit of the spectrograph across the centers of the umbrae and *parallel* to the radius of the solar image passing through the spot. The measurements were carried out upon both the K_2 and the K_3 lines over the limb edges and center edges of the penumbrae of the spot as shown in Table VI.

The typical appearance of the absorption line K_3 was that of a sinuous curve showing no abrupt change in direction at the outer limit of the penumbrae, but bending gradually back into the undisplaced line, as indicated by Plate V (*b*). Three of the four spots were followed from the east to the west limb; one was observed only near the east limb; but in every instance the wave-length on the edge of the penumbra toward the center was greater than on the edge toward the limb. In the former case the wave-length was longer than the corresponding wave-length in the arc, and in the latter the wave-length was always shorter than in the arc. The mean line-of-sight components for the four spots and the

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[1] Evershed, *Monthly Notices of the Royal Astronomical Society*, 69, 454, 1909.

[2] *Ibid.*, 70, 218, 1910.

TABLE VI

THE K_2 AND K_3 LINES OF CALCIUM OVER THE PENUMBRAE OF ECCENTRIC SPOTS

Slit parallel to the radius through the spot

SPOT	PLATE	DATE	LIMB EDGE		CENTER EDGE		REMARKS
			K_2	K_3	K_2	K_3	
No. 6869 On C.M. July 18.6, 1910	173c	1910 July 15	3933.650	.658	3933.673	.683	Line curved. 29 mm from East limb Return of 6862
No. 6874 On C.M. Aug. 9.5, 1910	182c	Aug. 4	.660	.637	.673	.668	Line curved. Spot 6 mm from East limb 1st apparition. Returned as 6880
	185c	Aug. 5	.653	.651	.702	.698	Line strongly curved. 17 mm from East limb
	187c	Aug. 6	.656	.651	.687	.690	Line curved. 30 mm from East limb
	195c	Aug. 9	.666	.661	.687	.687	On central meridian. 59 mm from South limb
	199c	Aug. 11	.664	.659	.662	.680	Line curved. 35 mm from West limb
	201c	Aug. 12	.651	.638	.680	.683	Line curved. 23 mm from West limb
			3933.658	.650	.680	.683	
No. 6880 On C.M. Sept. 5.8, 1910	219c	Aug. 31	.632	.637	.652	.669	Line sharply curved. 4 mm from East limb. Return of 6874
	221c	Sept. 1	.639	.639	.668	.668	Line curved. 12 mm from East limb
	223c	Sept. 8	.631	.624	.670	.686	Line strongly curved. 25 mm from West limb
	225c	Sept. 9	.633	.625	.681	.688	Line strongly curved. 15 mm from West limb
			3933.634	.631	.668	.678	
No. 6886 On C.M. Sept. 26.9, 1910	237c	Sept. 21	.645	.635	.665	.685	Line curved. 5 mm from East limb
	249c	Sept. 22	.649	.639	.686	.719	Line strongly curved. 13 mm from East limb
	255c	Sept. 24	.661	.654	.665	.693	Line curved. 40 mm from East limb
	256c	Sept. 26	.655	.653	.666	.687	
	261c	Oct. 1	.660	.640	.684	.702	Line curved. 8 mm from West limb
			3933.654	.644	.673	.697	

corresponding motions along the solar surface found by dividing by the sine of the angular distance from the center are given in Table VII.

This gives an average horizontal velocity of 2.0 km per second for the absorbing vapor and of 1.24 km per second for the emitting vapor on each side of the spot. For the absorbing vapor the mean result corresponds closely with the mean found by Mr. Evershed, namely 1.83 km per second. The results for the K_2 line are of less weight owing to the greater uncertainties in the measurement; but it is an interesting fact that the mean is decidedly less than in the case of the absorbing vapor, that every measurement points in the same direction, and that a similar difference appears, as will be seen later, in the measurements when the slit was perpendicular to the radius of the solar image passing through the spot. The data for the seven spots on which observations were obtained with the slit *perpendicular* to the radius are given in Table VIII.

TABLE VII

LINE-OF-SIGHT COMPONENTS AND MOTIONS ALONG SOLAR SURFACE

Spot No.	Unreduced		Reduced	
	K_2	K_3	K_2	K_3
6869	1.75 km	1.91 km	2.59 km	2.75 km
6874	1.68	2.59	2.29	3.51
6880	2.59	3.12	3.20	4.42
6886	1.45	4.04	1.83	5.33
Relative motion	1.87	2.91	2.48	4.00
On each side	0.94	1.46	1.24	2.00

With a slit normal to the radius vector through the spot the radial flow of the vapor would produce no Doppler effect, but the rotational or spiral movement of the vapor would displace the line on one side of the spot to the red and on the opposite side to the violet. After the spot passes the meridian the displacements on the two edges of the spot would be reversed; if the direction of rotation is such that the north edge is approaching when the spot is east of the meridian the north edge will be receding when the spot is west of the meridian. This was distinctly the condition shown by spot 6577 and its return as 6592, and also by spot 6874. Of the seven spots observed, counting each apparition, five showed relative displacements of the K_2 and the K_3 lines for the north and south edges. On Plate V (*c*) is shown an enlarged reproduction of a limited portion of the spectrum of spot 6874 with the slit at right angles to the radius vector. On the original negative the displacements on the opposite sides of the spot are very distinctly visible. By holding the reproduction on the level of the eye and looking along the line the relative displacements may be seen.

TABLE VIII

THE K_2 AND K_3 LINES OF CALCIUM OVER THE PENUMBRAE OF ECCENTRIC SPOTS

Slit perpendicular to the radius through the spot

Spot	Plate	Date	I			II			Remarks
			K_2	K_3	Edge	K_2	K_3	Edge	
No. 6577 On C.M. Dec. 8.3, 1908	144	1908 Dec. 13	3933.649	.639	N.	3933.711	.706	S.	At West limb. Returned as 6952
			.651	.648	N.	.706	.714	S.	
No. 6592 On C.M. Jan. 4.7, 1909	183	Dec. 30	.649	.635	S.	.677	.677	N.	At East limb. Return of 6577
			.659	.645	S.	.664	.666	N.	
			3933.652	.642		3933.690	.691		
No. 6610 On C.M. Feb. 2.2, 1910	237	1909 Jan. 28	.676	.683	N.	.686	.689	S.	8 mm from East limb
			.670	.663	N.	.669	.683	S.	Only observation
			.682	.676	N.	.680	.694	S.	
			.677	.671	N.	.684	.704	S.	
			3933.676	.673		3933.680	.693		
No. 6869 On C.M. July 18.6, 1910	173c	1910 July 15	.664	.656	N.	.691	.683	S.	29 mm from East limb
			.667	.660	N.	.681	.694	S.	Only observation
			3933.666	.658		3933.686	.689		
No. 6874 On C.M. Aug. 9.5, 1910	182c	Aug. 4	.668	.683	N.	.666	.650	S.	6 mm from East limb. Line curved. Returned as 6880
			.670	.685	N.	.671	.663	S.	
	185c	Aug. 5	.666	.683	N.	.661	.645	S.	17 mm from East limb. Line curved
	195c	Aug. 9	.676	.680	E.	.674	.672	W.	On central meridian 30 mm South of center
	199c	Aug. 11	.691	.692	S.	.664	.666	N.	35 mm from West Limb. Line curved
	210c	Aug. 14	.669	.672	S.	.652	.642	N.	5 mm from West limb. Line curved
			.672	.684	S.	.651	.638	N.	
			3933.673	.683		3933.663	.654		
No. 6880 On C.M. Sept. 5.8, 1910	219c	Aug. 31	.648	.652	N.	.659	.667	S.	4 mm from East limb. Return of 6874
	221c	Sept. 1	.656	.660	N.	.661	.661	S.	12 mm from East limb
	223c	Sept. 8	.681	.682	S.	.679	.681	N.	25 mm from West limb
			.671	.683	S.	.663	.667	N.	
			3933.666	.667		3933.664	.669		
No. 6886 On C.M. Sept. 26.9, 1910	237c	Sept. 21	.654	.666	N.	.663	.668	S.	5 mm from East limb
	249c	Sept. 22	.673	.677	N.	.672	.679	S.	13 mm from East limb
	261c	Oct. 1	.684	.700	S.	.681	.686	N.	8 mm from West limb
			3933.670	.681		3933.672	.678		

In the case of spot 6577 when it was near the west limb the north edge of the penumbrae gave the shorter wave-length. When it appeared at the east limb as 6592 the south edge gave the shorter wave-length. Considering the relative displacements at the two edges of the penumbrae as due to motions in the line of sight, it follows that when the spot was east of the meridian the penumbral calcium vapor was approaching on the south side and receding on the north side; and when the spot was west of the meridian the penumbral calcium vapor was approaching on the north side and receding on the south side—indicating in both instances a counter-clockwise rotation for this spot, which was situated in north latitude. In the case of spot 6874 observed during its passage from the east to the west limb, the rotation was in the same direction, but the spot was situated in south latitude. That these displacements are not due to an easterly and westerly drift on the north and south sides of the spot is shown by an observation on spot 6874 when on the meridian and south of the center by approximately one-third of the solar radius. In this position of the spot with the slit at right angles to the radius the displacements indicated approach of the penumbral vapor on the west side and recession on the east side, thus pointing to a true rotary movement. Of the three spots in the northern hemisphere two, 6610 and 6609, show a clockwise rotation; one, 6577, and its return, 6592, show a counter-clockwise rotation. We have then the two cases: spots in the same hemisphere showing opposite rotations and spots in the two hemi-

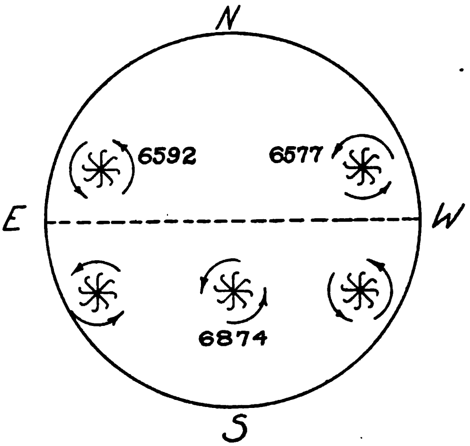

FIG. 1.—Spots in two hemispheres with same direction of rotation

spheres showing rotation in the same direction. In reference to his own observations Mr. Evershed says:[1]

> The opposite movement in the two hemispheres, although clearly shown in five instances, may not be an invariable rule. Referring to the magnetic field in spots, Professor Hale states that "although the larger spots in the northern and southern hemispheres of the Sun are usually found to be of opposite polarity, it frequently happens that spots of opposite polarity occur in the same hemisphere, sometimes in the same spot group."[2] One would expect, therefore, if the rotation movement bears a causal relation to the magnetic field, to find occasional instances of spots showing a clockwise rotation in the north, or vice versa.

Two of the seven spots observed, 6880 and 6886, showed no evidence of rotational motion. One of the two, 6880, was the return of 6874, which on its first apparition showed such motion unmistakably. The means of the line-of-sight components of the rotational motion and the corresponding velocities along the solar surface found by dividing by the sine of the angular distance from the center are given in Table IX.

TABLE IX

LINE-OF-SIGHT COMPONENTS AND MOTIONS ALONG SOLAR SURFACE

SPOT NO.	UNREDUCED		REDUCED	
	K_2	K_3	K_2	K_3
6577	4.50 km	5.11 km	4.50 km	5.11 km
6592	1.30	2.36	1.45	2.59
6610	0.30	1.52	0.38	1.67
6869	1.52	2.36	2.29	3.58
6874	0.76	2.21	1.19	2.92
Relative motion	1.68	2.71	1.96	3.17
On each side	0.84	1.36	0.98	1.59

This gives an average horizontal rotational velocity of 1.59 km per second for the absorbing vapor and of 0.98 km per second for the emitting vapor on each side of the spot. The combination of the rotational movement with the inflow and downward movement gives a real vortical motion inward for the calcium vapor in the case of these four spots.

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[1] Evershed, *Monthly Notices of the Royal Astronomical Society*, 70, 224, 1910.

[2] Hale, *Publications of the Astronomical Society of the Pacific*, 21, 205, 1909.

[*To be continued*]

A PRISM WITH CURVED FACES, FOR SPECTROGRAPH OR SPECTROSCOPE[1]

BY CH. FÉRY

I

Spectrographs have almost entirely replaced spectroscopes in chemical investigations; they have the important advantage over the latter instruments of recording the results obtained. The use of the classic micrometer is rendered unnecessary by photographing upon the same plate the spectrum of iron, the lines of which have been carefully standardized. Measurements of the wave-length of the lines of the metal or of the body studied are then made with great precision by measuring the plate on a small dividing engine. Since the lines of iron are very numerous and very fine, it is easy to obtain by interpolation the wave-length of the unknown lines which are present with those of the standard spectrum. This apparatus is especially recommended for the blue, violet, and ultra-violet regions, these being the radiations for which ordinary photographic plates are sensitive. Also in these regions the dispersion of nearly all refracting substances is considerable, and the large scale of the resulting spectrum is very favorable to precision of measurement.

The region easily photographed lies between 6600 and 2200 Å.U. To get these last wave-lengths it is necessary to make use of quartz, which does not absorb them.

II

The spectrograph usually employed consists of one or two 60° prisms of quartz, placed at minimum deviation for the middle of the region to be photographed, and cut so that the general direction of the beam is parallel to the axis of the quartz. If a single prism is employed it must be made by the combination of two 30° prisms, one of right-handed, the other of left-handed quartz, the indices of the two for right- and left-handed circularly polarized light being slightly different (Cornu prism).

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[1] Translated from the *Journal de Physique* (4) 9, 762, 1910; with additional remarks and illustrations from the author.

To increase the scale of the spectrum, since quartz has a dispersive power which is only about a third of that of flint glass, lenses of long focus are used (of the order of 0.50 meter to one meter), these lenses being cut perpendicularly to the axis, from crystals of opposite rotation. Usually these lenses are not achromatic; fluorite and Iceland spar are practically the only two materials which it is possible to associate with quartz without increasing the absorption in the ultra-violet; but these substances are rare and expensive, and Iceland spar has also a pronounced double refraction, troublesome in this connection. On account of the lack of achromatism the focus of the ultra-violet rays is much shorter than that of the visible rays, and the photographic plate must make an acute angle with the rays issuing from the second lens. This angle, which we shall calculate in the case of a spectrograph with only one prism, is evidently independent of the focal length of the lenses.

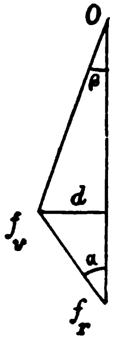

FIG. 1

Let O (Fig. 1) be the optical center of the combination of the two lenses between which the prism is placed, β the angle of dispersion between wave-lengths λ_v and λ_r, and f_v and f_r the focal lengths, violet and red, of the two radiations. We have

$$\frac{\tan\alpha}{\tan\beta}=\frac{\dfrac{d}{f_r-f_v}}{\dfrac{d}{f_v}}=\frac{f_v}{f_r-f_v}\,.$$

The difference in focal lengths, f_r-f_v, is easy to calculate for one of the lenses, by applying the known formula

$$\frac{1}{f_v}=(n_v-1)\Sigma\frac{1}{R}\,,$$

$$\frac{1}{f_r}=(n_r-1)\Sigma\frac{1}{R}\,.$$

For the joint effect of the two lenses the difference will evidently be twice as great, and equation (1) will become

$$\frac{\tan\alpha}{\tan\beta}=\frac{f_v}{2(f_r-f_v)}\,.$$

If we take, for getting this ratio, the two extreme radiations, of which the ordinary indices are, for quartz,

$$\text{Line } A \ \ldots \ n_r = 1.539$$

$$\text{Line } P \ \ldots \ n_v = 1.568$$

we find

$$\frac{\tan \alpha}{\tan \beta} = 9.25 .$$

Let us calculate β.

$$n_r \sin \frac{A}{2} = \sin \frac{A+\delta_r}{2} ,$$

$$n_v \sin \frac{A}{2} = \sin \frac{A+\delta_v}{2} ,$$

or for quartz

$$1.539 \times 0.5 = \sin \frac{A+\delta_r}{2} = \sin 50^\circ\!.02 ,$$

$$1.568 \times 0.5 = \sin \frac{A+\delta_v}{2} = \sin 51^\circ\!.60 .$$

The difference $51^\circ\!.60 - 50^\circ\!.02 = 1^\circ\!.58$ represents then $\frac{1}{2}(\delta_v - \delta_r)$. The angle of dispersion of the 60° prism is therefore

$$\beta = 2 \times 1^\circ\!.58 = 3^\circ\!.16$$

$$\tan \beta = .056$$

and

$$\tan \alpha = .056 \times \frac{f_v}{2(f_r - f_v)} = .056 \times 9.25 = 0.508 ,$$

therefore

$$\alpha = 27^\circ .$$

It is then at this small angle of 27°, or 63° with the normal to the plate, that the rays must meet the latter.

Moreover the foci of different colors are distributed on a curved surface, which makes it necessary to bend the plate or to employ films in order to get as near as possible to the form of the diacaustic. This large inclination of the photographic plate results in multiplying the dispersion by $\frac{1}{\sin 27^\circ} = 2.2$, but it offers the disadvantage of multiplying by the same factor the width of the lines, due to incorrect focusing. This observation explains the instability of the adjustment of spectrographs and shows how their different parts must be assembled in a rigid manner if one wishes to obtain a stable adjustment.

We may add to these some other criticisms arising from the cumbrousness and the high price of these instruments, and also from the loss of light caused by the passage of the rays in the different media. Finally, the adjustment of this apparatus, although well known, is nevertheless secured only by a long and delicate operation, and can be accomplished only by trial, in so far as the inclination of the plate is concerned, as well as the curvature which must be given to it.

III

In trying to avoid these different objections I have put together a spectrograph, based upon the employment of a prism with curved faces, especially constructed, of which I shall give a description.

It is admitted that to obtain a pure spectrum it is necessary to cause a bundle of parallel rays to fall on the face of a prism placed at minimum deviation. The source is a slit as narrow as possible, set parallel to the refracting edge, and placed at the principal focus of the collimating lens. A second lens, receiving the different parallel monochromatic bundles, gives in its focal plane a clear-cut image of the slit.

One may say, in a very general way, that to get a pure spectrum it is necessary and sufficient that the angles of incidence of the different rays be equal. The same should be true for the angles of emergence, for all the rays of the same wave-length. This condition is evidently fulfilled by the prism with the plane faces such as is usually employed, but one may imagine other solutions which satisfy this condition equally well, and which realize also those of minimum deviation, and of equality of the angles of incidence and emergence.

Let R be the radius of curvature of the front face of the prism $MPQN$ (Fig. 2). Draw the lines PC and QC making with the normals at the points of incidence the equal angles i. The slit is placed at the point of intersection, C, of these two lines. The condition mentioned will be fulfilled by all the rays issuing from C and falling on the prism.

Let us imagine that the monochromatic radiation chosen is such that we have

$$\sin i = n_\lambda \sin r.$$

An angle r will then correspond to the angle of incidence i, and on prolonging the two rays thus refracted at P and Q, we shall obtain at their point of intersection, B, the center of curvature of the second surface MN of a prism with curved faces. This second surface is silvered to increase its reflecting power, and the refracted rays, after all being reflected normally, will return to an exact focus on the slit itself.

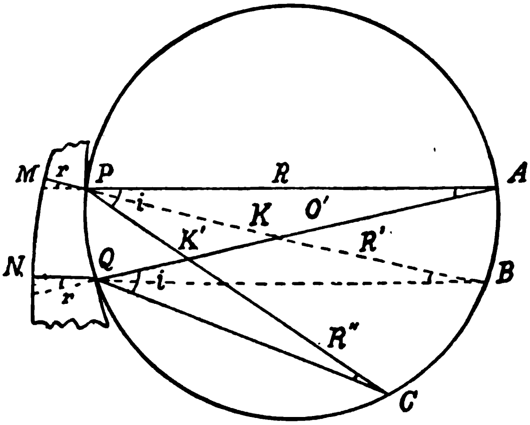

FIG. 2

If the radiation chosen were heterogeneous, the angles of refraction would be different for each color, but equal over the whole surface of the prism for any one color. After reflection from the surface MN the different rays of each monchromatic bundle would leave the prism with the same angle of refraction and would pass to an exact focus in the neighborhood of the slit.

It is easy to see from the figure that the angles A, B, and C are equal, for the two triangles which have for a common vertex the point K', where the lines PC and QA intersect, and as other vertices, the points P, A, C, and Q, are similar. They have an opposite angle at K' and two angles equal to i by construction. Therefore the angle at A is equal to the angle at C. In the same way it may be shown that the angle at A is equal to the angle at B, and also that all monochromatic bundles which are concentrated in the neighborhood of C converge at this same angle.

A circle may therefore be passed through the points P, Q, C, B, and A.

The result is that the slit, the spectrum, and the centers of curvature of the two surfaces of the prism are upon the same circle, drawn with the radius of the first surface as a diameter. Since the distance PQ is small with respect to the distances R, R', and R'', we have obviously

$$R' = R \cos r$$
$$R'' = R \cos i,$$

relations which make it possible to calculate the radii of curvature and the focal length of the prism, knowing its angle r and its index $n \sin r = \sin i$.

In order to simplify the mounting of the apparatus an angle of incidence i is chosen larger than that producing normal reflection at the silvered surface. To the angle of incidence $i+di$ there corresponds an angle of refraction $r+dr$, and all the rays of each monochromatic bundle are reflected from the face MN, undergoing a deviation of $180° - 2\,dr$; they all emerge at the same angle $i-di$, and converge to an exact focus upon the circle passing through the center of curvature of the two spherical faces of the prism.

It is easy to see, too, that the length of spectrum obtained is independent of the position of the spectrum on the circle.

This length is equal to δ (the angle between the extreme rays of the spectrum), multiplied by the distance of the prism from the photographic plate, and divided by the cosine of the angle at which the rays strike the photographic plate at VR (Fig. 3).

$$l = \frac{\delta \times PR}{\cos i},$$

but

$$PR = R' \cos i.$$

Therefore the length of the spectrum photographed is

$$l = \frac{\delta \times R' \cos i}{\cos i} = \delta R' = \text{Const.}$$

Astigmatism of the system.—The conjugate focus of a point on the slit obtained by a narrow bundle parallel to the edge of the prism is evidently not the same as that calculated above. This results in a certain amount of astigmatism, as in the concave grating of Rowland.

There are various consequences of this astigmatism.

It renders the employment of comparison spectra more difficult. I have overcome the difficulty by limiting the spectrum with a screen, movable vertically, and pierced by a horizontal opening.

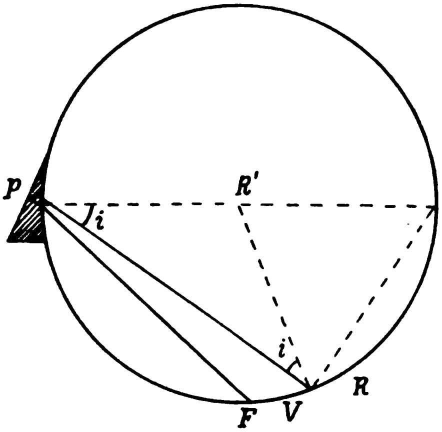

FIG. 3

By displacing this screen by an amount equal to the width of the opening which it carries, we may place upon the same plate the comparison spectrum, and this without changing the position of the image on the slit. The image of a point falling on the slit furnishes in fact a spectrum of lines.

The astigmatism presents, on the other hand, the advantage of not focusing the dust specks on the slit, and thus producing the disagreeable horizontal lines.

A spectrograph has been built embodying the foregoing principles. It is not cumbersome. The base is a casting which carries the slit, prism, and plate-holder, and may be covered by a case to prevent the entrance of dust into the apparatus, and to exclude outside light during an exposure.

The focus chosen is one meter, which corresponds for the quartz (a prism of 30°) to the same length of spectrum which would be given by lenses of 80 cm focal length and a prism of 60°. In reality the dispersion is even greater and the scale would correspond to that of a spectrograph having the lenses of one meter focal length, but in the last-named instrument, as we have seen, the inclination of the plate to the beam of light is 27°. Here it is 39°, or 51° to the normal.

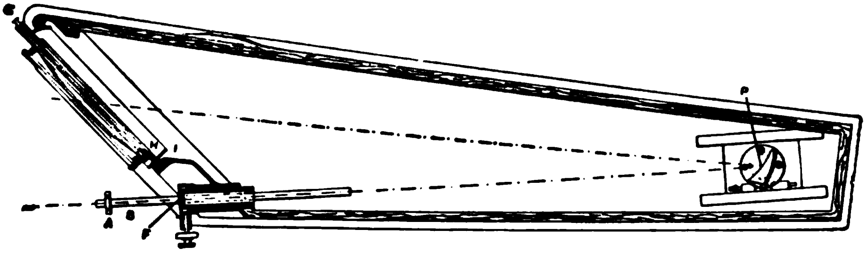

FIG. 4

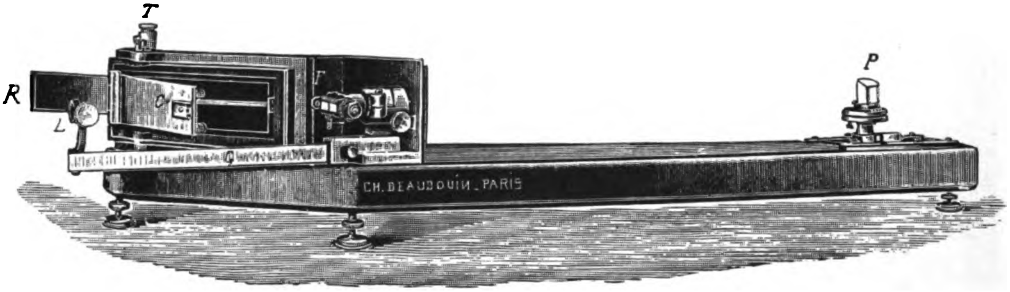

FIG. 5

The ordinary spectograph multiplies the dispersion, then, by

$$\frac{1}{\sin 27^\circ} = 2.2.$$

Here it is multiplied by only

$$\frac{1}{\sin 39^\circ} = 1.59.$$

The spectra given by the apparatus are about 220 millimeters long. The definition is very good throughout the whole extent of the plates used (24 centimeters in length).

IV

Application of this prism to the construction of a spectroscope. —It is sufficient to replace the plate-holder by a slide provided with a vernier and an eyepiece. The visible spectrum is about 9 cm long with a flint prism of one meter focal length. A vernier reading to $\frac{1}{20}$ mm therefore makes it possible to measure wave-lengths to one Ångström.

Eight instruments of this system have actually been constructed. One is employed at the Mint in Paris for testing alloys of gold and of silver. Another serves for the analysis of minerals containing radium. A third has been bought for studying the sources of the ultra-violet radiations utilized for the sterilization of water.

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Paris

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THE

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SPECTRUM OF COMET MOREHOUSE (1908 *c*)

By A. de la BAUME PLUVINEL and F. BALDET

The exceptionally fine weather during the autumn of 1908, and the considerable northern declination of this comet have enabled us to study its spectrum under very favorable circumstances. Our researches were made exclusively on negatives obtained with objective-prisms. In 1903, one of us had already called attention to the advantages presented by the objective-prism in investigating the spectra of faint comets;[1] and, since then, we have applied this method to the study of the spectra of several comets.

INSTRUMENTS USED AND PHOTOGRAPHS TAKEN

In the present case we have employed three different instruments:

1. At the beginning, when the comet was comparatively faint, we were obliged to utilize an objective-prism of large light-power, but necessarily of low dispersion (2.2 mm between F and H). The prism had an angle of 20° 18′, and it was placed in front of a portrait lens 0.08 m in diameter, whose focal length was 0.30 m. This instrument was already used in photographing the spectrum of comet 1902 *b* and that of comet 1907 *d*.

2. The comet having considerably increased in brightness, we were enabled, during October and November, to substitute a prism

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[1] *Comptes Rendus*, 136, 743, 1903.

TABLE I

DATE 1908—PARIS M.T.		EXPOSURE		EFFECTIVE LENGTH OF EXPOSURE	COMPARISON STAR	OBJECTIVE-PRISM	PLATE	REMARKS ON THE NEGATIVES
		Beginning	End					
October	3	12h 36m	15h 1m	2h 25m	*β Cephei*	No. 1	Panchr.	Bad comparison spectrum
	4	13 5	14 58	1 53	*γ Cassiopeiæ*	1	Panchr.	Fairly good
	4	15 49	16 47	0 58	*γ Cassiopeiæ*	1	Panchr.	Faint
	6	14 15	16 50	2 35	....	1	Pinac.	Intense, tail 6°
	11	7 7	7 37	0 30	....	1	L.V.	Fogged
	12	6 39	7 29	0 50	....	1	Pinac.	Good
	13	6 59	8 11	1 12	*Vega*	No. 2	Pinac.	Faint
	14	7 23	8 53	1 30	....	2	Pinac.	Fairly good
	15	8 41	9 31	0 50	....	2	Pinac.	Faint
	15	10 9	11 15	1 6	....	2	Pinac.	Faint
	16	6 40	10 30	3 40	....	2	Pinac.	Good
	17	10 0	12 2	2 2	*Vega*	2	Pinac.	Very satisfactory
	18	6 48	11 21	3 57	*Vega*	2	L.V.	Best spectrum for blue and violet
	19	6 27	12 23	5 56	....	2	L.V.	Very satisfactory
	20	9 25	10 25	1 0	....	2	Pinac.	Faint
	22	6 38	12 49	6 0	*Vega*	2	Pinac.	Very intense
	23	6 34	6 55	0 21	....	2	L.V.	Moderately good
	23	7 0	13 0	6 0	....	2	Pinac.	Very intense
	27	9 30	11 40	2 10	....	2	Pinac.	Moderately good
	28	7 46	11 15	3 29	....	2	L.V.	Very intense
	29	7 53	11 45	3 52	*Vega, Procyon*	No. 3	L.V.	Very faint; fogged
	30	6 20	11 20	5 0	*Capella*	3	L.V.	Good
	31	6 12	11 10	7 1	*Capella*	3	L.V.	Best spectrum for ultra-violet
November	1	6 27	8 30					
	6	6 4	8 4	2 0	....	3	L.V.	Good
	20	6 46	6 56	0 10	....	No. 2	L.V.	Good
	21	5 58	7 41	1 43	....	2	L.V.	Good
	27	5 42	7 25	3 7	*Capella*	2	Pinac.	Best spectrum from red to blue
	28	5 50	7 14					
	29	5 46	7 18	1 32	....	2	Pinac.	Faint; fogged

Abbreviations: Panchr. = Wratten Panchromatic plate; Pinac. = Wratten Pinacyanol plate; L.V. = Lumière "étiquette violette" plate; No. 1, with the prism of 20° 18′. No. 2, with the prism of 60°. No. 3, spar and quartz.

of 60° for the prism of 20° 18′, and thus to obtain higher dispersion (10.9 mm between F and H).

3. With the view of studying the ultra-violet, we also utilized a prismatic camera having a 60° prism of Iceland spar and a quartz object-glass 0.06 m in diameter with a focal length of 0.87 m (dispersion, 25.8 mm between F and H).

The spectroscopes were mounted on an equatorial telescope having an object-glass of 0.24 m aperture and 3.75 m focal length. This refractor, with a magnifying power of 140, was used as a guiding telescope. On 11 of the negatives we have printed the spectrum of a bright star, either on one side of the comet's spectrum or on both sides of it; and these comparison spectra enabled us to determine the wave-lengths of the comet's monochromatic radiations.

For the region extending from red to green, we used Wratten and Wainwright's "Pinacyanol" and "Panchromatic" plates; while for the blue and ultra-violet part we utilized Lumière's "étiquette violette" plates, particularly sensitive to the more refrangible rays of the spectrum.

We obtained in all 28 negatives, whose total exposure amounts to 72 hours 49 minutes. On p. 90 is an extract from our observational notebook.

MEASURING OF THE NEGATIVES AND DETERMINATION OF WAVE-LENGTHS

We have measured the negatives taken with the above instruments Nos. 2 and 3, the spectra obtained with instrument No. 1 not being sufficiently dispersive to give accurate results. A Zeiss comparator, with a microscope usually magnifying 4 diameters, was used in the measures. On each side of the monochromatic images of the comet were the wires of the micrometer. Two measures, one way and the other, were made; then the negative was reversed, and the same process repeated. Several plates were thus measured on various occasions separated by some weeks. The accuracy of the settings has never exceeded 10 μ, except for the doublet at λ 4256.9–4279.0, which was remarkably sharp and intense. The maximum discrepancies of all the measures of this doublet, on each of the 14 plates measured, do not exceed $\pm 4\ \mu$.

After trying different interpolation formulae, namely, Hartmann's, we finally adopted the following, which is quite sufficient in the present case:

$$\frac{1}{\lambda}=A+a\delta+b\delta^2+c\delta^3\ ,$$

δ being the distances of the monochromatic images from an arbitrary point, λ the wave-length corresponding to the distance δ, and A, a, b, c, constant coefficients. A first series of coefficients was calculated in order to interpolate between λ 4340.7 and λ 6563.1; and a second series was similarly computed for interpolation between λ 3770.8 and λ 4861.5.

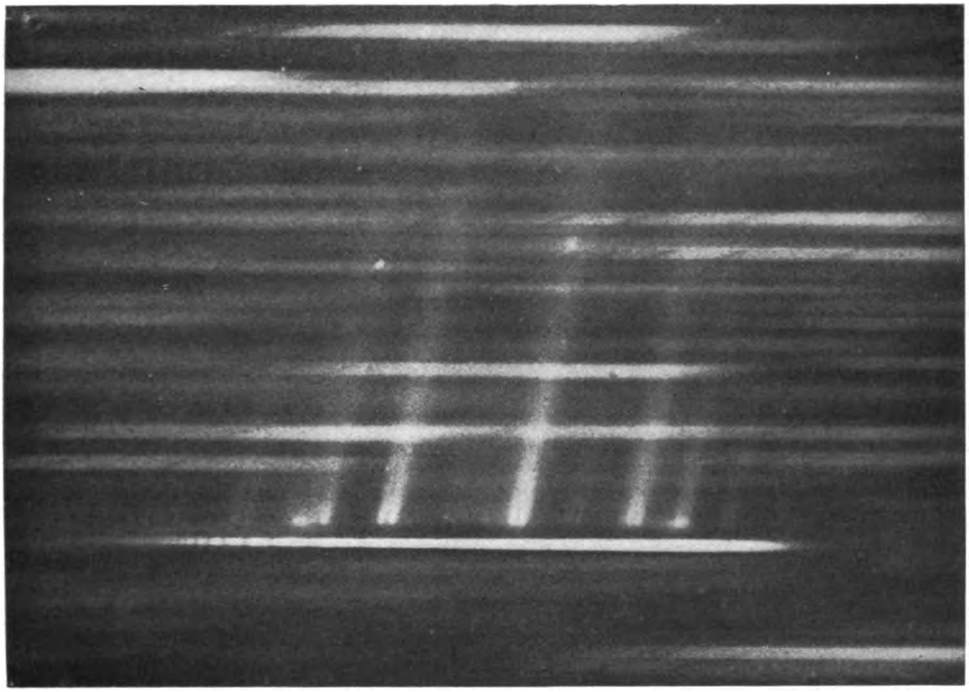
Toward λ 4000 the above maximum error of 10 μ in our settings entails an inaccuracy of 0.6 Å in the wave-length. As to the precision of the settings on the comparison spectra, we shall say that the positions of the hydrogen lines in the spectrum of *Vega* were found accurate to within 0.7 Å.

RESULTS

With the objective-prism of low dispersion we got an intense spectrum, showing seven distinct monochromatic images of the comet, though no continuous spectrum is visible. These images are located in the blue, violet, and ultra-violet. Hence comet Morehouse emitted only very refrangible rays, and was consequently blue. The spectrum extends to a considerable height, and the various images of the tail can be separated as far as 34′ from the nucleus. Beyond 34′, the monochromatic images of the tail are superposed, and the negatives show here nothing but a nebulosity which can be traced right up to the edge of the plate, that is to 6° from the head.

On the negatives taken with instruments Nos. 2 and 3, the monochromatic images of the comet are neatly doubled. This is clearly shown on Plate VI, which is an enlargement of a photograph taken with objective-prism 2 in the blue and violet. Four of these doublets are very strong and conspicuous, and in every interval between a pair and the one next there is a fainter doublet. Furthermore, in the yellow and ultra-violet, other feeble images are visible,

PLATE VI



THE SPECTRUM OF COMET MOREHOUSE

Photographed with 60° objective-prism on October 18, 1908, with exposure of 3^h 57^m
Enlarged 4.7 times from the original negative

which turn out to be also double. A close examination of the negatives has enabled us to recognize the existence of 21 doublets. Of these, 18 have been doubled with a definiteness sufficient to enable us to measure the positions of both their components; as to the other 3 doublets, their faintness did not allow us to do more than determine their middle part.

Having remarked that between two strong doublets there generally was a weak one, we were led to suspect the existence of two series of doublets—a series of bright doublets and a series of faint ones; and we have inquired whether the lines of these two series were not distributed in the spectrum in accordance with some known law.

After trying a formula for a line spectrum (Rydberg) and a formula for a band spectrum (Deslandres), we found that the doublets were distributed according to the second law of Deslandres, which is that "the intervals expressed in frequencies $\frac{1}{\lambda}$ between the heads of bands of the same series increase in arithmetical progression." This law may be defined by the relation

$$\frac{1}{\lambda}=\frac{1}{\lambda_0}+an+bn^2\,,$$

n being an entire number.

If we consider each of the two lines composing the doublets as heads of flutings, the intense doublets give two series, A_1 and A_2, and the faint ones yield two other series, B_1 and B_2, that is to say, altogether four series.

We had computed the wave-lengths of the heads of these four series by applying the above formula containing three coefficients, λ_0, a, b, which can be calculated on the basis of the wave-lengths of three observed lines. But as we have a large number of lines whose wave-lengths we have measured, we have treated our results by the method of least squares in order to obtain the most probable values of the coefficients.[1] The observed and computed wave-lengths are given in Table II.

---

[1] We have been assisted in this part of our inquiry by M. A. Kannapell, who is familiar with such kind of work.

We thus see that there is a satisfactory agreement between the observed and computed wave-lengths. The small discrepancies may probably be attributed to the fact that the law of Deslandres is only approximate, as well as to the difficulty inherent in the measures themselves.

TABLE II

n	λ Computed	λ Observed	Difference O.–C.	λ Computed	λ Observed	Difference O.–C.
			INTENSE SERIES			
		Series A_1			Series A_2	
1..........	3291.5	3294.	+ 2.5	3270.5	3269.	– 1.5
2..........	3444.6	3446.	+ 1.4	3425.5	3436.	+10.5
3..........	3615.7	3611.	– 4.7	3597.6	3586.	–11.6
4..........	3808.2	3803.4	– 4.8	3790.0	3782.6	– 7.4
5..........	4026.1	4023.3	– 2.8	4006.4	4003.4	– 3.0
6..........	4274.6	4279.0	+ 4.4	4251.5	4256.9	+ 5.4
7..........	4560.8	4575.8	+15.0	4531.4	4549.2	+17.8
8..........	4893.8	4879.0	–14.8	4854.1	4846.1	– 8.0
9..........	5285.7	......	......	5230.1	......	......
10..........	5753.4	......	......	5673.8	......	......
11..........	6321.5	......	......	6205.0	......	......
12..........	7025.9	7027.4	+ 1.5	6852.6	6848.4	– 5.2
			FAINT SERIES			
		Series B_1			Series B_2	
1..........	3372.3	3385.	+13.	3360.8	3357.	– 3.8
2..........	3528.2	3530.	+ 1.8	3517.0	3519.	+ 2.0
3..........	3705.2	3701.	– 4.2	3692.9	3687.	– 5.9
4..........	3907.2	3907.6	+ 0.4	3892.3	3898.2	+ 5.9
5..........	4139.8	4143.7	+ 3.9	4119.6	4114.0	– 5.6
6..........	4410.1	4411.9	+ 1.8	4381.5	4387.7	+ 6.2
7..........	4727.7	4721.9	– 5.8	4685.8	4695.3	+ 9.5
8..........	5105.4	5106.7	+ 1.3	5043.9	5021.1	–22.8
9..........	5562.0	5562.7	+ 0.7	5470.8	5482.2	+11.4
10..........	6124.1	......	......	5988.0	......	......

As to the three doublets, Nos. 9 and 11 of the strong series, and No. 10 of the faint series, they could not be neatly separated on account of their diffuseness, so that we have contented ourselves, as above stated, with taking their centers. Now, the position of these diffuse centers agrees with that of the middle of the computed doublets, as shown by Table III.

Calculation indicates that the doublet No. 10 of the strong series ought to be discernible at λ 5673.8–5753.4; but it was impossible to detect the slightest trace of it, probably because the pinacyanol plate used presents in this region of the spectrum a marked minimum of sensitiveness.

TABLE III

Series	λ Computed	Computed Middle	Observed Middle	Difference O.−C.
A_1...........	5285.7	5257.9	5259.8	+ 1.9
A_2...........	5230.1			
A_1...........	6321.5	6258.8	6254.5	− 4.3
A_2...........	6205.0			
B_1...........	6124.1	6056.9	6020	−37. †
B_2...........	5988.0			

† Very difficult measure.

The data of the above table have enabled us to draw the diagram on p. 96. In the upper figure we indicate the frequency of the radiations; in the lower one, their wave-length. This second illustration is in a way a schematic reproduction of one of our negatives. (The enlargement is 2.6.)

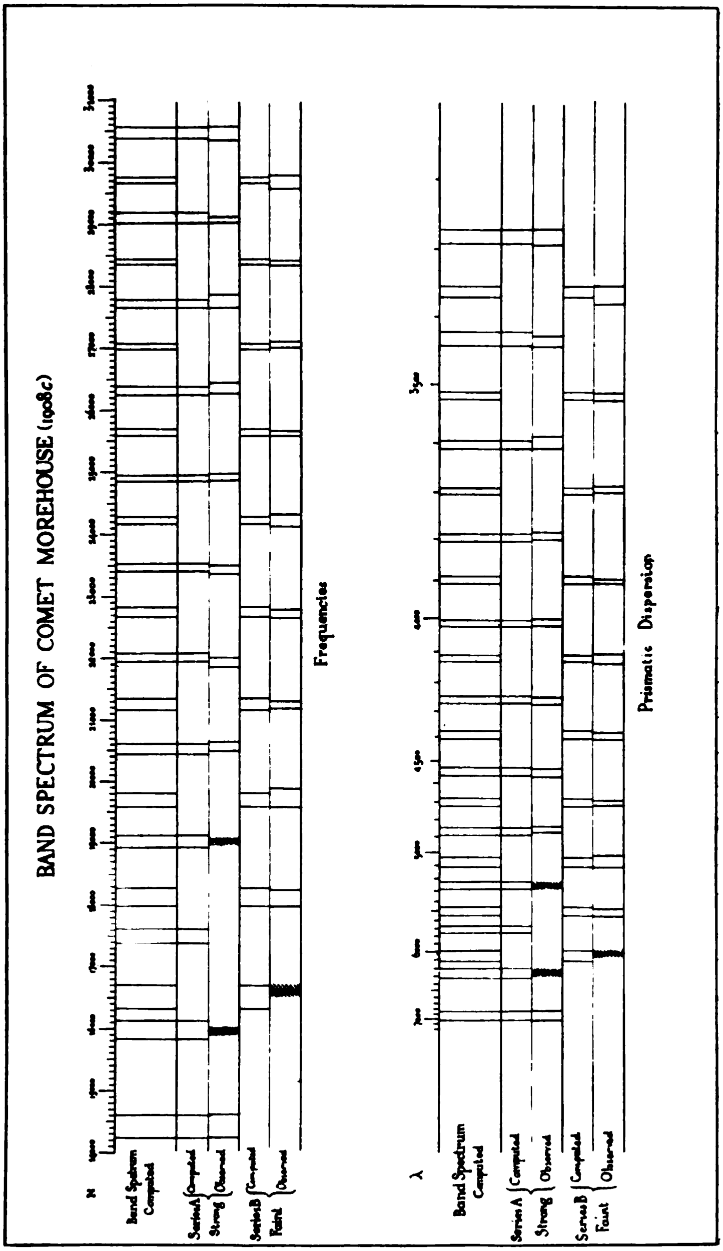
As a test of possible theories, we have examined whether the interval between the components of the doublets had not varied with time. On this point we have confined our measures to the sharpest doublets at λ 4256.9–4279.0, with the result that, on 14 spectra, taken between October 15 and November 29, the distance in question had not changed, so far as can be determined by the limits of accuracy of our measures, which, as already mentioned, attain here ±4 μ or ±0.3 Å.

Now, in addition to the doublets, we found on our negatives that the more conspicuous pairs are attended, on the violet side, by a third, complete monochromatic image of the comet, whose wave-lengths are the following:

TABLE IV

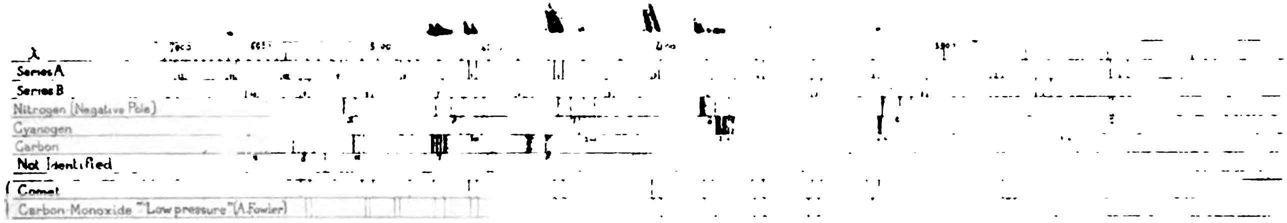
Doublet	λ	Doublet	λ
No. 5, Series A . .	3990.6	No. 2, Series B	3508
No. 6, Series A. . .	4236.3	No. 6, Series B	4373
No. 7, Series A. . .	4522.7	No. 8, Series B	4986.7

The number of these images being insufficient, we cannot know whether their intervals also obey Deslandres' law.



BAND SPECTRUM OF COMET MOREHOUSE (1908c)

PLATE VII



DRAWING FROM PHOTOGRAPH OF SPECTRUM OF COMET MOREHOUSE AND COMPARISON WITH THE SPECTRA OF DIFFERENT GASES

The intense and faint doublets, as well as the images just mentioned, are not the only monochromatic radiations found in the spectrum of the comet. A study of all our plates has enabled us to fix the position of a large number of condensations, of which some are found in the head only, the others being in the spectrum of both the head and the tail. We have drawn as faithfully as possible all the details visible in our spectra, and we give the resulting drawing in Plate VII. After obtaining a complete representation of the spectrum and of the wave-lengths of every radiation, we tried to identify the monochromatic images of the comet with the known spectra of various gases; and this comparison is given in the diagram of Plate VII, under the drawing, with which it coincides in wave-length.

PRESENCE OF THE SPECTRUM OF CARBON MONOXIDE

After having shown that the spectrum of comet Morehouse was essentially composed of bands, analogous to those yielded by a large number of gases, we have tried to identify it with some known spectrum. When we published a preliminary account of our inquiry,[1] we found no agreement between our spectrum of the comet and any published spectra, although the comet's radiations offered some analogy to those of various carbon compounds. But in 1909 Professor Fowler announced[2] that he obtained in his laboratory a spectrum which appeared almost identical with that of comet Morehouse; and he showed later that this new spectrum was that of carbon monoxide at very low pressure.[3] As soon as we heard of these interesting results, we compared the table of wave-lengths which Fowler gave with those of our spectrum of comet Morehouse.

The comparison in Table V will show the agreement between Fowler's wave-lengths and ours.

The accordance is particularly satisfactory for the lines of the two intense series, A_1 and A_2, which are, of course, more sharply defined on these plates than the lines of the faint series.

---

[1] *Comptes Rendus*, **148**, 759, 1909.

[2] *Monthly Notices*, **70**, 176, 1909.

[3] *Ibid.*, **70**, 484, 1910.

Yet we shall observe that there are two doublets for which the coincidence is not close. In the first place, Fowler's doublet at λ 3415–3429 does not agree with our doublet at λ 3430–3446. We are inclined to account for this discrepancy by the faintness of our spectrum in this region. It is possible that the most refrangible component of the doublet is not visible on our plates, and, further, that we have taken for the less refrangible component a radiation not belonging to the doublet. Secondly, Fowler's doublet at λ 4887–4916 does not agree with our doublet at λ 4846.1–4879.0. In this case, it would be hard to admit an error on our side, the image of the doublet being very sharp; and it is noteworthy that Fowler himself gives this doublet as "a possible pair."

TABLE V

CO Low Pressure (Fowler)	Comet	Series	Δλ Comet—*CO* (Fowler)	*CO* Low Pressure (Fowler)	Comet	Series	Δλ Comet—*CO* (Fowler)
λ 3415.0	λ 3436	A_2	−21.0	λ 4253.2	λ 4256.9	A_2	− 3.7
3429.0	3446	A_1	−17.0	4276.0	4279.0	A_1	− 3.0
3587.0	3586	A_2	+ 1.0	4545.4	4549.2	A_2	− 3.8
3602.0	3611	A_1	− 9.0	4570.5	4575.8	A_1	− 5.3
3693.0	3687	B_2	+ 6.0	4688.5	4695.3	B_2	− 6.8
3707.5	3701	B_1	+ 6.5	4715.0	4721.9	B_1	− 6.9
3781.0	3782.6	A_2	− 1.6	4887†	4846.1	A_2	+40.9
3797.6	3803.4	A_1	− 5.8	4916†	4879.0	A_1	+37.0
3891*	3898.2	B_2	− 7.2	5049	5021.1	B_2	+27.9
3907*	3907.6	B_1	− 0.6	5078	5106.7	B_1	−28.0
4001.3	4003.4	A_2	− 2.1	5473	5482.2	B_2	− 9.2
4020.4	4023.3	A_1	− 2.9	5510	5562.7	B_1	−52.7

* Indicated by Fowler as a possible pair.
† Indicated by Fowler as a faint band.

The spectrum of carbon monoxide at low pressure of Fowler and our spectra of comet Morehouse present another peculiarity which constitutes a proof of their identity. We found that on the spectra obtained with the objective-prism combination No. 3 the images of the brighter doublets at λ 4003.4–4023.3 and at λ 4256.9–4279.0 were slightly shaded toward the red, and that this shading extended to nearly one-half the interval of the doublet's two components. Now the doublets of carbon monoxide at low pressure also show this feature.

The number of our doublets is 21, while Fowler's are only 12. Hence there remain 9 doublets which were not observed in the laboratory. Furthermore, of the 12 doublets, 2 do not agree very satisfactorily, as already explained. But the accordance of the two spectra in their brightest and most conspicuous parts is too close to admit of any doubt that the doublets in the spectrum of comet Morehouse are practically identical with those of carbon monoxide at very low pressure.[1]

PRESENCE OF THE SPECTRUM OF NITROGEN

There is at λ 3914.7 an intense and complete image of the comet, neatly shaded on the violet side. Now, as far as the limits of accuracy of our measures go, this radiation coincides with the head of the most characteristic band in the spectrum of the negative pole of nitrogen. The presence of nitrogen in the gases of the comet thus seems highly probable; and, in order to corroborate this view, it is necessary to examine whether the other bands of the spectrum of this gas are also found in the comet. The spectrum of nitrogen obtained in the laboratory, at the cathode, consists of six bands. Of these, four, β, γ, δ, ϵ, coincide with complete monochromatic images of the comet; and the absence of the band α is probably due to the faintness of this radiation in our spectrum of the comet, and also to its location in the green. As to the group ζ, it would be difficult to affirm that it is found on our plates, superposed, as it is, on the less refrangible component of the doublet No. 1 in the A series. At any rate, as this image is notably brighter than that of the other component (a peculiarity which is not observable in the other doublets), it is very probable that there is, in this part of the spectrum, a superposition of the doublet's component and of the band ζ of nitrogen. Therefore, we think it safe to state that the cathodic spectrum of nitrogen is entirely recognizable in the comet's spectrum; and we will add that the gas seems to be as abundant in the head as in the tail.

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[1] It should be remembered that the distribution in pairs of the chief radiations emitted by a comet was first recognized by M. H. Chrétien in the spectrum of comet Daniel (1907 *d*) (*Comptes Rendus*, **145**, 549, 1907).

Subjoined is the comparison between the two spectra:

TABLE VI

Laboratory Spectrum		Comet's Spectrum		Remarks on the Identification
Group	λ of Band-Heads	λ of the Condensations in the Head	Images of the Tail	
ζ	3296.1	3294.	....	See the notes on this identification in the text
	3298.5	......	....	
ε	3548.2	3553.	From 3530	
	3563.5	3569.	....	
	3581.5	......	to 3586	Component λ 3586 of doublets No. 3, series A_2, is superposed
δ	3857.1	......	From 3856	The third group of *CN* and the doublet No. 4, series B, are superposed
	3883.9	......	....	
	3913.7	3914.7	to 3915	*The most characteristic image*
γ	4166.3	......	From 4144	The second group of *CN* is superposed
	4198.7	......	....	
	4236.3	......	....	
	4278.0	......	to 4279	Component λ 4279.0 of doublet No. 6, series A_1, is superposed
β	4515.3	......	From 4523	
	4553.8	......	....	The doublet No. 7, Series A, is superposed
	4599.4	4613.9	....	
	4651.2	......	....	The group γ of carbon and the doublet No. 7, series B, are superposed
	4708.6	......	to 4722	
α	5150.0	......	....	Not observed: minimum of sensitiveness of pinacyanol plate
	5227.5	......	....	

λ after Deslandres, for the groups δ, ε, ζ.
λ after Hasselberg, for the groups β, γ.
λ Ångström and Thalén, for the group α.

PRESENCE OF THE SPECTRUM OF CYANOGEN

At the beginning of the ultra-violet there are two contiguous dots of great brightness, surrounded by faint nebulosity. Notwithstanding their marked intensity, these dots are not attended by an image of the comet's tail; they coincide with the first two heads of the cyanogen band, and they occur in the spectra of all comets. The study of the plates further reveals the other cyanogen bands, but only in the spectrum of the head. The absence of the fifth band of cyanogen is explicable by the faintness of the region in which it is situated. We may note that the second band of cyanogen, which some authors could not detect, is plainly visible on our plates.

Also, the wave-lengths of the third group of lines are all greater in the cyanogen than in the comet's spectrum. Yet these lines are too bright in the comet to allow of an error capable of accounting for the discrepancy.

Our comparison between the comet's spectrum and that of cyanogen is given as follows:

TABLE VII

Laboratory Spectrum		Spectrum of Comet's Head	Remarks on the Identification
Group	λ of Band-Heads		
5th	3360.1	......	Not observed (see notes in the text)
4th	3584.1	From 3569	Well-marked condensation. The component λ 3586 of doublet No. 3, series A_2, and the group ϵ of Nitrogen are superposed
	3586.0	......	
	3590.5	to 3611	
3d	3855.1	3855.9	Dot well marked
	3861.9	......	Invisible
	3871.5	3870.4	Intense dot
	3883.6	3881.5	Intense dot
2d	4166.9	From 4164	Condensation shading toward the violet
	4167.8	......	
	4180.5	......	
	4181.0	......	
	4197.2	......	
	4216.1	to 4223	
1st	4505	From 4520	Diffused condensation
	4526	....	
	4548	....	
	4582	....	
	4607	to 4614	

λ after Deslandres, for the fifth group.

λ after Kayser and Runge, for the second, third, and fourth groups.

λ after Thalén, for the first group.

PRESENCE OF THE SPECTRUM OF CARBON

There is also in the spectrum of the comet's head only, a conspicuous condensation stretching between the lines of the doublet No. 7, series B, and even on either side of it. This extensive nebulosity is easily identified with the band in the Swan spectrum, while other groups of Swan's also occur in the comet.

We summarize here our observations of the coincidence of the Swan spectrum with that of the comet:

TABLE VIII

Laboratory Spectrum		Spectrum of Comet's Head	Remarks on the Identification
Group	λ of Band-Heads		
β	4311.0	4305	Image faint
	From 4365.0 to 4381.9	4372	Image faint
γ	From 4682.0 to 4736.0	From 4637 to 4746	Intense condensation, extending in the interval comprised between these wave-lengths
α	From 5097.5 to 5164.0	From 5107 to 5163	Diffused condensation, extending in the interval comprised between these wave-lengths
δ	From 5466.0 to 5633.0	....	Not observed; minimum of sensitiveness of pinacyanol plate
ε	From 5953.5 to 6187.3	 6024	Diffused condensation, whose middle only is here given

λ From Kayser and Runge, for the group from 4365.0 to 4381.9.

λ From Ångström and Thalén, for the groups β, γ, α, δ, ε.

RADIATIONS NOT IDENTIFIED

The condensations of which we have spoken do not constitute the whole of the comet's spectrum, for at least 12 other monochromatic images can be traced. All these are faint, though fairly well defined; and we were not able to determine their origin.

They may be described as follows:

TABLE IX

λ	Description
3629	Image of the head only
3641	Image of the head and tail
3721	Image of the head only
3932	Image of the head, with faint tail
3949.2	Image of the head, with faint tail
3969.1	Image of the head only
4023.4 4067.7	Condensation in the head and in the tail, extending in the interval comprised between these wave-lengths
4458.5	Image of the head only
4475.7	Image of the head only
4492.6	Image of the head only
4771.9	Image of the head and tail
5368.7	Image of the head only

ABSENCE OF A CONTINUOUS SPECTRUM

On none of our negatives could we find any trace of a continuous spectrum. It is true that on many plates there is a band extending over the images of the head, and having the appearance of a continuous spectrum. But a more careful examination of the negatives shows that in each case this band is nothing but the spectrum of a star, accidentally superposed on the spectrum of the head. The proof of this is that the band in question looks sharply delimited on both edges, and that its width is identical with that of the numerous stellar spectra which occur on the plate. Furthermore, the beginning of the band varies in position with the comet's spectrum on the various negatives, which shows plainly that it is in no wise related to the comet.

In certain cases, the superposition of a stellar spectrum on that of the comet's has helped us to detect certain faint condensations in the latter. It is a known fact that a feeble action of light produces a more visible effect on a slightly fogged plate than on one never having received any previous luminous action whatever.

The absence of a continuous spectrum is especially noticeable in the following four regions of our plates:

From λ	3800	3914	4415	4770
to λ	3840	4003	4470	4849

SPECTRUM OF SOME REMARKABLE PARTS OF THE TAIL

We know that the matter composing the tail of comet Morehouse was at one time the seat of violent movements. Frequently, luminous masses seemed ejected from the nucleus backward, and it was possible to follow their displacement in the tail. One of us has studied these movements and measured their speeds.[1] Now we deem it interesting to note that these luminous clouds, although coming from the head, did not show the spectrum of the head, when once in the tail; and thus it was that we could not detect in their spectrum the radiations either of cyanogen, or of carbon.

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[1] *Comptes Rendus*, 147, 1033, 1908.

POSSIBILITY OF CHANGE IN THE SPECTRUM

We have examined the whole series of our plates in order to see if any modifications occurred in the comet's spectrum. From October 14 to 28 no change could be noted. Later, however, on November 21, when we were able to obtain a new negative with spectroscope No. 2, we found that three images at λ 4458.5, 4475.7, 4492.6, which were previously blended in the same condensation, could now be neatly separated. But as this peculiarity is recorded on only one plate, we dare not assert that we have to deal here with a real change in the spectrum of the comet.

SUMMARY

The results of our investigations may be summed up as follows: the spectrum of comet Morehouse was composed: (*a*) of four band spectra, which we attribute: (1) to carbon monoxide; (2) to nitrogen; (3) to cyanogen; (4) to carbon; and (*b*) of some faint radiations which it was not possible to identify.

Cyanogen and carbon were present only in the head; while carbon monoxide and nitrogen were the elements constituting the whole comet. The spectrum of carbon monoxide at low pressure was the most conspicuous one; then came, in order of decreasing brightness, that of nitrogen, that of cyanogen, and, lastly, the spectrum of carbon.

In conclusion, we wish to express our indebtedness to M. C. Flammarion for having allowed us to mount our spectrographs on the equatorial of his Observatory at Juvisy, and to M. F. Quénisset for his kind collaboration in obtaining the plates.

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THE DISCOVERY OF ECLIPSING VARIABLE STARS

By JOEL STEBBINS

It has often been pointed out that all spectroscopic binary stars would be eclipsing variables for observers properly situated in space; also that in those spectroscopic binaries thus far discovered, the fainter components must be relatively of considerable size. It is known that those systems which appear as eclipsing variables from the earth are of low density as compared with our sun, but we should expect that the most tenuous systems would, as a class, furnish most of the eclipsing stars. Also, if in a certain number of short-period spectroscopic binaries, there is not a certain proportion of eclipsing stars, then it should be possible to assign a lower limit to the density of these objects as a class.

Let us first investigate the density of the stars of shortest period, assuming that we have to deal in each case with two separate bodies, approximately spherical in form, whose relative orbit is circular. The radius of the larger component may be taken as unity, irrespective of whether or not this body is the brighter or more massive of the pair. Further let

d_0 = the mean density of the system, referred to the sun,
κ = the radius of the smaller body,
a = the distance between centers,
R = the ratio of the astronomical unit to the sun's radius,
P = the period in days,
T = the earth's sidereal period of revolution.

Then we have the well-known equation

$$d_0 = \frac{a^3 T^2}{R^3 P^2 (1+\kappa^3)} ,$$

and introducing the numerical values, $T = 365.256$ days, and $1/R = \sin 15' \; 59''.63$,

$$d_0 = [8.1282] \frac{a^3}{P^2(1+\kappa^3)} , \qquad (1)$$

the constant being a logarithm. Equation (1) gives at once the mean density of a system when P, κ, and a are known, but it also determines the lower limit of density for any binary star of known period. Since by hypothesis we are concerned with two bodies, we have $a^3 > 1 + \kappa^3$ and therefore

$$d_0 > \frac{[8.1282]}{P^2}.$$

Putting $d_0 = 1$ we then find $P > 0.116$ days; or a binary system with a period of 0.116 days must be more dense than the sun. Since the density of air on the solar standard is 0.00093, it similarly follows that a binary which revolves in 3.80 days is more dense than air. These limits are on the assumption that the system is composed of an indefinitely small satellite revolving close to the primary, but if we assume $\kappa = 1$, the limiting periods are doubled, and become 0.232 and 7.60 days for the solar and atmospheric densities respectively. It is thus evident that though short-period stars may be quite rare, there is a lower limit to the density which is by no means comparable with the nebular condition.

In the cases just considered, the two components are close together, but if we increase a the computed density is much greater. Remembering that κ is always the radius of the smaller body, we have $1 + \kappa^3 < 2$, and in place of equation (1) we may write the inequality

$$\frac{a^3}{d_0} < \frac{2P^2}{[8.1282]}. \tag{2}$$

We now put $d_0 = 1$, and compute a for different values of P, which give

TABLE I

$P =$	0^d25	0^d5	1^d0	2^d0	5^d0	10^d0
$a <$	2.10	3.34	5.30	8.41	15.5	24.6

That is, a binary star with period of 0.25 day either has a mean density greater than that of the sun, or else the distance between centers is less than 2.10 times the radius of the larger body. The period of 0.25 day is not exorbitantly small, for in Campbell's

"Second Catalogue of Spectroscopic Binary Stars"[1] there are four stars with periods known to be less than one day, the shortest being that of *β Cephei*, which Frost found to be 0.1904 day. Also in Hartwig's latest list of variables[2] there are 14 stars of the *Algol* type with periods shorter than a day. There are in addition more than 30 stars of short period, and of the *Antalgol* type, with equally rapid changes, but while most of these sytems are no doubt spectroscopic binaries, it is not certain that the components are entirely separated.

At present it is conventional to include in the class of *Algol* variables all stars which are normally of constant brightness, but which suffer a diminution of light at regular intervals. Stars of the *β Lyrae* type have two unequal minima, and continuous variation in brightness. It is probable that future observations will demonstrate that very few of the *Algol* stars are perfectly constant at maximum light, and we should expect a series intermediate in type between a star with no secondary minimum, and one with two equal minima. In view of the results from radial velocity determinations, the general term *eclipsing variables* seems appropriate for all stars listed at present under the *Algol* or *β Lyrae* types.

The alternative that rapidly moving spectroscopic binaries must be either dense or close together, brings us to the question of the probability of eclipses of these stars as seen from the earth. Of course the magnitude of the eclipse depends upon the relative size of the fainter body. Whatever may be our preconceived notions on the subject, the fact is that, in those eclipsing systems where the relative dimensions have been inferred from a thoroughly determined light-curve, the "companion" has in most cases been found to be the larger. For the present purpose we shall not be far in error if we assume that in the short-period systems the components are of the same size. Stroobant[3] has pointed out that if the normal *Algol* system be considered as composed of a bright and a dark body with equal radii, and of relative separation, $a=8$, then the probability is 1/7.7 that the orbit of such a system, situated at random, would be inclined so as to give us an eclipse of 0.5 stellar magnitude, or more. In other words, for every *Algol* star that is discovered there must be about 7 other such systems whose orbit planes do not pass near the sun. The arbitrary limit of 0.5 magnitude was adopted by Stroobant because practically all of the known eclipsing stars have a range greater than this amount. While this limit is all right for the random discovery of *Algol* stars, an experienced visual observer can detect a much smaller change in the light of a star whose period is known, and it seems that a systematic study of spectroscopic binaries would undoubtedly result in the discovery of more eclipsing variables. With the selenium photometer attached to our 12-inch telescope the measures are ordinarily limited to stars brighter than the third magnitude, but for stars as bright as magnitude 2.0 it is no exaggeration to state that a variation of 0.10 magnitude may be called conspicuous. Recently we made an observation which was discordant by 0.07 magnitude from previous results for the same star, and as will be shown later this enabled us to prove that the star is a variable. For the purpose then we may adopt the limiting amount of eclipse to be such that a star with a dark companion will vary by 0.10 magnitude, and if the components are of equal brightness, the same inclination of orbit will produce a range of about 0.05 magnitude, which is still within the power of the instrument.

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[1] *Lick Observatory Bulletin*, **6**, 17, 1910.

[2] *Vierteljahrsschrift der astronomischen Gesellschaft*, **45**, 341, 1910.

[3] *Bulletin de l'Académie royal de Belgique*, 1909, 329.

In a random distribution of the orbit planes we have to determine the probability that the inclination will lie between 90° and i, where i is the limiting angle for an eclipse of assigned amount. The area of the zone of the hemisphere, within which one of the poles of the orbit must lie, is equal to cos i times the total area of the hemisphere. Hence if we compute the value of i necessary for an eclipse of assigned magnitude, the probability that such an eclipse or one greater will occur is given by cos i. Consider the projected disks of the two bodies, and let

M_0 = the obscured area of the bright disk at minimum,
J_0 = the ratio of the light at minimum to that at maximum,
$2\eta_0$ = the angle at the center of the bright disk subtended by the common chord.

For $\kappa = 1$ the following equations are readily established:

$$2\eta_0 - \sin 2\eta_0 = M_0 = \pi(1 - J_0) ,$$

$$\cos i = \frac{2\cos \eta_0}{a} .$$

On the assumption of one perfectly dark component and a loss of 0.10 magnitude, we have $J_0 = 0.912$, and with $a = 5$ there follows $i = 70\overset{\circ}{.}8$, cos $i = 0.33$. Therefore, of those binaries with equal radii and $a = 5$, one third are eclipsing systems whose minima may be detected. For $a = 10$ we find cos $i = 0.16$, and the proportion of variables is one-sixth. If we compare these results with Table I, it is evident that there must be eclipsing stars among the binaries of short period, unless the systems are quite dense, or the companions very small. We now put these results in the form of three alternative statements, some one of which must be true.

1. Among the spectroscopic binaries of very short period there are a considerable number of eclipsing variables, which have a range of 0.05 magnitude or more.
2. These systems as a class are more dense than the sun.
3. The fainter components are much smaller than the bright components.

Let us now go back and note upon what tacit assumptions these alternatives are based. Equation (1) holds in general only for spherical bodies with a symmetrical distribution of density, but if two bodies are near enough to be distorted from the spherical form, the value of a would be so small as to produce a high probability of eclipses. In the more exaggerated cases of two stars in contact or partially coalesced, the systems which vary in light must be the rule rather than the exception. It is well known that all of the *Algol* stars whose spectra have been determined are of "early" type, B, A, and a few of class F, according to the Harvard classification. Likewise it is known that the spectroscopic binaries of shortest period are of the same spectral types. If the density of the *Algol* stars is typical of others of the same spectrum, we cannot admit a high density for the rapid spectroscopic binaries. To the other alternative, that the fainter bodies are much smaller than the primaries, we have the objection already mentioned, that where the relative size can be determined the companion is usually the larger body. Also where the spectra of both components are visible, we apparently have stars of nearly the same size, though it seems to be the rule that the fainter body is the less massive. However, in systems which show only one spectrum and which have large values of $a \sin i$, there cannot be a large disparity in the masses without giving the primary an enormous mass as compared with the sun. In the case of *δ Orionis*, where $P=5.73$ days, and $a \sin i=7{,}900{,}000$ km, if we assign as low a relative mass as 1/4 to the secondary, there results for the bright body, $m \sin^3 i=60\odot$.

The discovery of eclipsing systems among spectroscopic binaries is a perfectly definite problem of observation. For any star the time of principal minimum is easily predicted from the spectroscopic elements, being the instant when $u=90°$, and if two spectra are visible there will ordinarily be a second minimum at $u=270°$. It is natural to look for such variables among the orbits which have a large $a \sin i$; in fact, the size of this element is perhaps an additional measure of the probability of an eclipse.

Another consideration is that there is a strong tendency for stars of the *Algol* type to occur near the galaxy. This is true of the 88 stars in Hartwig's list for 1911, almost none of which are far from the Milky Way. I learn from Professor Pickering that a study of the distribution of all known variables is being completed at Harvard, and no doubt this peculiarity of the location of eclipsing variables will be fully discussed. Since there is known to be a preponderance of first-type stars in the Milky Way, and the few *Algol* variables of known spectra are in the same class, the general spectral type of the *Algol* stars is doubtless indicated by their preference for the galaxy.

Whatever weight the foregoing conclusions have in the judgment of others, to myself they seem well founded, for the good reason that the first *two* stars for which I was able to make a thorough test have turned out to be eclipsing variables. In fact, it was this surprising circumstance which led me to consider the probability of eclipses being visible from the earth. The first star, to which reference has already been made, is *β Aurigae*, the results for which are in the following article; the second is the well-known suspected variable δ *Orionis*. The study of the latter is not completed, but it may be stated that there are two unequal minima observed close to the times predicted from Hartmann's spectroscopic elements. The extreme range of δ *Orionis* is not far from 0.10 magnitude, and as I intend to continue observing this star until a thorough light-curve is obtained, this statement as to its variation is sufficient for the present. Of course we have other stars on our observing program, but during the past season we have been more concerned with the determination of good light-curves than with the wholesale search for new variables. It may be worth while to call attention to a favorable case among the bright stars, namely, α *Virginis*. This star is *H. R.* 5056, magnitude 1.21, spectral type B2, period 4.01 days, two spectra visible, and (a_1+a_2) sin $i=18,000,000$ km. Unfortunately there are two obstacles in the way of an exhaustive study of α *Virginis*, the lack of a suitable comparison star, and the close approach of the period to 4 days. At the present writing the hypothetical eclipses occur in the day time, and it will be several months before they take place while the star is near the meridian during the night.

It is always dangerous to predict the nature of future discoveries, but from the preceding considerations it is evident that the spectroscopic binaries furnish the means of a new departure in photometric observation. At present the discoverers of variable stars examine presumably hundreds of objects for each new variable that is found. It appears that a systematic study of the spectroscopic binaries at the favorable epochs would result in the discovery of at least a few eclipsing systems, though it is not probable that the writer or anyone else will repeat the experience of finding two new variables in the first two stars observed.

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A NEW BRIGHT VARIABLE STAR, β *Aurigae*

By JOEL STEBBINS

In the spring of 1910, when the study of the light-curve of *Algol* with the selenium photometer was nearing completion, I made a list of bright stars which are known to be short period spectroscopic binaries, and among those which seemed to promise a possible variation in light was β *Aurigae*, *H.R.* 2088, magnitude 2.07. This star was then too low in the west for satisfactory measures, and it was necessary to wait six months, until September 20, 1910, when observations were begun and continued every clear night for a month without finding any change of significance. On October 23 I was surprised by a good observation which placed the star 0.07 magnitude fainter than it had been on previous nights. The spectroscopic period being 3.96 days, if this were an eclipse it would be repeated two nights later, since the spectrum plates reveal two components of about equal intensity. This discovery was followed up, and the eclipses were observed at every opportunity. I had intended to measure the star's light throughout the entire period of the orbital variation, and it happened that the times of eclipse were not computed until after the discovery with the photometer. When the computation was made, and it was seen that the minima were occurring close to the predicted times, it was determined to do everything possible to secure the best light-curve. Here at last is the ideal case for which astronomers have been looking, an eclipsing variable star with the spectra of both components visible, and for which there are reliable determinations of parallax. A good light-curve would give us not only the absolute diameters of the components, but also the surface intensity of each. The spectrum of β *Aurigae* is Class A p, and up to the present it has not been directly determined whether the first-type stars have surfaces which per unit area are more or less luminous than the sun.

The method of observation with the selenium photometer has been described in a former paper,[1] and since that work only a few minor improvements have been made, which, however, have increased the accuracy of the results. In the first arrangement, the extra-focal star image, 7 mm in diameter, was made to fall upon a selenium surface of 26×18 mm, and the guiding was by means of the regular finder of the 12-inch refractor. The holder of the selenium cell has since been made anew, and the guiding is now accomplished by means of a small total reflecting prism, which is inserted in the center of the converging beam, about 16 cm in front of the focus. This prism intercepts not more than 2 per cent of the light, and an eyepiece and crosswires placed in the direction of the reflected rays furnish a satisfactory means of keeping the main star image at the same position on the selenium. With the better guiding system it was safe to diaphragm the effective selenium surface to a diameter of 10 mm, which reduces the effect of the sky near full moon. The arrangement of the ice-pack about the selenium has also been modified, with the result that the temperature control is improved, especially in warm weather.

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[1] *Astrophysical Journal*, 32, 185, 1910.

Another change from previous practice was to alternate more rapidly from star to star. On *Algol* the normal plan was to make exposures upon the comparison star and variable to the number of 4, 8, and 4, while for *β Aurigae* this was changed to 2, 6, and 2 exposures. The brighter the star the fewer the readings that are required to measure its light with the same probable error, but it was with considerable hesitation that I decided upon a schedule requiring more time upon one star than the other. In the long run, if the same time is allotted to each star, there is an equal probability of passing cloud or haze affecting the measures of each object, but if more time is consumed in observing the variable, there is a possibility of a preponderance of errors in one direction. In the present work, a set of measures, 4 exposures on the comparison star and 6 on the variable, required a minimum of 12 minutes, or we observed at the rate of 5 sets per hour. As compared with visual observations this is something of a snail's pace; but as I have noted before, if we could admit occasional errors of one- or two-tenths of a magnitude, bright stars could be measured with the selenium photometer at the rate of one star per minute, or as fast as a telescope can be manipulated.

The adopted comparison star for *β Aurigae* is *β Tauri*, which is 17° distant, but this disadvantage is in part offset by the high altitude at which these stars culminate, within 5° and 12° of the zenith. The correction for absorption was based upon Mueller's Potsdam table,[1] which for zenith distances up to 70° may be represented within 0.01 magnitude by the formula

$$\text{Reduction to zenith} = 0.23 \text{ mag. } (\sec z - 1).$$

The zenith distance of each star was computed for each half hour of sidereal time, and the absorption correction is given in Table I.

TABLE I

Sidereal Time	Differential Absorption	Sidereal Time	Differential Absorption
	mag.		mag.
$1^{h}0$	+0.013	$5^{h}0$	0.000
1.5	+0.006	5.5	+0.001
2.0	0.000	6.0	+0.004
2.5	−0.002	6.5	+0.006
3.0	−0.003	7.0	+0.011
3.5	−0.003	7.5	+0.018
4.0	−0.002	8.0	+0.027
4.5	0.000	8.5	+0.037

It is evident that under favorable conditions these two stars may be compared continuously for more than six hours, with no correction for absorption as great as 0.020 magnitude. However, it is my experience that the average night at this station has a coefficient of absorption somewhat greater than 0.23 magnitude, and the tabular correction has been multiplied by a factor varying from 1.0 to 2.0, which was derived from additional observations of *β Orionis*, taken for the purpose on all nights when the differential absorption was more than 0.003 or 0.004 magnitude. In all cases the observations were discontinued at sidereal time 8.0 hours. Whatever method is used to eliminate or correct for the absorption, there is always the danger from sudden changes in transparency of the air, and in spite of our utmost care I believe that a considerable portion of the outstanding errors in the results for *β Aurigae* are due to atmospheric causes. One of the defects of the selenium photometer

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[1] *Die Photometrie der Gestirne*, Leipzig, 1897, p. 515.

TABLE II

OBSERVATIONS OF β *Aurigae* NEAR MINIMA

Date	G.M.T.	Phase	Difference of Magnitude	Absorption	Remarks
1910			mag.	mag.	
October 1........	$20^{\mathrm{h}}42^{\mathrm{m}}$	$-5^{\mathrm{h}}85$	0.366	−0.003	
	21 14	− 5.32	.356	− .002	3 sets
October 7........	21 00	43.43	.361	− .001	
	21 33	43.98	.347	.000	3 sets
October 9........	20 31	− 4.10	.320	− .002	
	21 02	− 3.58	.315	− .001	3 sets
October 23........	20 36	46.90	.426	.000	
	21 07	47.42	.423	.000	3 sets
October 25........	20 56	0.18	.406	.000	
	21 22	0.62	.392	+ .002	
	21 52	1.12	.405	+ .005	
	22 22	1.62	.386	+ .008	
	22 48	2.05	.402	+ .014	
	23 13	2.47	.369	+ .019	
October 27........	17 20	44.58	.338	+ .002	
	17 52	45.12	.331	− .002	
	18 18	45.55	.390	− .002	
	18 43	45.97	.419	− .003	
	19 08	46.38	.405	− .002	
	19 33	46.80	.458	− .001	
	19 58	47.22	.434	.000	
	20 24	47.65	.472	.000	
	20 55	48.17	.415	+ .001	3 sets
November 2......	16 14	− 2.58	.348	+ .023	
	16 38	− 2.18	.364	+ .011	
	17 03	− 1.77	.374	+ .002	
	17 29	− 1.33	.430	− .003	
	17 55	− 0.90	.444	− .005	
	18 20	− 0.48	.459	− .006	
	18 46	− 0.05	.413	− .005	
	19 26	0.62	.425	− .002	
	20 07	1.30	.411	.000	
	20 32	1.72	.380	+ .002	
	20 58	2.15	.398	+ .005	
	21 25	2.60	.354	+ .007	
	21 54	3.08	.382	+ .011	
	22 24	3.58	.358	+ .016	
	22 50	4.02	.336	+ .024	
November 8......	17 51	48.02	.435	− .006	3 sets
November 10......	16 14	− 0.65	.406	+ .006	
	16 40	− 0.22	.450	.000	
	17 06	0.22	.476	− .003	
	17 36	0.72	.429	− .004	
	18 03	1.17	.412	− .004	
	18 28	1.58	.390	− .004	
	18 54	2.02	.374	− .001	
	19 20	2.45	.364	.000	
	19 45	2.87	.366	.000	
	20 12	3.32	.348	+ .002	
	20 41	3.80	.360	+ .006	
	21 10	4.28	.360	+ .010	

TABLE II—*Continued*

Date	G.M.T.	Phase	Difference of Magnitude	Absorption	Remarks
			mag.	mag.	
1910 November 10......	$21^h 37^m$	$4^h.73$	0.378	+ .016	
	22 10	5.28	.363	+ .027	3 sets
November 24......	15 41	49.70	.386	.000	
	16 06	50.12	.366	− .001	
	16 31	50.53	.366	− .002	
	16 56	50.95	.367	− .003	
	17 22	51.38	.351	− .002	
	17 47	51.80	.380	− .001	
	18 12	52.22	.356	.000	
	18 38	52.65	.353	.000	
	19 04	53.08	.360	.000	
1911 February 9........	13 47	− 5.07	.378	.000	
	14 12	− 4.65	.358	+ .002	
	14 38	− 4.22	.378	+ .004	
	15 03	− 3.80	.348	+ .006	
	15 28	− 3.38	.330	+ .012	
	15 53	− 2.97	.396	+ .017	
	16 18	− 2.55	.386	+ .026	
February 15.......	12 36	42.68	.376	− .001	
	13 02	43.12	.379	.000	
	13 26	43.52	.432	+ .001	
	13 51	43.93	.318	+ .004	
	14 16	44.35	.319	+ .009	
	14 41	44.77	.398	+ .012	
February 21......	13 08	− 2.88	.388	.000	3 sets
February 23......	12 42	44.68	.341	.000	
	13 06	45.08	.370	+ .001	
	13 31	45.50	.354	+ .003	
	13 58	45.95	.352	+ .005	
	14 22	46.35	.476	+ .008	
	14 48	46.78	.418	+ .012	
	15 14	47.22	.442	+ .019	
	15 38	47.62	.434	+ .026	
March 13.........	12 55	1.67	.382	+ .008	
	13 20	2.08	.371	+ .012	
	13 45	2.50	.366	+ .020	
	14 16	3.02	.373	+ .032	
March 19.........	13 13	50.90	.330	+ .012	
	13 38	51.32	.341	+ .018	
	14 02	51.72	.342	+ .026	
March 23.........	13 18	51.95	.344	+ .025	
	13 44	52.38	.372	+ .035	

OBSERVATIONS BETWEEN MINIMA

1910 September 20.....	21 02	15.58	.361	− .005	
27.....	20 49	88.35	.343	− .003	
28.....	20 50	17.33	.343	− .005	
29.....	20 55	41.42	.350	− .005	
October 6.....	20 52	19.28	.360	− .002	
16.....	20 57	69.32	.344	.000	

TABLE II—*Continued*

Date		G.M.T.	Phase	Difference of Magnitude	Absorption	Remarks
1910				mag.	mag.	
October	18.....	$21^{h}01^{m}$	$22^{h}33$	0.344	0.000	
	28.....	20 23	71.65	.348	.000	
	30.....	20 11	24.42	.343	.000	
November	21.....	19 33	76.60	.346	+ .004	
	25.....	20 38	78.65	.346	+ .021	
	29.....	16 29	75.45	.334	− .004	
	29.....	17 32	76.50	.365	− .001	
December	7.....	15 57	76.83	.365	− .004	
	13.....	14 47	29.58	.329	− .001	
	20.....	14 57	7.68	.363	− .003	
	20.....	16 01	8.75	.382	− .002	
	24.....	16 25	10.12	.342	.000	3 sets
	26.....	13 54	55.60	.350	− .001	
	26.....	14 58	56.67	.366	− .003	
	26.....	16 01	57.72	.351	− .001	
1911						
January	3.....	15.52	59.47	.363	.000	
	3.....	16 54	60.50	.356	+ .004	
	4.....	18 03	85.65	.353	+ .013	
	8.....	14 10	82.72	.352	− .003	
	8.....	15 17	83.83	.355	− .001	
	22.....	14 52	39.23	.343	.000	
	22.....	15 57	40.32	.350	+ .006	
	23.....	14 43	63.08	.352	.000	
	23.....	16 01	64.38	.342	+ .008	
	23.....	17 07	65.48	.341	+ .026	
	29.....	16 39	18.93	.332	+ .015	
February	1.....	14 06	88.37	.348	+ .001	
	10.....	13 58	19.12	.345	+ .002	
	20.....	13 07	68.17	.337	+ .001	6 sets
	22.....	13 09	21.13	.345	+ .002	6 sets
	22.....	14 24	22.38	.366	+ .008	6 sets
	24.....	13 03	69.03	.349	+ .001	
	24.....	15 02	71.02	.320	+ .017	
March	2.....	14 24	24.28	.364	+ .013	6 sets
	14.....	13 17	26.03	.344	+ .013	4 sets
	14.....	14 08	26.88	.322	+ .030	4 sets
	22.....	13 33	28.20	.355	+ .023	

is that stars must be observed in succession rather than simultaneously, but I am sure that we have not reached the limit of accuracy attainable on good nights, or even on fair nights if the stars are close together.

In Table II are listed the observations of *β Aurigae* taken between September 20, 1910, and March 23, 1911. The times were reduced to the sun, and the phase based upon Baker's spectroscopic elements, from which the computed time of eclipse is 1910 October

21, 21^{h} 46^{m} G.M.T. The difference of magnitude is always in the sense: magnitude of *β Aurigae* minus magnitude of *β Tauri*. The correction in the absorption column has already been applied to the difference of magnitude. A set of measures comprises 6 exposures on the variable, and 4 on the comparison star, and near minima each tabulated difference of magnitude is the mean of 2 such sets, except as noted, while between minima each result is usually the mean of 5 sets.

Before the observations were completed, it became evident that the measures at large west hour angles gave residuals in one direction, proving that the average absorption correction was too small. This was investigated by first discussing those observations between minima where the correction was not more than 0.010 magnitude. These selected results were arranged according to phase and grouped into the normals of Table III. For this purpose the observations from 4 to 6 hours from minima were also used, as it was seen that the eclipses lasted only about 3 hours on each side of minimum. It was assumed that the two branches of the entire curve are the same, and one-half the period, or $47^{h}52$, was subtracted from the phase of each observation following the second minimum. In Table III, the branch of the curve is indicated in the first column, and the last column gives the residuals from a comparison with the final light-curve.

TABLE III

NORMAL MAGNITUDES BETWEEN MINIMA, SMALL ABSORPTION

Branch of Curve	Phase	Difference of Magnitude	Absorption	O.−C.
		mag.	mag.	mag.
I, II.........	$5^{h}89$	0.358	+0.001	+0.001
I............	8.85	.362	− .002	+ .006
II...........	10.43	.360	− .001	+ .005
II...........	15.13	.350	+ .004	− .001
I............	17.34	.350	− .003	.000
I............	20.91	.350	.000	+ .001
II...........	21.32	.343	+ .001	− .006
I............	25.46	.346	+ .002	− .002
II...........	27.01	.349	− .002	.000
II...........	31.20	.354	− .001	+ .003
II...........	39.33	.349	− .001	− .007
I............	40.32	.348	.000	− .009
I, II.........	42.76	.363	.000	+ .005

The magnitudes in Table III are practically free from the effect of absorption, and show a decided variation in the light of β *Aurigae* between minima, caused presumably by the two components being elongated in the line joining them. If we assume that each body is a prolate spheroid with major axis coincident with the line of centers, this variation may be expressed by the formula

$$J = J_0 \sqrt{1 - \epsilon^2 \sin^2 i \cos^2 \phi}\ , \qquad (1)$$

where J is the light intensity at any instant, J_0 the intensity at maximum, ϵ the eccentricity of figure of the ellipsoids, i the inclination of the orbit, and ϕ the phase angle or true anomaly measured from minimum. For each normal magnitude of Table III, it is easy to compute the relative light, J, and then solve for the two unknowns J_0 and $\epsilon \sin i$, $\sin i$ being nearly unity; but in the present case it is sufficient to use the approximate relation

$$m = m_1 + \frac{c}{2}(1 - \cos 2\phi)\ , \qquad (2)$$

where m is the magnitude at any time, m_1 the hypothetical magnitude at conjunction if there were no eclipse, and c the ellipticity of the spheroids, $c = \frac{a-b}{a}$ in an ellipse with semi-axes a and b. It is not necessary to convert magnitudes into light-intensities because of the convenient relation that 1 per cent change in light corresponds very nearly to a variation of 0.01 magnitude, more exactly 0.0108 magnitude. Each normal gives an observed m in (2), and forming 13 observation equations, then the normal equations, and solving, we have

$$\begin{aligned} m_1 &= 0.359 \text{ mag.} \pm 0.002 \text{ mag.} \\ c &= 0.011 \qquad \pm 0.003 \end{aligned}$$

This determination of c is the most difficult photometric measurement I have yet attempted, but there is no doubt that the observations show this ellipticity. On the basis of constant light between minima the residuals give $[vv] = 0.000488$, and the addition of the term in c reduces this to $[vv] = 0.000268$. The probable error of one normal magnitude of Table III is ±0.003 magnitude, which is sufficiently small to justify the attempt to evaluate the ellipticity. In view of all the facts we may adopt

$$c=0.01$$

without assigning a probable error.

The adopted light-curve between minima was computed from m_1 and c substituted in (2), and is shown in Fig. 1. Each magnitude of Table II was next compared with the curve, and it was found that the results with large absorption may be brought into accordance with the others by multiplying the correction by 1.35. Therefore all of the magnitudes in Table II were altered by the addition of 0.35 times the first absorption correction.

In Table IV are the adopted normal magnitudes, and in each case the absorption is the average total correction which has been applied. Near minima each of the normals is ordinarily the mean of 3 magnitudes of Table II, or a total of at least 6 sets of measures, while the normals between minima comprise about 15 sets. The last column gives the residuals from a comparison with the final computed light-curve.

Although a few of the normals comprise an extra number of sets, there seems no good reason for assigning them greater weight. Observers without experience with the selenium photometer can hardly realize the idiosyncrasies of this instrument. In most classes of work one measure is about equal to another taken under the same conditions, but with selenium if we assign weights inversely as the squares of the probable errors, we find that some observations should have at least 10 times the weight of others. The question is wholly the disturbance going on within the selenium, and it is not at all unusual to find the probable error of a single measure as small as 1 per cent, and then a few minutes later to see the errors rise to 2, 3, and occasionally 5 per cent. Again, it is evident that one observation with a clear sky is worth a score of measures taken through passing cloud, haze, or smoke, and though a most careful watch was kept of the sky, as well as of the successive galvanometer readings, it is almost certain that some of the measures are impaired by changes in the absorption. For these reasons I have not made a fine point of assigning weights, and the adopted normals between minima or near minima are considered as all about equally reliable.

TABLE IV

Normal Magnitudes near Minimum I

Phase	Difference of Magnitude	Absorption	O.–C.
h	mag.	mag.	mag.
−4.31	0.355	+0.004	−0.003
−2.70	.381	+ .019	+ .017
−1.76	.391	+ .005	− .003
−0.68	.437	− .002	+ .008
−0.03	.422	− .002	− .013
+0.49	.431	− .001	− .001
+1.00	.415	− .001	− .006
+1.50	.396	+ .002	− .009
+1.80	.380	+ .004	− .013
+2.09	.394	+ .014	+ .012
+2.47	.371	+ .018	+ .001
+2.83	.369	+ .017	+ .009
+4.05	.366	+ .020	+ .008
Near Minimum II			
43.88	0.362	+0.004	+0.003
45.23	.352	+ .001	− .023
45.82	.387	.000	− .010
46.50	.435	+ .008	+ .014
46.97	.439	.000	+ .008
47.42	.436	+ .017	+ .001
47.98	.436	− .003	+ .004
50.12	.372	− .001	+ .005
51.90	.358	+ .014	.000
Between Minima			
8.85	0.362	−0.002	+0.006
17.28	.346	+ .002	− .004
19.84	.350	+ .001	+ .001
23.35	.356	+ .007	+ .008
27.67	.343	+ .022	− .006
40.32	.348	.000	− .009
56.66	.355	− .002	− .001
61.02	.357	+ .002	+ .004
66.01	.344	+ .016	− .006
70.26	.342	+ .006	− .006
76.34	.352	− .002	+ .003
81.73	.353	+ .008	+ .001
87.46	.349	+ .005	− .007

In general the observations during the eclipses were continued for many hours on a few nights, whereas the results between minima depend upon shorter periods of observing on a greater number of nights, and it is therefore impossible to make the entire series homogeneous. Roughly speaking, each normal near minima is the result of something more than one hour's continuous observing, and between minima the normals represent about 3 hours of work. These happen to be the approximate times required for securing the similar observations of *Algol*, and it may be of interest to compare the probable errors of the previous work with those found from the residuals of Table IV.

TABLE V

	β Aurigae	*Algol*
	mag.	mag.
Probable error of a normal magnitude near minima...........	±0.007	±0.023
Probable error of a normal magnitude between minima	±0.004	±0.006

The Harvard magnitudes are *β Aurigae*, 2.07, *Algol*, 2.1; while the Potsdam values are *β Aurigae*, 2.21, *Algol*, 2.4; and therefore the new variable is slightly brighter than *Algol* itself, though not enough to increase the accuracy of the results. The larger errors near minimum for *Algol* were due to the greater diminution of light, and the residuals between minima depended upon a light-curve which is more complicated than that of *β Aurigae*. Nevertheless, it is certain that the present work represents a substantial increase in accuracy over the former, and this is especially true of the selected normals of Table III, where as noted the probable error is ±0.003 magnitude. The residuals which have been discussed were of course derived from the light-curve computed from the adopted elements, which we now proceed to obtain.

THEORY OF THE SYSTEM OF *β Aurigae*

We may assume without further argument that *β Aurigae* is an eclipsing system, especially since the observed minima agree closely with the times predicted from the spectroscopic elements. This star, the binary character of which was discovered at Harvard, has been studied spectroscopically for 20 years, and the results have been ably brought together and summarized by Dr. R. H. Baker.[1] As his results will be used from time to time in what follows, they are given here in such form as needed.

[1] *Publications of the Allegheny Observatory*, I, 163, 1910.

SPECTROSCOPIC ELEMENTS BY BAKER

$P = 3.960027$ days $+ 0.000010\ (t - 1906)$

$e = 0.00$

$T = 1905$ September 11.7324 G.M.T.

$(a_1 + a_2) \sin i = 11{,}981{,}000$ km

$$m_1 = \frac{2.21\odot}{\sin^3 i}$$

$$m_2 = \frac{2.17\odot}{\sin^3 i}$$

We are first concerned with the variable period, P. Since T refers to the passage of the more massive component through the ascending node, eclipse I will take place in the circular orbit at the time $T + \frac{P}{4}$. This time was brought forward to October 21, 1910, with the period 3.960052 days, and the light-ephemeris computed upon the basis of a present period, 3.960077 days. The phase of the photometric observations was reduced to the sun, though this correction was presumably neglected in deriving the spectroscopic elements. After plotting the normals of Table IV, I was quite unable to assign any time of minimum better than that predicted, any discrepancy being certainly not more than one- or two-tenths of an hour, and we may therefore conclude that the spectroscopic and photometric results are in entire accordance. Assuming Baker's period, the light-elements of *β Aurigae* are:

Minimum I $= 1910$ October 21, $21^{h}\ 46^{m}$ G.M.T. $+ 3^{d}\ 23^{h}\ 02^{m}\ 30\overset{s}{.}7 \cdot E$
$= $ J. D. $2418966.907 + 3\overset{d}{.}960077\ E$
Min. II $-$ Min. I $= 1\overset{d}{.}980 = 47\overset{h}{.}52$,

where minimum I refers to the eclipse of the more massive component.

We have now to determine the elements of an *Algol* variable from an eclipse variation of less than 0.08 magnitude, which, to the writer at least, seemed a difficult task. It is interesting to note what might have been our conclusions concerning this system, if our eyes were, say, ten times as sensitive to light-changes, and this variable had been discovered before the advent of the spectroscopic measures. Assuming a bright body and a small dark companion, the observations can be well represented by a period of revolution of 1.98 days and a radius of the satellite about 1/4 that of the primary. It is not probable that the approximate elements of other *Algol* variables are so much in error, for an eclipsing star with equal minima cannot have a range of more than 0.75 magnitude, and most of the *Algol* stars thus far discovered have a variation more than a whole magnitude.

It is evident that almost any hypothesis as to the relative radii can be made to fit the observations, but there are considerations which seem to show that the components are of the same size. The spectroscope reveals that the masses are nearly equal, also that the spectra are of the same type, and so nearly of the same intensity that various observers have made mistakes in identification. The photometric observations make the two minima equal within 0.01 magnitude, which means the same surface-intensity for either component. From the equality of the spectra we may conclude that the total light from each body is the same, and dividing total light by surface-intensity, the radii must be equal, within say 5 or 10 per cent. With this simplification of the problem, since the orbit is circular, the only elements to be found are a, the radius of the relative orbit, and i, the inclination. In all strictness the unit for a is the shortest radius of either body. Consider the projected disks, which are assumed as circular during an eclipse, and let

P = the period,
t = the time in hours from minimum,
ϕ = the true anomaly, counted from minimum,
θ = the projection of ϕ in the apparent orbit,
ρ = the distance between centers of the projected disks,
2η = the angle subtended at each center by the common chord,
M = the area common to the two disks,
J = the light sent to the earth, the unit being the sum of the total light of the two components.

To obtain preliminary elements we adopt a value for the light, J, at minimum, and also for the duration of the eclipse. Let a zero subscript represent the value of each quantity at minimum, and ϕ_1 the value of ϕ at first or last contact, and we have for the determination of a and i:

$$
\begin{aligned}
2\eta_0 - \sin 2\eta_0 &= M_0 = 2\pi\,(1 - J_0)\ , \\
a &= \frac{2\sqrt{1-\cos^2\eta_0\,\cos^2\phi_1}}{\sin\phi_1}\ , \\
\cos i &= \frac{2\cos\eta_0}{a}\ .
\end{aligned}
\tag{3}
$$

The equations for the light-ephemeris from adopted elements are

$$
\begin{aligned}
\phi &= \frac{360^\circ}{P}\,t\ , \\
\operatorname{ctn}\theta &= \cos i \operatorname{ctn}\phi\ , \\
\rho &= \frac{a\sin\phi}{\sin\theta}\ , \\
\cos\eta &= \frac{\rho}{2}\ , \\
M &= 2\eta - \sin 2\eta\ , \\
J &= 1 - \frac{M}{2\pi}\ ,
\end{aligned}
\tag{4}
$$

and the observation equations for the corrections, δa and δi, from the residuals, δJ, take the form

$$\frac{\rho\sin\eta}{\pi a}\,\delta a - \frac{a^2\sin 2i\cos^2\phi\sin 2\eta}{2\pi\rho}\,\delta i = \delta J\ .$$

As there are only two unknowns, the least-square solution is very easy.

Applying these equations to the case in hand, the normals near the two minima were combined into normals for a composite minimum, and the magnitudes were converted into observed light, J, the unit being the light for difference of magnitude, $m_1 = 0.359$. The eclipse was assumed to last 6 hours, or $\phi_1 = 11\overset{\circ}{.}36$, and the light at minimum, $J_0 = 0.930$. The values of a and i thus derived were seen to be very close to the best that the data would yield, but the corrections δa and δi were computed from (5), and the two sets of elements are in Table VI.

TABLE VI

ELEMENTS OF β *Aurigae*

	I	II
a	6.83	6.86±0.35
i	77°.22	77°.19±0°.48
$[vv]$	0.000453	0.000392

The weights and probable errors of a and i were computed in the usual way from the coefficients of the normal equations, and the final residuals.

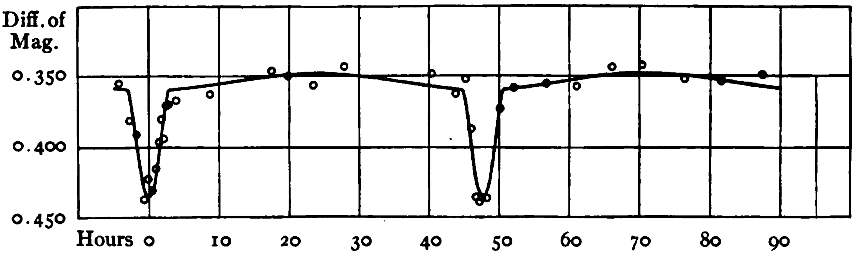


FIG. 1.—The light-curve of β *Aurigae*

The next step was to compute the light-curve during eclipse, and this is given in Table VII, together with that part of the curve between minima which was previously derived. The computed duration of each minimum is $5^h.90$, and since the two branches of the curve are assumed to be identical, the phase for the second half is obtained by adding $\frac{P}{2}=47^h.52$ to the phase in the table.

TABLE VII

LIGHT-CURVE OF β *Aurigae*

PHASE	DIFFERENCE OF MAGNITUDE		PHASE	DIFFERENCE OF MAGNITUDE	
	From β *Tauri*	From Minimum		From β *Tauri*	From Minimum
h.	mag.	mag.	h.	mag.	mag.
±0.0	0.435	0.000	10.0	0.355	0.080
±0.5	.432	.003	15.0	.351	.084
±1.0	.421	.014	20.0	.349	.086
±1.5	.405	.030	23.76	.348	.087
±2.0	.385	.050	25.0	.348	.087
±2.5	.369	.066	30.0	.350	.085
±2.95	.359	.076	35.0	.353	.082
+5.0	.358	.077	40.0	.357	.078

This light-curve is shown in Fig. 1, where each plotted circle represents a normal magnitude of Table IV.

COMPARISON WITH THE SUN

From the spectroscopic data it is a simple matter to express the dimensions of *β Aurigae* in terms of the sun, whose radius is taken as 695,000 km. From $i=77^\circ.19$ the true value of a_1+a_2 is 12,280,000 km, from which follow the quantities in Table VIII.

TABLE VIII

R	Radius of each component	2.58 ⊙
m_1	Mass of first component	2.38
m_2	Mass of second component	2.34
d	Density of each component	0.14

These results show nothing extraordinary about the system of *β Aurigae*. The radii and masses are not extreme, and the density is comparable with that of other *Algol* stars. It should be noted that the mean density, 0.14, follows also from the light-curve without reference to the spectroscopic measures.

We are now ready to consider what is perhaps the most important element of all, the surface-intensity of each component. This requires a knowledge of the absolute annual parallax of *β Aurigae*, and in Table IX are the results adopted by Kapteyn and Weersma.[1]

TABLE IX

PARALLAX OF *β Aurigae*

Observer	π	Weight
Flint	0″.000	2
Tikhoff	+0″.023	3
Adopt	+0″.014	

This small parallax is perhaps confirmed by the researches of Hertzsprung[2] and Ludendorff[3] on the extended system of *Ursa Major*.

---

[1] *Publications of the Astronomical Laboratory at Groningen, No. 24*, 1910.

[2] *Astrophysical Journal*, **30**, 135 and 320, 1909.

[3] *Astronomische Nachrichten*, **183**, 113, 1909.

Ludendorff derived from the proper motion of β *Aurigae* the parallax, $\pi=0''.023$, and from the radial velocity, $\pi=0''.019$. Of course these values of the parallax depend upon the assumption that β *Aurigae* is moving parallel to and with the same velocity as the other members of the group. It would be interesting indeed if we could generalize as to the characteristics of the others from the results for β *Aurigae*, but this star is quite isolated from the remaining members, having the least parallax, and the greatest computed luminosity of all.

We might adopt $\pi=0''.02$, but it seems better to assign $\pi<0''.03$ and compute the lower limit of luminosity from the equation

$$\log 2l=-2 \log \sin \pi-0.4 \ (M-S) ,$$

where l is the luminosity or total light of each component, π the parallax, M and S the stellar magnitudes of the star and sun, which we adopt as 2.07 and −26.6 respectively. Representing the surface-intensity of each body by σ, there follows

$$l > 80\odot ,$$
$$\sigma > 12\odot ,$$

or, roughly speaking, each component of β *Aurigae* emits per unit area almost certainly more than 12 times, and possibly 25 times as much light as the sun. To test this result, the parallax corresponding to $\sigma=\odot$ was computed, and found to be $\pi=0''.104$. We may therefore conclude definitely that each component of this star, which is of spectrum Class A p, is very much more intense than the sun. This is not surprising, for there is evidence that the same thing is true of the brighter component of the *Algol* system. According to current theories of stellar evolution perhaps we should expect the surfaces of Sirian stars to be of greater intrinsic brilliancy than the surfaces of solar stars—the position of the maxima in the spectral energy-curves seems almost conclusive evidence on this point; but it is nevertheless desirable to check theory by observation wherever possible.

In the foregoing calculations it has been tacitly assumed that each component is uniformly intense over its entire surface, but it is evident that no such uneven illumination could be detected as that found for the companion of *Algol.* There was the possibility that the discovery of the secondary minimum of *Algol* with the selenium photometer may have been due to the extra sensibility of selenium to the light of a red star, but in the present case of *β Aurigae*, where the minima are equal, we should expect the range of magnitude to be quite independent of the wave-lengths by which the instrument is affected.

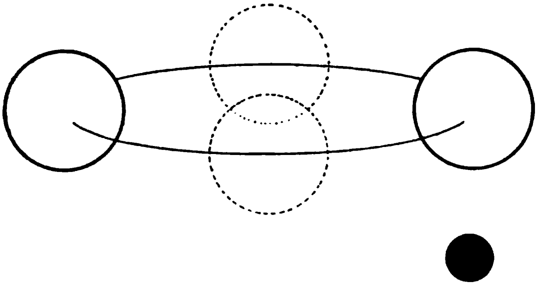


FIG. 2.—The System of *β Aurigae* as viewed from the earth

Fig. 2 shows the system of *β Aurigae* as viewed from the earth, and for comparison the sun has been drawn to scale and as a dark body. The ellipticity of figure, $c=0.01$, is not discernible to the eye, and the masses are so nearly equal that the two components are represented as moving in the same orbit.

SUMMARY

1. It has been found that *β Aurigae* is an eclipsing variable star which is brighter than *Algol* itself. The variation caused by eclipses amounts to 0.076 magnitude, with an additional change of 0.011 magnitude due to the ellipticity of figure of the two components, making a total range of 0.087 magnitude.

2. Within the limits of observational error, the times of light-minimum are in exact accordance with the times predicted from the spectroscopic elements by Baker.

3. The principal facts concerning this system are:

FROM THE LIGHT-CURVE

	Radius of each component	1.00
a	Distance between centers	6.86 ± 0.35
c	Ellipticity of figure	0.01
i	Inclination of orbit	$77^\circ.19 \pm 0^\circ.48$
P	Total period	$95^{h}.04$
	Duration of each eclipse	$5^{h}.90$

FROM $\pi < 0''.03$, Sun $= -26.6$ Mag., β *Aurigae* $= 2.07$ Mag., $(a_1 + a_2) \sin i = 11{,}981{,}000$ km

R	Radius of each component	$2.58\odot$
m_1	Mass of first component	2.38
m_2	Mass of second component	2.34
d	Density of each component	0.14
l	Total light of each component	$>80.$
σ	Surface-intensity of each component	$>12.$

4. The determinations of parallax make it seem certain that the components which are of spectrum Class A p, have each a surface-intensity many times greater than that of the sun.

I am indebted to Mr. Percy F. Whisler for most efficient assistance in making all of the observations of β *Aurigae*, and also to the Rumford Committee of the American Academy of Arts and Sciences for successive grants of $350 and $200 in support of this and other work with the selenium photometer.

UNIVERSITY OF ILLINOIS OBSERVATORY
URBANA, ILL.
April 21, 1911

MOTION AND CONDITION OF CALCIUM VAPOR OVER SUN-SPOTS AND OTHER SPECIAL REGIONS. II

By CHARLES E. ST. JOHN

PROMINENCES IN PROJECTION

Occasionally spectrum plates have been obtained in which the K absorption line has shown extraordinary appearances. These have been obtained at various times, and sometimes by successive exposures near the same point of the solar surface, so that the reality of the appearances is well established and the phenomena which they represent must be of quite frequent occurrence in the sun. The data for the plates showing such peculiarities are given in Table X.

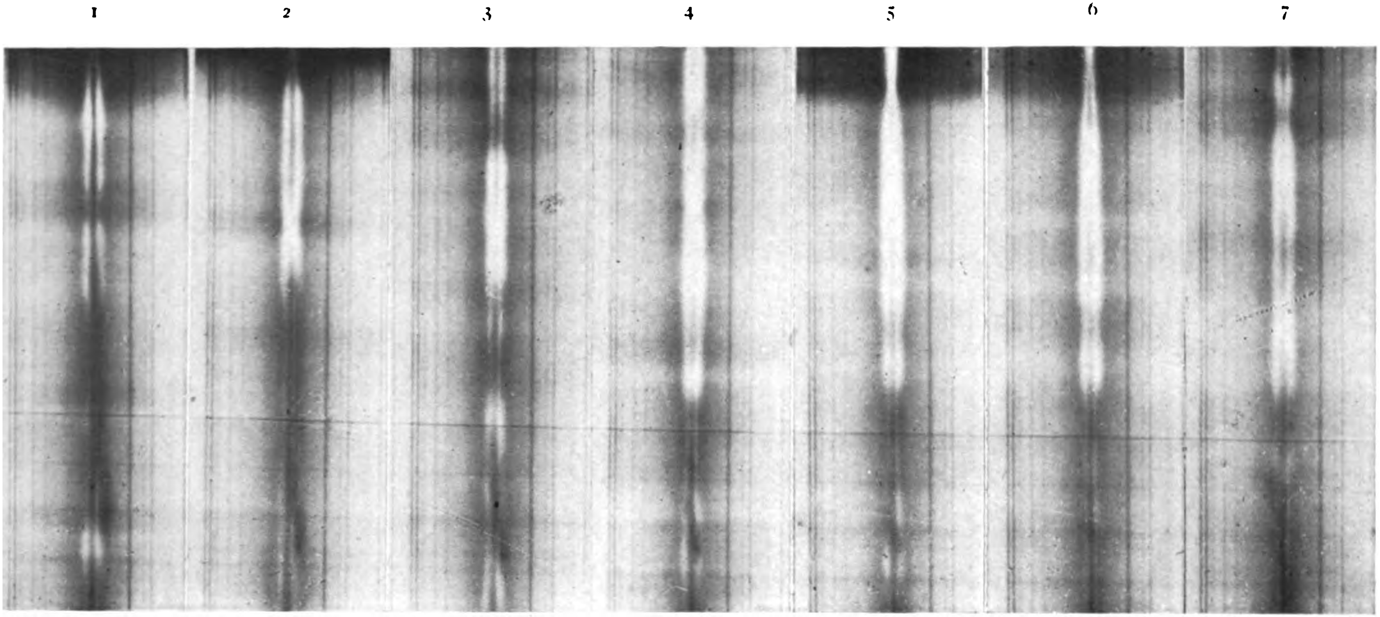
The characteristic appearance in each case upon which the present interpretation is based is the undisturbed condition of the absorption line K_3. This occupies its usual position, as shown by its wave-length agreeing with the normal value for this part of the disk, and is bounded on the red and violet edges by the emission line K_2. In some cases the violet component of K_2 shows a narrow absorption line superposed upon it through a distance of some millimeters. In other instances there is a broad absorption line on the violet side of the emission line. The approximate wave-length of this abnormal line shows a high ascending velocity in the vapor to which it is due. From the fact that the ordinary absorption line is undisturbed it appears that the phenomena here manifesting themselves by the displaced absorption line originate in great masses of calcium vapor having no direct connection with those levels of the solar atmosphere where the ordinary absorption line is produced and moving independently of that part of the atmosphere. That is, they are detached masses of vapor, such as prominences on the solar hemisphere turned earthward would present. The phenomenon bears a certain analogy to that observed by Frost and by Slipher in the spectra of spectroscopic binaries in which the H and K lines were undisplaced, and of which the suggested interpretations are an extended atmosphere of

TABLE X

Abnormal K Line over Special Regions

Plate	Date	Normal K_3 Line	Abnormal K Line	Velocity of Ascent	Remarks
	1909			km	
216......	Jan. 19	3933.675	3933.464	15.5	V K_2 double over a region of the solar surface 50,000 km in extent. K_3 undisturbed. Bounded by the bright components of K_2. 50 mm from the East limb
226......	Jan. 20	.680	.470	15.0	V K_2 double over a region 100,000 km in extent. K_3 regular and undisplaced. 50 mm from East limb
247......	Jan. 29	.683	....	....	2 mm steps from east to west. Over dark hydrogen flocculus north of spot 6608. Exposure 2, normal conditions
247......	Jan. 29	.679	....	....	Exposure 3. R K_2 partially washed out on red edge. V K_2 broad. K_3 sharply bounded on V K_2 side. Faintly bounded by R K_2
247......	Jan. 29	.679	.484	13.9	Exposure 4. V K_2 broad. Divided by absorption line. K_3 undisturbed and bounded by the two components of K_2
247......	Jan. 29	.681	.412	19.4	Exposure 5. K_3 undisturbed. Bounded by K_2 on both edges. Broad dark line to the violet of V K_2
247......	Jan. 29	.678	.349	24.2	Exposure 6. K_3 undisturbed. Bounded by K_2. Broad dark line to violet of V K_2
247......	Jan. 29	.681	.378	22.0	Exposure 7. K_3 undisturbed. Extraordinary absorption line similar to exposure 6.
257......	Jan. 30	.676	.456	16.1	Exposure 1. V K_2 double over region of solar surface. 15,000 km in extent. K_3 undisturbed. Bounded by K_2 on each edge
257......	Jan. 30	.670	.467	15.2	Exposure 2. 15,000 km west of Exposure 1. V K_2 double over region 35,000 km in extent. K_3 undisturbed. Bounded by K_2 on both edges
276......	Feb. 22	.684	.466	15.3	V K_2 double over a region 80,000 km in extent. K_3 undisturbed. At center. Time 10 A.M.
286......	Feb. 22	.676	.460	15.8	Same region as in No. 276, but 4 hours later

PLATE VIII



THE K LINE OF CALCIUM AT 2 MM INTERVALS IN THE NEIGHBORHOOD OF A DETACHED MOVING MASS OF VAPOR

Enlargement: 5 times from original negative with 18-foot spectrograph and Snow telescope. 1909, January 29

Exposures 1, 2, Normal condition

Exposures 5, 6, Abnormal absorption line to the violet of and distinct from K_3

calcium surrounding the system or detached masses of calcium vapor in space. Plate VIII is a reproduction of a photograph taken with 2 mm steps over such a moving mass. It is possible to see in the upper third of the reproduction, on exposures 5, 6, and 7, the characteristics referred to: the undisturbed K_3 line, the two components of the K_2 line, and the absorption line displaced to the violet. A fine example of this effect is shown in the beautiful reproduction, Plate 46, of the Meudon *Annales*, Tome IV. Under the title "Sur un filament extraordinaire,"[1] MM. Deslandres, d'Azambuja, and Burson report on this interesting case in which the displacements indicated vertical velocities exceeding 100 km per second. The phenomenon was of short duration, ending an hour and a half after the maximum velocity had been gained. It was not attributed to a prominence viewed in projection against the disk, but otherwise the interpretation,

> Tout se passe comme si, la couche K_3 restant à peu près dans son état ordinaire, une grande masse de gaz au-dessus, distincte de la première, s'élevait irrégulièrement à de grandes vitesses,

is the same as that given here.

It seems more than probable that the dark flocculi or *filaments* are closely connected with prominences when the remarkable Plates 43 and 45, Tome IV of the Meudon *Annales*, are examined. These reproduce K_3 spectroheliograms taken four days apart, upon which a long, dark flocculus, or chain of such, extends across the southern hemisphere of the sun. On Plate 43, where the dark flocculus reaches the east limb, it appears to blend into a large prominence, and at the west limb at the point touched by the flocculus, a small prominence is shown. On Plate 45, taken four days later, the flocculus has advanced from the east limb and the associated prominence has disappeared, while at the west limb the prominence has increased in height and appears still on the line of the dark flocculus. The whole appearance is as if the dark flocculus were a mass of calcium vapor high above the chromosphere, revealing itself on the disk by absorption and at the east and west limbs as prominences. Such masses of calcium vapor would be expected, under this point of view, to show as they do at times the enormous upward velocities obtained in the case of prominences. As the projected masses settle back again the motion would be slower. Measurements of the K_3 line over such a dark flocculus appearing on the $H\alpha$ spectroheliogram of August 4, 1910, gives over the flocculus λ 3933.681, and around the flocculus λ 3933.675, indicating a downward movement in both cases which was slightly greater over the region of greater absorption. The same point of view is held by Mr. Michie Smith, who states that these dark calcium flocculi when photographed near the limb are found to agree in position with the prominences.[1]

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[1] *Comptes Rendus*, 150, 1638, 1910.

Mr. Evershed[2] considers the very conspicuous dark flocculus shown on certain H_2 spectroheliograms taken at Kodaikânal, Feb.–Apr. 1910, as due to absorption produced by the extensive prominence then transiting the sun's disk, and says:

> When these dark markings are seen extending to the sun's limb they are found almost invariably to end in a prominence, but the latter in many cases are somewhat insignificant in height.

The detailed spectrographic study of such regions of absorption is a part of the program at Mount Wilson.

DISCUSSION

The evidence for and against general or local systems of circulation of the chromospheric calcium vapor deduced from the data of this paper may be considered from the point of view of the absorbing and emitting layers separately and together. In the case of the absorbing vapor the wave-length measurements over possibly related regions are:

At the center	3933.680 (63)[3]
Circumfacular regions	.678 (134)
Flocculi	.679 (176)
Penumbrae	.681 (120)
Umbrae	.681 (11)
In the arc	.667[4]

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 25, 7; *Astrophysical Journal*, **27**, 225, 1908.

[2] *Astrophysical Journal*, **33**, 6, 1911.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 48, 41, 1910; *Astrophysical Journal*, **32**, 76, 1910.

[4] *Contributions from the Mount Wilson Solar Observatory*, No. 44, 10, 1910; *Astrophysical Journal*, **31**, 152, 1910.

The most striking feature of the assembled data is the remarkable agreement of the separate means with the general mean 3933.679 from five hundred separate exposures. It appears that on the whole the high-level calcium vapor of the chromosphere is descending over these different regions of the solar surface with a surprisingly uniform velocity on the average. In seeking for the source of supply we may look to outer space as Professor Newall has suggested, or to the inner regions of the sun. In speaking of the possibility of attributing the main characteristic spectroscopic phenomena of the sun to matter constantly streaming in from without Professor Newall[1] says:

> We may divide the attracted matter into three categories: (I) molar matter, in masses large enough to be drawn in in spite of light-pressure, having diameters longer than about 2½ times the wave-length of light; (II) molecular matter, in masses small enough to escape, in virtue of the diffractional effects, the repulsion of light, having diameters less than about one-tenth of the wave-length of light; and (III) light-driven matter, in masses intermediate between molar matter and molecular matter, and subject to the repulsion of light.
>
> The reversing layer of the Sun is provided by molecular matter, constantly streaming across what we may call a critical envelope concentric with the Sun, and probably is in a slightly modified state of radiative equilibrium. The constant flux of matter here involved is provided by those constituents of the planetary whirl of molecular matter circulating round the Sun, which are directed inwards upon the Sun in virtue of properly directed collisions.

If there were no evidence of any source of supply from the inner regions of the sun, the practical agreement of the mean velocities of the descent of the absorbing layer of calcium over such diverse regions of the solar surface as those over which the above measurements were made might well be adduced as tending to support the hypothesis of a constant influx from outer space. The rotation of this absorbing layer shows a small polar retardation and a high equatorial velocity relative to the reversing layer, but still an organized rotation of low value compared to planetary velocities. There is evidence, however, that calcium vapor is rising from the interior over the general surface of the sun and over particular regions. As previously quoted in this paper from *Contribution* No. 48 the measurements of the wave-length of the K_2 line of calcium due to emitting vapor was found to be 3933.641 Å over the general surface of the sun as against 3933.667 Å in the arc, indicating a mean upward velocity of 1.97 km per second. This vast ascending mass of calcium vapor would seem to be sufficient for the continual renewal of the supply, though perhaps in a less simple manner than the inflow from outer space. Over limited but still extensive areas of the solar surface great masses of relatively cool calcium vapor are ascending with very high velocity. These have been attributed to prominences seen in projection. These masses of vapor must diffuse laterally and eventually return to the solar surface under the action of gravitation, being free from the effects of light-pressure owing to their molecular condition. The supply from this source, probably having its origin in eruptions of much denser vapor from the interior, would hardly account for the layer of absorbing vapor enveloping the whole sun. This finds a sufficient and more probable supply in the vapor producing the K_2 line of calcium which is ascending over the general surface.

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[1] Newall, Presidential Address, *Monthly Notices of the Royal Astronomical Society*, **69**, 343, 1909.

The quantitative data with a bearing upon a circulation of the calcium vapor shown by measurements of the K_2 line are the wave-lengths of this line over particular regions as shown below:

Over general surface.......	3933.641	(20)[1]
Over flocculi.	.664	(175)
Over penumbrae..........	.665	(120)
Over umbrae.............	.682	(141)
In the arc................	.667	

This shows a rise of emitting vapor over the general surface with a velocity of 1.97 km per second and a descent of this vapor in the umbrae of spots with a velocity of 1.3 km per second, a total range of 3.27 km per second. The condition over the flocculi indicated by these measures is an intermediate one, possibly, on the whole, one of slow ascent. . Taken in connection with the inflow shown in Table VII (p. 75) the evidence for a system of circulation of the emitting vapor is quite complete, in which the supply of vapor descending in the umbrae of spots is that rising mainly from the immediate surroundings of the flocculi and their peripheral portions and flowing toward the umbrae where it is drawn suddenly and rapidly inward.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 48, 21, 1910; *Astrophysical Journal*, **32**, 56, 1910.

Attention has been called to the difference between the velocities of inflow and rotation obtained from wave-lengths of the K_2 and K_3 line. The results from Tables VII and IX are:

	K_2	K_3
Inflow........	1.24 km	2.00 km
Rotation......	0.98	1.59

Everything occurs as though the disturbance begins in the upper layer of the chromosphere and works down into the emitting layer, to which it communicates a motion of inflow and rotation, the lower stratum always lagging behind the upper. In the great majority of cases the K line over the umbrae is bright and single, but on a few plates taken even when the seeing was excellent the absorption line K_3 is present, while on later plates of the same spot it had disappeared. In such a case the presence of the absorption line cannot be due to photospheric light falling upon the slit of the spectrograph. The absorption line appears to be at times a real spot phenomenon and there is some evidence that it belongs to the earlier life history of a spot.

Some light is thrown upon the bright reversal of the K line over the umbrae of spots by comparing the relative intensities of the umbral line and the K_2 and K_3 components over the penumbrae. For this purpose there was at hand the photometer devised by Hartmann[1] for the measurement of the intensities of photographically blackened surfaces. This enables one to isolate an extremely small portion of the surface and to match its intensity against the photographically produced wedge. The measurements are given in Table XI, in divisions of the wedge-scale, which increase with increasing density in the negative.

A striking result is the relatively weak intensity of the bright umbral line compared with that of the K_2 and K_3 components immediately outside the umbra. Under ordinary visual examination of the negatives one gets the impression that the umbral line is much brighter than K_3 and almost equal in brightness to K_2 while in fact it is slightly less bright than K_3 on the average (Plate V, *a*). The dark background of the umbra brings the line into high relief, just as in the laboratory experiments on the sodium reversal the line appears to decrease in brightness on passing from the dark background to the intense continuous background of the arc. Since the temperature of the spot umbra is lower than that of the surroundings, the bright umbral line is probably produced by the same vapor as that to which the K_2 line is due, there being a continuous flux of the vapor into the umbra. The line is narrower and less bright than the K_2 line over the flocculi and penumbra, as would naturally follow from the diminished pressure and lower temperature obtaining at the center of the spot vortex. The resulting fall in the temperature of the absorbing vapor in the spot umbrae would reduce its radiation below that of the vapor to which the umbral line is due, as the bright line is frequently double the width of the K_3 line, and not always less intense as may be seen from the last three lines of Table XI. Therefore, it does not seem probable that the upper layer of calcium plays an important rôle in the production of the reversal. The absence of the K_3 line over the umbrae may be due to the diminished quantity of the absorbing vapor which was earlier drawn inward, while that supplied by the lateral inflow mainly descends in the region of the penumbrae and fails to reach the umbral region in large quantity during periods of rapid downdraught.

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[1] *Astrophysical Journal*, **10**, 321, 1899.

TABLE XI

Relative Intensities of the Components of the K Line

Over Umbrae			Over Penumbrae	
Reversal	V K_1	R K_1	K_2	K_3
40.4	28.1	27.5	45.4	43.2
35.6	25.7	26.3	45.6	41.2
48.1	37.2	36.8	58.9	53.1
47.0	36.0	35.1	55.2	48.0
41.1	28.2	28.3	46.3	41.2
44.4	29.5	29.6	57.1	....
44.9	28.5	28.3	46.2	44.0
49.6	30.2	29.7	56.0	48.4
50.9	47.3	47.0	66.4	50.3
44.7	32.3	32.1	53.0	46.5

Plate V (*a*) reproduces the typical condition over spot umbrae, though in many cases the reversal is relatively more intense than here shown. The curve (*c*) in Fig. 2 is drawn from the data in Table XI and represents the K line over the umbra of a spot. The widths of the K_2 and K_3 lines in Fig. 2 are drawn to scale and show the relative widths under the different conditions to which the curves refer, but the width of the K_1 line is only suggested. The ordinates of the curves reproduce the relative intensities of all the components in terms of the wedge-scale. The intensities of the absorption line on the two sides of the reversal as shown by these measurements are practically equal, individually, and on the average. Jewell[1] found that:

> The absorption toward the red from the reversal was considerably darker and broader than toward the violet, thus giving evidence of considerable relative velocity between the emission line and the broad absorption line or shading.

The present measurements show that the bright umbral line is shifted to the red, which would tend to cause the absorption in the K line to be more conspicuous toward the violet. A compensation might result from the displacement of the center of gravity of the K_1 line also to the red owing to the greater pressure under which the line is produced.

The inflow of the calcium vapor and the downdraught in the umbrae appear to be the usual condition of the spot, but the penumbral or extra-penumbral vortex is not developed around every spot, nor does it always persist in the case of a given spot when once developed. This is in harmony with the visual observations of Secchi, who considered the vortex an exceptional case:

> Si cela arrive quelquefois, c'est assez rare, car, sur trois cents taches et plus qu'on observe dans le cours d'une année, il y en a sept ou huit seulement qui présentent d'une manière bien tranchée la structure spirale qui devrait caractériser les tourbillons. . . . . Non seulement les taches ne présentent pas toutes la forme de tourbillons, mais, de plus, cette forme, lorsqu'elle existe, ne persiste pas plus d'un jour ou deux, tandis que les taches elles-mêmes peuvent subsister longtemps encore après avoir perdu la forme spirale.[2]

The spiral structure is not shown around all spots on the $H\alpha$ spectroheliograms, though the spots themselves are magnetic fields as evidenced by the Zeeman effect. It is probable that in many

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[1] *Astrophysical Journal*, 3, 105, 1896.

[2] Secchi, *Le Soleil*, Part I, p. 89.

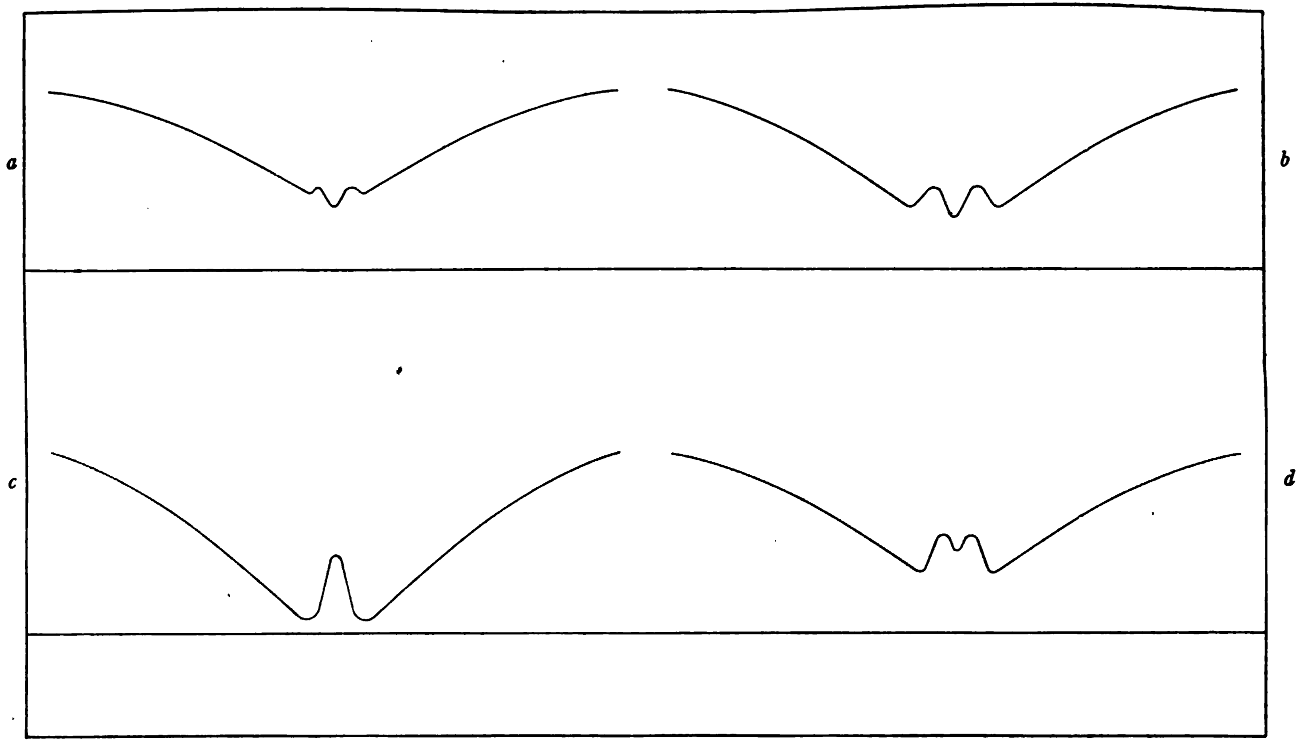

FIG. 2.—Relative intensities of the components of the K line of calcium

a, At the center of the solar disk
b, At the sun's limb
c, Over sun-spot umbrae
d, Over flocculi

instances the region throughout which the vortex is strongly developed does not extend very far outside the umbra, and the surrounding gaseous masses are not sufficiently involved to be arranged in stream-lines. It is surprising that, in view of the spectroscopic evidence of the inflow and downdraught of the calcium vapor, spectroheliograms taken with the H and K lines do not show more indication of stream-lines which are sometimes such marked features over vast areas of the $H\alpha$ spectroheliograms. This may be due to calcium vapor being present in broader and more continuous masses over the flocculi, and being more uniformly distributed around spots than the hydrogen. If an electro-magnetic instead of a hydrodynamic cause be assumed for the line-of-force structure on $H\alpha$ images, a suggestion made by M. Deslandres may be the key to the explanation:

> Chaque ion solaire mobile subit le champ électrostatique et électromagnétique de tous les autres. Comme l'effet n'est pas le même pour les atomes des masses différentes, on aurait là une explication des différences que présentent les images du calcium et d'hydrogène dans l'atmosphère solaire.[1]

Stream-line structure does occasionally occur on H_2 spectroheliograms according to both Hale and Evershed. In speaking of this Hale says:

> We have seen, however, that apparent lines of force having about the same curvature as the dark $H\alpha$ flocculi are frequently shown by H_2 between spots of opposite polarity, though this line fails to give most of the finer details of the $H\alpha$ structure. The difference may be due in large part to the greater effect of convection currents at the H_2 level.[2]

And Evershed writes:

> In good spectroheliograph plates of the calcium flocculi the regions of the penumbrae in quiet spots often show radial stream-lines and these are sometimes curved. In general, however, the more brilliant masses of flocculi of the surrounding region show no tendency to either a radial or a spiral structure.[3]

RELATIVE INTENSITIES OF THE K LINES

In the study of the conditions which produce the complex constitution of the H and K lines it appeared probable that a knowledge of the relative intensities of the components of the K line would be useful, and particularly the intensity of K_2 in comparison with the intensity of the continuous spectrum in the case of the very brilliant flocculi. For this purpose the Hartmann photometer was found very convenient and satisfactory.

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[1] *Comptes Rendus*, **150**, 72, 1910.

[2] *Publications of the Astronomical Society of the Pacific*, **22**, 77, 1910.

[3] *Monthly Notices of the Royal Astronomical Society*, **70**, 224, 1910.

One of the most striking appearances in a series of exposures extending from the limb to the center of the sun in which the exposure-times are such as to give equal intensities for the continuous spectrum is the great intensity of the K_2 components at the limb, and their gradually decreasing intensity with increasing distances from the limb. At the limb these components are very conspicuous, while at the center they are apparently very weak, except over flocculi, but still distinctly visible. Two typical plates were selected from a long series, upon which the intensity of the K_1, K_2, and K_3 lines was measured with the mean results in Table XII, which are expressed in divisions of the wedge-scale and represent the ordinary conditions at the center and limb of the sun.

TABLE XII

RELATIVE INTENSITIES OF THE K LINE AT LIMB AND CENTER

Plate	Position	K_1	K_2	K_3	K_2-K_1	K_2-K_3	K_1-K_3
329........	Center	54.1	54.9	50.6	0.8	4.3	3.5
462........	Limb	50.4	54.8	47.9	4.4	6.9	2.5

Two points revealed in obtaining this series of measurements are surprising, namely: the equal intensities of the K_2 line at the limb and center, and the difficulty of detecting with the photometer the difference between K_2 and K_1 at the center. When the lines are all in the field of view of the photometer, the K_2 line is distinctly seen and the dissymmetry of its components is clearly apparent, but when a small portion of the K_2 line is isolated by the photometer and settings of the wedge have been made for equality, it is very difficult to observe any change when the inner edge of the K_1 line is brought into the field of view. The results given in the table are the means of many settings. The apparent brightness of the K_2 line at the center is due largely but not entirely to contrast effect owing to the proximity of the strong absorption line K_3.

The increased intensity of the K_2 line at the limb is mainly apparent. The absolute intensities on these limb and center plates were practically equal, but the contrast effect at the limb was greatly increased by the greater strength of the absorption lines K_1 and K_3 between which the K_2 components lie. The effect is also enhanced by the greater width of the lines at the limb. In the cases of K_1 and K_3 the absorption at the limb, as would be expected from the greater effective depths of the absorbing layers, is greater than at the center for each line. But the increment in the case of K_1 exceeds that in the case of K_3, as they have become more nearly equal. At the center the K_2 line is just measurably brighter than the background produced by the K_1 line. At the limb the darkening of the background, particularly in the case of K_1, raises the K_2 line into relief without any great increase of its intensity. The results are shown graphically in Fig. 2, (*a*) at the center, (*b*) at the limb. The dissymmetry of the components of the emission line at the center is a marked feature, V K_2 being broader than R K_2. At the limb the components are equal in width and intensity. The curves are to scale and represent the relative widths of K_2 and K_3 and the relative intensities of all three components.

When the slit of the spectrograph crosses the very brilliant flocculi the density of the emission line K_2 appears to the eye, sometimes, to equal the density of the continuous spectrum. The photographic intensity of the spectrum decreases rapidly just beyond the K line due to absorption in the ultra-violet region by the glass of the lens. It was thought possible that the instrumental absorption might so reduce the intensity of the continuous spectrum on the violet side that a false result might be obtained if that alone were used. Comparison points were therefore taken on both sides of the K line at distances of about 12 Å with the following results expressed in the wedge-scale:

TABLE XIII

K_2 COMPARED WITH CONTINUOUS SPECTRUM

Plate	λ 3920	K_1	K_2	λ 3945	Continuous–K_2
197.........	69.5	39.3	59.2	70.7	10.9
267.........	59.5	28.6	48.5	60.9	11.7
296.........	57.6	39.2	51.2	58.2	6.7

In each case the intensity of the strong emission line was markedly less than that of the continuous spectrum even on the violet side. The usual calcium spectroheliogram is taken with H_2 or K_2, using one or both components, upon which the flocculi appear bright against the less intense background produced by the ordinary intensity of the line. The strength of the absorption line photographed with a long slit varies greatly along its length, becoming less intense as an absorption line when the slit crosses a flocculus, under which conditions it approaches the brightness of the emission line and may even disappear as an absorption line. The measurements in Table XIV were made at 2 mm steps along the line, including in the range of measurements two small flocculi.

TABLE XIV

INTENSITIES ALONG THE K_3 LINE

			Over Flocculus						Over Flocculus			
K_3	49.5	54.9	60.3	63.6	62.0	60.9	53.2	51.7	56.4	60.7	56.2	51.1
K_2	....	....	62.8	66.4	62.9	....	....	....	....	62.7	....	....

As seen, radiation has increased over the flocculi in both the K_2 and K_3 lines. The curve, (*d*), Fig. 2, shows the cross-section of the K line over such a flocculus. In comparing the ordinates of this curve with those of (*a*), it is seen that the radiation in the case of all three components has increased, the increase being the greatest for K_3. Spectroheliograms taken with the K_3 absorption line should show areas of great relative intensity corresponding to the flocculi shown on K_2 spectroheliograms; owing to the brightness of the K line over the umbrae, spots should not in general appear; and the dark flocculi, due to masses of absorbing vapor high above the chromosphere, should be much more conspicuous features than upon the K_2 spectroheliogram, since this cooler vapor, unless in very rapid motion, would greatly increase the absorption at the center of the K_3 line and affect the K_2 components in less degree. In speaking of the possible photography of the disk with the K_3 line Hale and Ellerman say:[1]

> If successful, photographs taken in this way will probably show the calcium prominences as dark regions projected upon the disk.

---

[1] *Publications of the Yerkes Observatory*, 3, Part I, p. 19.

These are the appearances found by M. Deslandres in his skilful application of the spectroheliograph to work with the K_3 line:

> Cette couche supérieure, étudiée avec le calcium et l'hydrogène a comme caractères; l'extension jusqu'à elle des facules de la surface, mais avec des formes différentes, la disparition presque absolue des taches et l'apparition des lignes noires, souvent très longues, qui sont les filaments.[1]

THE EMISSION LINE K_2

There is complete consensus of opinion in the case of K_3 namely: that it is produced by absorption in the upper and cooler regions of the chromosphere. As to the bright reversals H_2 and K_2, Hale and Ellerman say:

> From a strict application of Kirchhoff's law it would appear that the calcium vapor in the lower chromosphere is actually hotter than the calcium vapor which lies above and below it. It seems improbable that the law can be rigorously applied in this case, and hence it may be necessary to attribute the strong radiation of the intermediate layer to causes other than temperature alone.[2]

Mr. Evershed speaks of the bright H_2 and K_2 lines as representing the emission of the relatively hot calcium vapor rising immediately above the ordinary faculae.

And Hale, in his *Stellar Evolution* (p. 86), says:

> The bright H and K lines, referred to in the last paragraph, were found in close association with the faculae, and it appeared probable that much of the highly heated calcium vapor, to which these bright lines are due, rises from the interior of the Sun through the faculae.

M. Deslandres considered the light of the chromosphere to have an electric origin and wrote of the brilliant flocculi overlying the faculae:

> Dans la chromosphère, les phénomènes électriques sont plus intenses au-dessus des points élevés de la surface.[3]

And later he has attributed the increased radiation over the bright regions to the compression produced by the descent of the vapors from the higher levels:

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[1] *Annales de l'Observatoire d'Astronomie Physique de Paris* (Meudon), **4**, 103, 1910.

[2] *Publications of the Yerkes Observatory*, **3**, Part I, p. 16.

[3] *Comptes Rendus*, **138**, 1377, 1904.

> En général, sur les parties brillantes, les déplacements sont vers le rouge; et la vapeur descend dans l'atmosphère. Comme alors elle se comprime et s'échauffe, l'augumentation de son éclat est naturelle.[1]

A somewhat analogous explanation of the bright reversals over the general disk was suggested by the writer,[2] namely, an increase of radiative power produced by the mutual interaction of the rising emitting vapor and the descending absorbing vapor, their relative velocity being found to be about 3 km per second. A small rise of temperature would probably be sufficient to account for the increased radiation, as Mr. King's work with the furnace has shown that the H and K lines are high-temperature lines in the sense that, with rise in temperature, they increase markedly in intensity in comparison with the arc lines, and without decided widening.[3] The temperatures available in Mr. King's experiments with the electric furnace did not exceed 3000° C. At solar temperatures the same increment of temperature would be less effective in increasing radiation, but that this characteristic of the H and K lines still obtains at solar temperatures is indicated by their great intensity in the sun as compared with that of λ 4226.9, which at low furnace temperatures is stronger than H and K. This explanation would have less force over the faculae where the calcium vapor has little if any upward velocity, while over these regions the radiation of the calcium vapor is the most intense.

A very different cause is assigned for the origin of the bright lines H_2 and K_2 by Professor Julius in his "Spectroheliographic Results Explained by Anomalous Dispersion."[4] This demands a density-gradient at right angles to the line of sight, which he finds in the cross-section of solar vortices:

> For it [the density] is a minimum in the axes of vortices; and the average direction of the whirl-cores, lying between the Earth and the central parts of the Sun in the surfaces of discontinuity, differs but little from our line of sight. The rays of the Sun thus reach us after having traveled a great distance along

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[1] *Annales de l'Observatoire d'Astronomie Physique de Paris* (Meudon), **4**, p. 77.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 48, 37–39; *Astrophysical Journal*, **32**, 72–74, 1910.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 32, 1908; *Astrophysical Journal*, **28**, 389, 1908.

[4] *Astrophysical Journal*, **21**, 278, 1905.

lines making small angles with the levels of slowest density variation in a lamellar, partly tubular, structure.

Accordingly, the bright lines H_2 and K_2 come from photospheric light of wave-length very near to that of the central absorption line, whose divergence has been changed to convergence by the tubular structure and thus reaches the earth with increased intensity. A serious difficulty involved in this explanation is that it demands that the axis of the tubular structure be always directed toward the earth. If the direction of the whirl-cores differs but little from our line of sight when they occur in the central portions of the sun, the direction must be nearly at right angles to our line of sight when they rise from the peripheral regions since they are approximately normal to the solar surface; but the spectroheliograph shows the flocculi near the sun's limb reduced in width as they would be by foreshortening if they were really bright surfaces. It is very easy to convince oneself by the examination of a series of daily spectroheliograms that the flocculi appear as bright surfaces attached to the sun would appear in projection when carried across the line of sight by the sun's rotation.

It is evident that the bright reversal of the K line over the general disk and the increased intensity of the emission line K_2 over the facular region in comparison with the intensity over the general surface have not yet received a generally accepted explanation. An explanation of these reversals is here based upon the different radiation coefficients for the different parts of the line, and the molecular scattering of light. For the lower layer there is general agreement among solar spectroscopists, namely, that the density of this layer is the basis for the great width of the line, and that for the central portion of the line, at least, the absorption is complete. The temperature of this layer being less than the source of the continuous spectrum background, a broad line darker than the continuous spectrum is produced. In the case of the next layer, the so-called emitting layer, its pressure and temperature are both reduced in reference to the underlying layer. The decrease of pressure results in a narrower line, but its lower temperature does not necessarily imply a decreased radiation, as on nearing the center of the line the coefficient of radiation rapidly increases, and if the gain due to a higher coefficient of radiation exceeds the loss due to decrease of temperature the result would be a relatively narrow bright line superposed upon the center of the K_1 line. This result would be favored by the greater scattering of the light coming from the lowest layer, which would tend to darken the background by reducing the light in the K_1 line.

Spectroheliograms taken with the first slit on the K_1 line, at decreasing distances from the center of the line, and then with the slit on the K_2 line, in the opinion of solar observers, represent different levels in the solar atmosphere, rising in level as the slit approaches the center of the line. The area of a flocculus increases progressively, reaching its maximum area, according to M. Deslandres, when photographed by the light from the K_3 line. These appearances have been interpreted as indicating a tree-like structure of the rising column of calcium vapor. A diagram illustrating this view is given by W. J. S. Lockyer in *Nature*, **69**, 611, 1904. This implies that the calcium vapor is rising over the faculae, an assumption which the wave-length measurements obtained in this investigation do not appear to justify.

An explanation of the effects obtained at different levels by employing the parts of the K line in taking monochromatic photographs is here based upon the selective absorption in different parts of the K line which determines the depths in the solar atmosphere from which light of the different parts of the line reaches us, and upon the great sensitiveness of the H and K lines to changes of temperature or excitation. The faculae may be considered as regions from which a disturbance is propagated into the overlying gaseous masses. These may be regions of higher surface-temperature, absolutely or from their being elevated above a portion of the surrounding vapors, or as Professor Newall suggests:

> Under the influence of local disturbances, regions of the Sun may emit corpuscles which succeed in penetrating the lower strata of the reversing layer, and cause higher strata to glow as if their effective temperature were higher than lower strata, and thus bright flocculi arise.[1]

In either of the above cases the disturbing influence would tend to spread radially from the center of disturbance and increase the radiation coefficient of the calcium vapor perhaps selectively even within the K line. Mr. King has shown in his work with the electric furnace that the intensity of the H and K lines is very susceptible to temperature changes without corresponding changes in the width. For a narrow range of wave-length, represented by the width of K_3, the radiation increment may be assumed to be large; through a range of wave-length corresponding to the difference between the widths of K_3 and K_2 the increment would be less; while for wave-lengths between the boundaries of K_2 and K_1 the increment would be extremely small. The diagram in Fig. 3 will illustrate the manner in which the area of a flocculus would vary for different parts of the H and K lines.

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[1] *Monthly Notices of the Royal Astronomical Society*, **69**, 343, 1909.

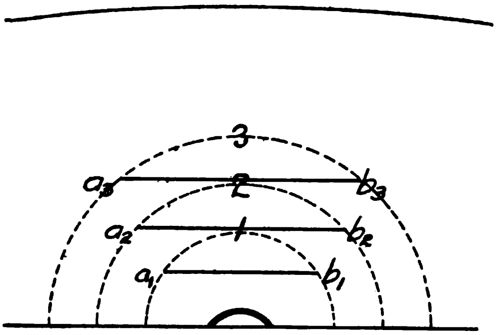

FIG. 3

The distance of the line a_3 b_3 below the upper surface indicates the depth to which we see into the solar atmosphere by the light of the K_3 line; the half-circle marked 3 represents the limit of the appreciable effect upon the K_3 line produced by the disturbance propagated from the facula. As the first slit of the spectroheliograph, set upon the K_3 line, moves over the region an area much larger than the facula would be bright relatively to that photographed by the neighboring parts of the K_3 line. For wave-lengths differing slightly from K_3 the increase of radiation would not be so great for equal excitation, but the depth from which the light comes would be greater. The level therefore from which the light of the K_2 line comes, is lower owing to the decreased absorption. At this depth the excitation may be intense enough to be effective upon the radiation of this wave-length, thus increasing the intensity of K_2 in comparison with the intensity of the adjoining parts of the line. The area photographed by the K_2 line would be less than that photographed by the K_3 line, but still considerably larger than the facula as indicated in the diagram by the line a_2 b_2 and the corresponding circle.

In the case of the K_1 line the light comes from a still greater depth, but for wave-lengths differing so much from the center of the line the increased excitation is very much less effective in increasing radiation than in the case of K_2 and K_3, and the distances from the center of disturbance at which the excitation is sufficient to increase the radiation in K_1 over the facula in comparison to the radiation in the neighboring parts of the line are much less, and the areas photographed by setting the first slit of the spectroheliograph at increasing distances from K_3 would approach the area of the facula as a limit.

There is probably a piling up of calcium vapor over the penumbrae of spots and the surrounding faculae, evidenced both by the greatly increased width of the K_2 line over these regions and the absence of appreciable motion of ascent or descent of the emitting vapor as shown by the wave-length determinations, though it is rising rapidly over the general surface. It is difficult to believe that it is not rising through the faculae as elsewhere. The descent of the upper portions of the layer toward the umbrae of spots and the great thickness of the layer would tend to mask the rising vapor that is probably being added to the mass from below and perhaps crowding it slowly upward, the down-flow into the umbrae being supplied by the inflow from the surroundings of the flocculi. That there is an intimate connection between the faculae and the accompanying flocculi is indicated by their community of form and location. Such a nodal condition as is indicated by the normal wave-length of the K_2 line over the faculae surrounding spots might be favorable to an accumulation of calcium vapor as a kind of back-water effect where opposing currents tend to annul each other. The observations upon which this paper is founded were made upon completely developed spots. It would probably give added light to the subject to study the motions of calcium vapor over developing faculae and spots, and be particularly interesting to observe the initial conditions in the case of a large flocculus.

From the point of view of the combined action of absorption and scattering, Professor Schuster[1] has considered the radiation of a gaseous layer of uniform temperature and of a layer in which the temperature variation is such that the radiation of a black body varies directly with the distance from the front surface. In the latter case, the only one which approximates solar conditions, he shows that for certain values of the coefficients of absorption and scattering homogeneous radiations may appear bright with a dark center. Bright in this sense means brighter than the continuous spectrum, but the solar H_2 and K_2 lines are less bright than the continuous spectrum. Professor Schuster finds that for certain values of the coefficients the intensity of homogeneous radiations rises to a maximum which is greater than the intensity of the continuous spectrum, then falls for larger values of the coefficient of absorption which obtain as the center of the line is approached. The problem to be solved, however, is that in which the intensity-curve for the line has a minimum corresponding to K_1, a maximum corresponding to K_2, and a second minimum corresponding to K_3, all less than the continuous spectrum, and occurring with progressively increasing coefficients of absorption and progressively decreasing thicknesses of the effective layers. Its solution requires a knowledge of the coefficient of scattering, and of the radiation-gradient which probably differs greatly for the wave-lengths concerned in the production of the K_1, K_2, and K_3 lines; as Professor Schuster points out, the radiation-gradient depends not only on the temperature-gradient but also on the wave-length.

GENERAL CONCLUSIONS

1. In the great majority of sun-spots the calcium vapor is descending in the umbrae with velocities varying from 0.68 km per second to 2.2 km per second. This result is obtained from the usual bright reversal over the umbrae and also from the absorption

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[1] *Astrophysical Journal*, 21, 1, 1905.

line when present. Occasionally the calcium vapor is rising in the umbrae.

2. Over the penumbrae of spots the calcium vapor which is the source of the bright line K_2 has little, if any, vertical motion. The calcium vapor producing the absorption line is descending with approximately the same mean velocity as over the general disk.

3. Over flocculi surrounding spots the emitting vapor shows a very slight but doubtful upward motion. The mean wave-length for K_2 over the central flocculi is 3933.665 Å compared with 3933.667 Å in the arc, an agreement within the errors of measurement. The absorbing vapor, the source of the K_3 line, is descending with the same mean velocity as over the general disk.

4. Over circumfloccular regions the upper layer of absorbing calcium vapor is descending with practically the same mean velocity as over the general disk, and does not show evidence of a local system of circulation involving the flocculi and their immediate surroundings.

5. A radial motion of the calcium vapor inward across the penumbrae is shown both by the emitting and absorbing vapor. The velocity is higher for the absorbing vapor than for the emitting vapor.

6. A rotary motion of the calcium vapor around the umbrae of spots is an occasional phenomenon, and is shown by the displacements of both the absorption line and the bright line. The velocity is higher in the case of the absorbing vapor than in the case of the emitting vapor.

7. The combination of the radial motion inward and the rotary motion results in a spiral or a vortical motion converging upon the umbra. The direction of rotation does not depend upon whether the spot is north or south of the sun's equator. The disturbance appears to originate in the high-level absorbing vapor and to work downward, involving the emitting layer.

8. The phenomenon of a dark absorption line appearing in one of the K_2 components, or beyond their outer edges, while the K_3 line is undisturbed, is interpreted as being caused by detached masses of relatively cool calcium vapor high above the chromo-

sphere, and in more or less rapid motion, that is, by prominences projected against the sun's disk.

9. In addition to the general circulation of the calcium vapor shown by the ascent of the emitting vapor and the descent of the absorping vapor over the general surface of the sun, there appears to be a local system in which the emitting vapor rises around the flocculi, flows across the penumbra, and is then drawn downward into the umbra with or without vortical motion. There is very slight evidence, if any, of its rising from the interior of the sun through the faculae, except in the case of eruptions.

10. The intensity of the bright K_2 line over flocculi is less than that of the continuous spectrum. The apparent weakness of the K_2 line in the spectrum of the center of the sun is due mainly to the slight difference between it and the background of the K_1 line. Its apparent increase of intensity at the limb is due in the main to the greater absorption in the K_1 and K_3 lines which raises the K_2 line into relief without much actual increase of intensity.

11. The relative intensities of the K_1, K_2, and K_3 lines may be explained by the great differences in their radiation coefficients and the consequent different depths in the solar atmosphere from which light of their respective wave-lengths reaches the surface owing to selective absorption. The occurrence of the bright emission line K_2 would be favored by the scattering of the light from the still lower layer.

12. The results for different levels obtained by the spectroheliograph find a possible explanation through considering the underlying faculae as sources of disturbance propagated into the overlying masses of calcium vapor and increasing in different degrees the radiation coefficients of the components of the K line.

MOUNT WILSON SOLAR OBSERVATORY
March 1911

AN INCLOSED ARC FOR SPECTROSCOPIC WORK

By JAMES BARNES

During the past few years the author has been engaged in the study of the spectra of a number of substances in the arc burning in gases under reduced pressure. Different kinds of vessels for holding the arc were constructed, but none gave perfect satisfaction. Often when long exposures were necessary, the heat developed loosened some of the joints so that the pressure could not be kept constant. When certain electrodes, as aluminium, were used, the arc easily went out and could not be started again, due to the fact that the air which entered the vessel united with the hot poles, forming a nonconducting film on their surfaces. Opening the vessel and its readjustment took considerable time, and often the exposed photographic plate was spoiled by the many changes.

In the vessel described in this paper these troubles and many others of less importance have been eliminated. The arc can be kept burning as long as the electrodes remain intact. It is hoped that the apparatus may be of some use along other lines than those indicated below.

DESCRIPTION

Fig. 1 shows a vertical section of the vessel. The rim, *a*, of a thick-walled bell-jar about 25 cm high and 15 cm in diameter is ground to fit an iron plate, *b*. Through the plate run a number of tubes; *c* is connected to an exhaust pump, pressure gauge, and gas reservoirs; d_1 and d_2 are the ends of a lead pipe wound round a sheet-iron cylinder, *e*, and there are two more, *f*, which will be described later. An opening is bored in the jar and a short glass tube, *g*, is sealed into it by means of a preparation called "Caementium." This tube closed by a quartz plate gives a ready exit for the radiations from the arc.

The upper electrode, *h*, is held by a brass tube fixed into the neck of the jar with the above preparation and sealing-wax. This tube contains an inner pipe, *l*. Connecting *k* with *d* by means of rubber tubing and *l* with a source of running water the whole vessel is kept cool. A thin iron disk is placed at *m* to protect the upper part of the vessel from radiation.

The lower electrode, *j*, is held at the end of the long arm of a lever made of an iron bar pivoted at *n*. This electrode is raised to make the arc by holding in the hand near the short arm of the lever a small electro-magnet. When the magnet is removed the electrode falls back to its original position. The screw, *o*, regulates the distance between the poles. To prevent sparking at the pivot a loose wire, *r*, connects the lever with the base to which is fixed the negative binding post.

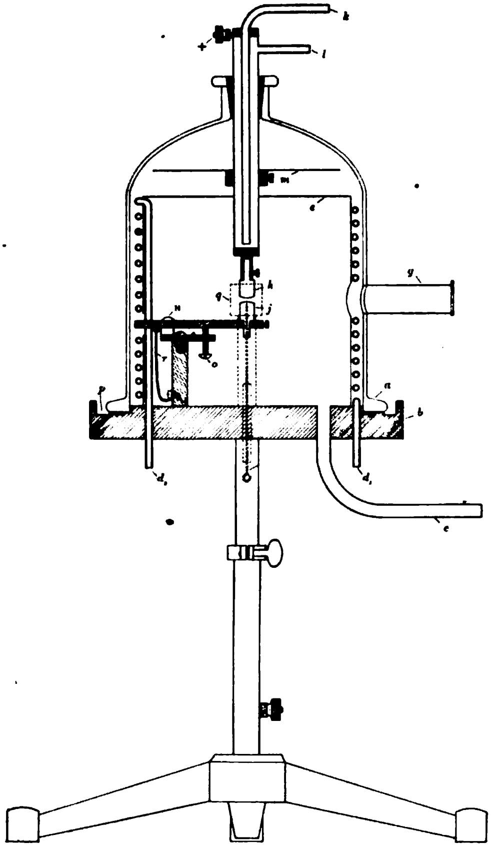

Fig. 1

In order to have no leakage at the ground joint it is necessary to cover the surfaces with a thin layer of grease. A preparation called "Albany grease" was found to be very satisfactory for this purpose. For pressures below a millimeter it is better to cover them with a stopcock lubricant such as that suggested by Travers,[1] and to fill the cavity, *p*, with mercury. By this means and a Gaede pump working continuously, very low pressures can be reached. Care must be taken to have both the ground surfaces perfectly clean before the application of the grease.

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[1] *Study of Gases*, p. 24.

Another way for starting the arc suggested itself during the observations and it led to a few interesting observations. Two small square aluminium plates, *q*, were placed in the vessel in such a position that the poles of the arc rested between them. Wires attached to these plates and sufficiently insulated with ebonite and glass tubes were led through the iron base at *f* and then connected with the terminals of a large induction coil. The discharge from this coil passing through the gap between the poles at once starts the arc when the pressure is sufficiently low. This is true for all the substances so far tried, viz., aluminium, magnesium, calcium, copper, carbon, and iron; in the case of the last three substances it is interesting to note that this method of relighting the arc is successful only when the poles have cooled down so that they do not glow. There is no delay in remaking the arc with the other three substances as electrodes. The current used for the arc was direct, from a 220-volt circuit.

OBSERVATIONS

The arc under reduced pressure gives spectra which are not only purer than those obtained from the arc in air, but the lines are narrower and sharper. This is of much importance for exact measurement of wave-lengths. The narrowness of the lines is best shown by the interference bands they produce when passed through an interferometer. A Fabry and Perot instrument, of the type discussed in a former[1] note, was used and the interference pattern focused on the slit of a Hilger constant-deviation spectroscope. It was found that the bright lines of the iron, copper, and brass arcs *in vacuo* give interference fringes which are much more distinct than those obtained from the same arcs in air. In the case of substances like cadmium it is almost impossible to obtain any interference pattern in any of the lines, but when an iron electrode is bored out and filled with the metal, and then used for one electrode in the arc under reduced pressure, the pattern appears very clearly. The interference fringes are just as sharp as those obtained by using an Heraeus cadmium quartz lamp. This cadmium arc runs very steadily, and with proper adjustment of the current-strength will run for hours on one filling. Through the kindness of Dr. Pfund, I was able to observe the interference pattern produced by the radiations of iron, and iron filled with barium carbonate in the arc *in vacuo* after passing through his[1] apparatus employed for the determination of standard wave-lengths. The observations corroborated those obtained with the less elaborate apparatus; the sharpness of the red and green lines of barium was very marked. I wish to express my thanks to Dr. Pfund for the privilege of using his apparatus and for his assistance in making the observations.

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[1] *Nature*, **80**, 187, 1909.

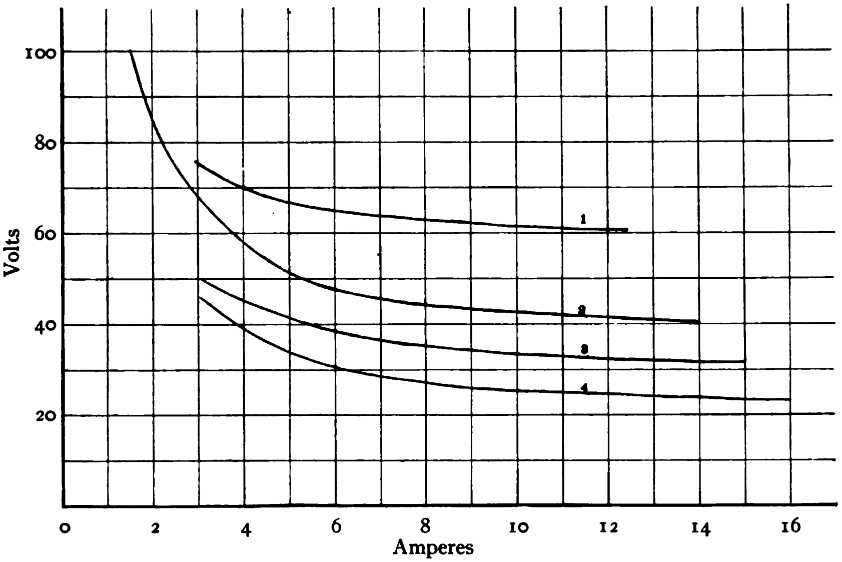

FIG. 2.—1. Carbon arc, atmospheric pressure. 2. Carbon arc, 1 cm pressure. 3. Magnesium arc, atmospheric pressure. 4. Magnesium arc, 1 cm pressure.

This method of obtaining a large number of very bright monochromatic lines by using a number of substances for electrodes should be of use in experiments for the determination of indices of refraction by interference methods.

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[1] *Astrophysical Journal*, **28**, 197, 1908.

A series of observations on the fall of potential across the arc with varying current-strengths was made. Fig. 2, curves 1 and 2, shows the average results of these observations upon the carbon arc in air at atmospheric pressure and at a pressure of 1 cm, respectively. Curves 3 and 4 are those for magnesium. It will be noticed that the voltage across the arc in air is always larger than that across the arc *in vacuo* for the same current-strength and material. The slopes of 1 and 3 are approximately the same, and of 2 and 4 are the same; the rate of change of voltage with current is, however, greater for the arcs *in vacuo* than for the arcs in air, so that for 12 amperes there is a difference of almost 20 volts between the two arcs in the case of carbon and 10 volts in the case of magnesium. These differences are large compared with the total fall of potential. Many times the arc has been seen to leave one terminal and end about 10 cm away on some part of the apparatus. It appears to be a centimeter or more in thickness and is somewhat like a vacuum-tube discharge in a large tube when the pressure is between 1 and 0.5 cm. In this arc the difference of potential between the terminals is over 125 volts and the current-strengths between 1 and 2 amperes.

Bryn Mawr College
April 1911

THE SPECTRA OF ALUMINIUM, COPPER, AND MAGNESIUM IN THE ARC UNDER REDUCED PRESSURE

By JAMES BARNES

The study of the spectra of the elements in the arc and spark under various conditions has been the subject for a very large amount of investigation. The effects produced by reducing the pressure of the gas surrounding the arc has not been thoroughly investigated except in the case of magnesium[1] and calcium,[2] where it was found that at pressures of a few millimeters of mercury the relative intensities of many lines are quite different from their relative intensities in the arc in air at atmospheric pressure. Some lines and bands disappear completely, others make their appearance. An interesting illustration of this is the important line λ 4481 of magnesium. This line is one of the strongest in the spectrum of the arc *in vacuo*, while it is one of the weakest in the arc in air when the current is greater than 3 amperes. Similarly with the calcium bands with heads at λ 6382 and λ 6389, they appear with considerable intensity in the arc *in vacuo*. These flutings are an important feature of sun-spot spectra.

The present paper contains the results of an investigation on the spectra of aluminium and copper in this arc, together with a few further observations on the magnesium spectrum.

APPARATUS AND MEASUREMENTS

The spectra were observed and photographed in the first order of a Rowland concave grating of six feet radius. The dispersion was about 9.33 Å to the millimeter. The apparatus for obtaining the arc under reduced pressures was much improved over that used in earlier work and has been described in the preceding article.[3] Both electrodes were made from rods of the metal and were about 1 cm in diameter. The current used was direct, and was taken from a 220-volt circuit. Its strength depended upon the material of the electrodes used.

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[1] Fowler and Payn, *Proc. R.S.*, **72**, 254, 1903; J. Barnes, *Astrophysical Journal*, **21**, 74, 1905; E. E. Brooks, *Proc. R.S.*, **80**, 218, 1908.

[2] J. Barnes, *Astrophysical Journal*, **27**, 152, 1908, and **30**, 14, 1909.

[3] *Astrophysical Journal*, **33**, 154, 1911.

The wave-lengths of the lines of the aluminium band, which have been measured as accurately as possible, are expressed in terms of the new standards. The wave-lengths of the iron lines employed were taken from Kayser's recent work,[1] "Standards of Third Order of Wave-Lengths on the International System." The measurements were made in the usual way from plates containing the metal and iron spectra by means of a Gaertner micrometer-microscope graduated to 0.001 mm. These plates were taken by exposing one-half of the slit to the metal arc and the other half to the iron arc. Since the plate-holder was not touched during the entire exposure, any displacement of the spectra relative to one another was eliminated. A number of settings was always made on a line and the mean taken. At least two plates were used.

For comparing easily and quickly the spectrum *in vacuo* with that in air, plates were taken by moving the holder so that one spectrum was above the other. This produced a slight shift in one direction or the other. The reproductions on Plate IX were taken in this way.

ALUMINIUM

There is an unmistakable difference in the relative intensities of many of the lines in the arc *in vacuo* compared with their intensities in the arc in air, all other conditions, especially the current-strength, being kept constant. This was about 3 amperes; larger currents caused the electrodes to flow. When the pressure of the air surrounding the arc was about 0.5 cm, the so-called spark lines[2] made their appearance, some with quite considerable intensities, such as $\lambda\lambda$ 4663, 3587, and 2631. Many of these lines in the spark are weakened when self-induction is added to the circuit.[3] The line λ 2816 is worthy of special mention on account of its increase in the vacuum arc. It has also other properties in common

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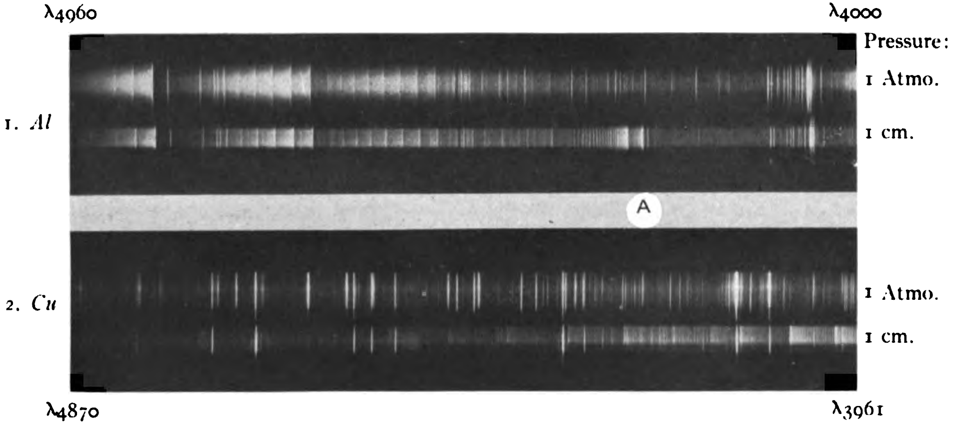
[1] *Astrophysical Journal*, **32**, 217, 1910.

[2] Steinhausen, *Zeitschrift für wissenschaftliche Photographie*, **3**, 45, 1905.

[3] A. S. King, *Astrophysical Journal*, **19**, 237, 1904.

PLATE IX



Spectra under Reduced Pressure

with λ 4481 of magnesium, such as the change of intensity with current-strength.

Within recent years much interest has centered about the four groups of bands lying between λ 5211 and λ 4471. Lockyer suggested that these radiations were due to a compound of aluminium and oxygen. Berndt[1] came to the same conclusion. Arons,[2] Hemsalech,[3] and Hartley[4] believe that they are not due to the oxide but to the metal itself. My observations show that these bands occur just as intense in the arc in air at a pressure of a millimeter or two as they do in air at atmospheric pressure, so that I am inclined toward the latter view. Basquin,[5] working with a rotating arc in hydrogen, observed for the first time a beautiful fluting with four heads at $\lambda\lambda$ 4241, 4260, 4288, and 4353. These bands, which do not appear in air, come out very strongly in the arc *in vacuo*, as is shown at *A* on Plate IX, Fig. 1. The following table gives their wave-lengths. The iron lines used as standards were $\lambda\lambda$ 4260.484, 4254.334, 4299.243.

*4241.21
4241.71
4242.26
4243.08
4244.09
4245.25
4246.58
4248.05
4249.65
4251.36
4253.24
4255.20
4257.38
*4259.68
4260.10
4260.45
4261.08
4261.76
4262.55
4263.52
4264.58
4265.80
4267.20
4268.80
4270.64
4272.69
4274.95
4277.64
4280.65
4283.88
4287.17
4290.62
4294.23
4298.03
4302.06
4306.24
4310.72
4315.46
4320.51
*4353.27
4354.05
4355.07
4356.45
4358.34
4360.46
4361.10
4362.07
4363.37
4365.03
4367.08
4369.54
4371.36
4374.95
4378.95

* **Indicates heads.**

COPPER

The copper arc *in vacuo* is almost as brilliant as it is in air. With the current used, about 8 amperes, both poles became red hot and remained so throughout the exposures. This current-strength was found necessary; when more resistance was turned into the circuit the arc quickly went out.

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[1] *Annalen der Physik*, 4, 788, 1901.

[2] *Ibid.*, 1, 700, 1900.

[3] *Ibid.*, 2, 331, 1900.

[4] *Trans. Roy. Dublin Soc.*, 9, 85, 1908.

[5] *Astrophysical Journal*, 14, 8, 1901.

Copper has without doubt a larger number of lines affected by the reduction of pressure than any of the elements so far tried. The change in the intensities of the lines in the region between λ 4870 and λ 3960 are shown on Plate IX, Fig. 2. Different from aluminium and calcium, only the faintest outlines of bands besides those of nitrogen appear on the plates of longest exposure in the places where they have been found by Basquin (*loc. cit.*) in the arc in hydrogen, and by King[1] in the electric furnace. The time of exposure for these plates was about 30 minutes and a wide slit (0.2 mm) was used. These bands are so faint that the author is not sure they are the ones found by the above observers. On the plates taken with 1 and 2 minutes' exposure, which is quite sufficient for good photographs of the line spectra, not a trace of the bands could be seen. The following lines show clearly a decrease of intensity in the arc *in vacuo*. Those with an asterisk are weakened the greater amount.

4866.4
4794.2
4767.7
*4697.6
*4675.0
4642.8
*4587.2
*4540.0
4507.6
*4378.4
4259.6
*4177.9
*2392.7
2293.9
*2263.2

It will be found on reference to Kayser's *Handbuch der Spectroscopie*, **5**, pp. 401 f., that all of these lines are found in the arc in air, many of them broad and hazy; a few are found in the spark, but always with a smaller intensity than in the arc. Mention should be made of λλ 4675, 4587, 4378, and 4177, for they are just visible on the plates of longest exposure, while they are among the strongest in the arc in air.

MAGNESIUM

The current used was about 4 amperes. The effect of the reduction of pressure upon the intensities of the arc lines lying between λ 4703 and λ 4058 and upon the spark line λ 4481 have been recorded (*loc. cit.*). The author wishes to extend these observations to include the following lines, whose relative intensities are considerably increased in the vacuum arc, viz.: λλ 2790.9 2798.1, 2928.9, 2936.8, and 3107.1; the first four are intense spark lines. The two pairs at λλ 4433.6, 4428.2, 4390, and 4385, which Brooks calls the "F and P" lines, also appear strongly.

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[1] *Astrophysical Journal*, **21**, 250, 1905.

The so-called oxide and hydride bands appear on the plates of the spectrum of the arc in air at a few millimeters pressure.

CONCLUSIONS

The results of the observations upon the spectra of the elements which have been investigated so far in the arc under reduced pressure may be divided into two classes.

1. The bands which have been attributed to compounds of oxygen and hydrogen with aluminium, magnesium, and calcium all appear very clearly in this arc. Whether or not they are true oxide or hydride bands one must admit is still an open question. For it can always be claimed that there is enough oxygen left in the vessel surrounding the arc or sufficient hydrogen occluded in the metals to produce these bands. It might be of interest to state in this connection that the hydrogen lines could not be found on the plates, while the nitrogen bands were very clear.

2. With regard to the lines, the effect of the reduction of the pressure is in general to increase the intensity of the spark lines of aluminium and magnesium, and to diminish many of the arc lines of copper. Whatever the explanation of this may be, one can state that the arc *in vacuo* is a step between the arc in air and the spark, although the potential-drop between the electrodes in this arc is always smaller than that in air for the same current-strength. The lines mentioned in this paper, which are those whose intensities change the most, do not belong to the Principal or Subordinate Series in Kayser's arrangement.

In conclusion I wish to express my thanks to two of my students, Miss Frehafer and Miss Howson, for assistance in making these observations.

Bryn Mawr College
April 1911

AN INQUIRY INTO THE VARIATION OF THE SPECTROSCOPIC BINARY κ *PAVONIS*

BY ALEX. W. ROBERTS

When the binary character of κ *Pavonis* was certified to by Wright,[1] its variation became matter of very careful inquiry. In order to obtain measures of this star—and also of *l Carinae* and *v Puppis*, both bright spectroscopic binaries—as free from systematic error as possible, a mode of observation was adopted that, I think, has proved satisfactory.

Briefly stated, the variable and its comparison stars are observed in a plane mirror, so mounted that the field can be rotated through any angle. Four observations, one in each quadrant, are taken by eye estimates. These determinations are then reduced for scale-value and standard value. One observation is considered to be the mean of the four determinations.

In order to free the mind from any bias, no ephemeris of the star was constructed and frequently observations were dropped for some weeks in order to eliminate any conception regarding its variation that might unconsciously have arisen during the progress of the observations. The star's period seems subject to secular change, and since this variation has not been sufficiently determined, I shall confine myself, in the present paper, to a discussion of the 1910 observations. These exhibit the chief features of variation that it is the purpose of this paper to consider.

The following table sets forth these observations. In column 1 is given the date of observation; in column 2, the Julian date of observation; in column 3, the date of observation reduced to mean curve, taking as mean period

9.09106 days;

in column 4 is found the observed magnitude; column 5 gives the magnitudes computed from the mean curve for the year 1910, and column 6 gives the (O.—C.) residuals.

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[1] *Lick Observatory Bulletin*, **3**, 3, 1904; *Astrophysical Journal*, **20**, 141, 1904.

1910 OBSERVATIONS OF κ *PAVONIS*

Date	Julian Day	Reduced Date	Obs. Mag.	Comp. Mag.	O.−C.
	24187				
April 2^{d} 9^{h} 52^{m}	64.41	2419028.05	$3^{\mathrm{m}}69$	$3^{\mathrm{m}}86$	$-0^{\mathrm{m}}17$
3 11 25	65.48	029.12	3.85	3.71	+0.14
11 50	65.49	029.13	3.85	3.71	+0.14
12 30	65.52	029.16	3.86	3.72	+0.14
4 9 10	66.38	030.02	4.24	4.21	+0.03
7 9 45	69.41	023.96	5.28	5.21	+0.07
10 9 10	72.38	026.93	4.84	4.90	−0.06
10 40	72.44	026.99	4.84	4.89	−0.05
11 50	72.49	027.04	4.73	4.87	−0.14
11 8 25	73.35	027.90	4.08	3.98	+0.10
10 25	73.43	027.98	3.97	3.92	+0.05
15 10 55	77.46	022.92	4.92	4.98	−0.06
30 7 56	92.33	028.69	3.87	3.67	+0.21
May 3 8 10	95.34	022.61	4.85	4.90	−0.05
9 5	95.38	022.65	4.82	4.91	−0.09
10 3	95.42	022.69	4.87	4.91	−0.04
11 40	95.49	022.76	4.97	4.94	+0.03
4 9 25	96.40	023.67	5.11	5.15	−0.04
6 8 55	98.37	025.64	5.08	5.13	−0.05
10 0	98.42	025.69	5.06	5.12	−0.06
7 8 50	99.37	026.64	5.03	4.98	+0.05
9 10	99.38	026.65	4.97	4.98	−0.01
14 55	99.62	026.89	4.93	4.91	+0.02
15 25	99.64	026.91	4.91	4.91	0.00
	24188				
8 9 15	00.39	027.66	4.28	4.19	+0.09
10 15	00.43	027.70	4.23	4.15	+0.08
11 5	00.46	027.73	4.24	4.13	+0.11
9 7 35	01.32	028.59	3.74	3.67	+0.07
8 30	01.35	028.62	3.82	3.67	+0.15
9 25	01.40	028.67	3.73	3.67	+0.06
10 7 20	02.31	029.58	3.86	3.94	−0.08
8 30	02.35	029.62	3.88	3.92	−0.04
10 0	02.42	029.69	3.98	4.00	−0.02
12 8 50	04.37	022.55	(4.60)	4.89	(−0.29)
9 35	04.40	022.58	4.76	4.89	−0.13
13 6 55	05.29	023.47	5.15	5.11	+0.04
14 7 15	06.30	024.48	5.24	5.24	0.00
8 30	06.35	024.53	5.29	5.24	+0.05
15 9 25	07.39	025.57	5.20	5.16	+0.04
10 10	07.42	025.60	5.10	5.15	−0.05
16 9 56	08.41	026.59	5.05	4.98	+0.07
12 40	08.53	026.71	5.02	4.97	+0.05
June 1 6 30	24.27	024.27	5.20	5.23	−0.03
7 55	24.33	024.33	5.19	5.24	−0.05
9 12	24.38	024.38	5.21	5.24	−0.03
10 15	24.43	024.43	5.20	5.24	−0.04
2 6 43	25.28	025.28	5.26	5.20	+0.06
7 40	25.32	025.32	5.21	5.20	+0.01
9 26	25.39	025.39	5.21	5.19	+0.02
3 6 35	26.27	026.27	5.00	5.01	−0.01
7 50	26.33	026.33	5.01	5.01	0.00
10 30	26.44	2419026.44	5.01	5.00	+0.01

1910 OBSERVATIONS OF κ *PAVONIS—Continued*

Date	Julian Day	Reduced Date	Obs. Mag.	Comp. Mag.	O.−C.
	24188				
June $3^d 15^h 30^m$	26.65	2419026.65	4^m96	4^m97	-0^m01
4 10 0	27.42	027.42	4.43	4.39	+0.04
5 7 30	28.31	028.31	3.68	3.71	−0.03
8 35	28.36	028.36	3.66	3.69	−0.03
9 30	28.40	028.40	3.69	3.68	+0.01
6 5 45	29.24	029.24	3.66	3.75	−0.09
6 45	29.28	029.28	3.70	3.76	−0.06
8 5	29.34	029.34	3.67	3.79	−0.12
10 0	29.42	029.42	3.72	3.83	−0.11
9 8 10	32.34	023.25	5.07	5.06	+0.01
9 20	32.39	023.30	5.06	5.07	−0.01
9 50	32.41	023.32	5.09	5.08	+0.01
11 6 10	34.26	025.17	5.25	5.21	+0.04
6 58	34.29	025.20	5.21	5.21	0.00
14 5 0	37.21	028.12	3.78	3.81	−0.03
15 5 40	38.24	029.15	3.67	3.71	−0.04
9 3	38.38	029.29	3.67	3.77	−0.10
10 25	38.43	029.34	3.67	3.79	−0.12
19 11 45	42.49	024.31	5.19	5.23	−0.04
15 55	42.66	024.48	5.22	5.24	−0.02
July 25 6 55	78.29	023.75	5.27	5.17	+0.10
26 6 25	79.27	024.73	5.24	5.24	0.00
30 6 55	83.29	028.75	3.63	3.67	−0.04
8 45	83.36	028.82	3.62	3.67	−0.05
9 40	83.40	028.86	3.62	3.68	−0.06
10 30	83.44	028.90	3.66	3.68	−0.02
Aug. 3 5 16	87.22	023.58	5.23	5.14	+0.09
6 0	87.25	023.61	5.15	5.14	+0.01
7 25	87.31	023.67	5.15	5.15	0.00
8 36	87.36	023.72	5.15	5.16	−0.01
9 36	87.40	023.76	5.12	5.17	−0.05
5 5 0	89.21	025.57	5.10	5.16	−0.06
6 30	89.27	025.63	5.07	5.14	−0.07
8 50	89.37	025.73	5.08	5.11	−0.03
9 55	89.42	025.78	5.06	5.09	−0.03
11 5	89.46	025.82	5.05	5.08	−0.03
12 30	89.52	025.88	5.05	5.07	−0.02
6 8 13	90.34	026.70	4.89	4.97	−0.08
9 25	90.39	026.75	4.96	4.96	0.00
10 38	90.44	026.80	4.99	4.95	+0.04
7 5 25	91.22	027.58	4.20	4.24	−0.04
7 35	91.32	027.68	4.06	4.16	−0.10
8 55	91.37	027.73	4.00	4.13	−0.13
15 0	91.63	027.99	3.73	3.90	−0.17
10 5 10	94.22	030.58	4.54	4.54	0.00
6 30	94.27	030.63	4.59	4.57	+0.02
8 43	94.36	030.72	4.65	4.61	+0.04
9 45	94.41	030.77	4.59	4.63	−0.04
11 5 45	95.24	022.51	5.03	4.87	+0.16
7 40	95.32	022.59	4.98	4.90	+0.08
9 40	95.40	022.67	5.00	4.91	+0.09
10 30	95.44	022.71	4.93	4.92	+0.01
11 15	95.47	022.74	4.94	4.93	+0.01
15 15	95.64	2419022.91	4.94	4.97	−0.03

1910 OBSERVATIONS OF κ *PAVONIS—Continued*

Date	Julian Day	Reduced Date	Obs. Mag.	Comp. Mag.	O.−C.
	24189				
Sept. 13^{d} 7^{h}30^{m}	28.31	2419028.31	3^{m}61	3^{m}71	−0^{m}10
8 50	28.37	028.37	3.64	3.69	−0.05
22 7 10	37.30	028.21	3.86	3.76	+0.10
9 10	37.38	028.29	3.71	3.71	0.00
Oct. 31 6 35	76.19	030.74	4.79	4.62	+0.17
7 28	76.31	030.86	4.75	4.65	+0.10
Nov. 1 6 25	77.27	031.82	5.02	4.93	+0.09
6 55	77.29	031.84	4.96	4.93	+0.03
8 0	77.33	031.88	5.04	4.95	+0.09
2 7 30	78.31	023.77	5.08	5.18	−0.10
8 6 30	84.27	029.73	4.08	4.03	+0.05
9 6 25	85.27	030.73	4.73	4.61	+0.12
7 50	85.33	2419030.79	4.60	4.64	−0.04

Average error of a single observation = ±0^{m}057

Grouping together the dates given above in column 3, we obtain the following mean magnitudes of κ *Pavonis*, using 1910 observations only.

MEAN LIGHT-CURVE: 1910

No. of observations	Date	Mean Mag.	Mag. from Light-Curve	O.−C.
	24190			
4	22.56	4^{m}84	4^{m}88	−0^{m}04
4	22.66	4.89	4.91	−0.02
4	22.73	4.96	4.93	+0.03
4	22.85	4.97	4.96	+0.01
4	23.34	5.09	5.09	0.00
4	23.63	5.16	5.15	+0.01
5	23.79	5.18	5.18	0.00
5	24.34	5.20	5.23	−0.03
4	24.56	5.25	5.24	+0.01
5	25.27	5.23	5.20	+0.03
5	25.60	5.11	5.15	−0.04
5	25.78	5.06	5.09	−0.03
4	26.41	5.02	5.00	+0.02
4	26.66	4.96	4.97	−0.01
4	26.79	4.98	4.95	+0.03
4	26.97	4.83	4.88	−0.05
5	27.61	4.24	4.23	+0.01
5	27.85	4.00	4.03	−0.03
4	28.17	3.76	3.77	−0.01
4	28.34	3.65	3.69	−0.04
4	28.57	3.75	3.67	+0.08
5	28.80	3.68	3.67	+0.01
5	29.16	3.78	3.72	+0.06
4	29.31	3.68	3.77	−0.09
5	29.61	3.90	3.94	−0.04
4	30.49	4.51	4.50	+0.01
5	30.78	4.69	4.64	+0.05

The light-curve obtained from the data given in columns 2 and 3 is represented in Fig. 1, and the magnitudes indicated by this mean curve find a place in the foregoing tables as the theoretical computed magnitudes. The striking peculiarity of the light-curve of *κ Pavonis* is what appears to be a secondary phase, nearly midway between principal minimum and principal maximum. This secondary phase is very persistent in all the observations of *κ Pavonis*. In those taken during 1907 it is very marked. If, now, we regard *κ Pavonis* as varying uniformly after the simple type of most short-period variables, we may readily compute the amount of departure of this secondary depression from the fundamental curve. That is, we may regard this simple fundamental curve as the real basis of the star's variation, and the secondary aberration as due to causes of a different order. This fundamental curve, shown as a continuous curve in Fig. 1, may be defined as:

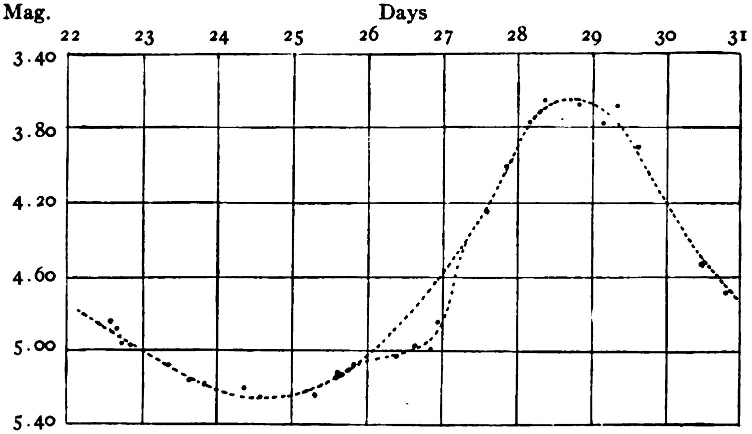

FIG. 1.—Light-curve of *κ Pavonis*, 1910. Julian Day 2419000+

$$\begin{aligned}4^{\mathrm{m}}.62 &+ 0^{\mathrm{m}}.74 \cos(\ \theta - \ 53^\circ\, 45') \\ &+ 0.18 \cos(2\theta - 265^\circ\, 35') \\ &+ 0.07 \cos(3\theta - 142^\circ\, 38') \\ &+ 0.03 \cos(4\theta - \ 85^\circ\ \ 0') \\ &+ 0.01 \cos(5\theta - 141^\circ\ \ 0'),\end{aligned}$$

where θ is the time after 2419023 expressed in angular measure. The secondary departure from this primary curve becomes:

$2419025^{d}.78$	$-0^{m}.02$
26.41	+0.14
26.66	+0.18
26.79	+0.28
26.97	+0.21
27.59	0.00.

Representing this in the form of a curve, we have the secondary minimum indicated in Fig. 2. Now a curve of this form suggests eclipse. It is indeed a valid and reasonable explanation. Spectroscopic inquiry has proved the duplicity of *κ Pavonis*. It would seem, therefore, that superimposed upon the variation of short-period character, there is, for a portion of the star's period, a diminution in brightness due to the partial eclipse of the secondary star by its primary.

The 1910 observations indicate the following elements of eclipse:

Amplitude of eclipse	$0^{m}.24$
Duration of eclipse	2.0 days
Minimum—central eclipse	2.1 days
Central eclipse—maximum	1.9 days

Taking $2419026^{d}.8$ as the date of central eclipse, that is, the instant when the stars are in the line of sight, and the velocity of approach zero, maximum approach should occur at,

$2419029^{d}.1$

and maximum recession at,

$2419024^{d}.5$

It will be of no ordinary interest to discover if spectroscopic research confirm these deductions.

Assuming these conclusions to be justified, then in 1910 maximum brightness of *κ Pavonis* took place 0.3 day before maximum approach and minimum brightness 0.2 day after maximum recession.

In the opening portion of this paper, I implied that an unascertained secular change in the period of *κ Pavonis* prevented me from combining all the observations made at Lovedale—considerably over a thousand—into one mean curve. But it may be of interest to state what the 1907 observations reveal regarding the secondary minimum of *κ Pavonis*. Considering this minimum as due to eclipse, then the following facts emerge:

Amplitude of eclipse	$0^{\text{m}}.26$
Duration of eclipse	2.1 days
Minimum—central eclipse	2.1 days
Central eclipse—maximum	2.0 days
Maximum approach	$2417928^{\text{d}}.6$
Maximum brightness	2417928.3
Difference	0.3 day
Maximum recession	2417924.0
Minimum brightness	2417924.2
Difference	0.2 day

The curious will notice that the relation of maximum and minimum phases to quadrature passage is exactly the same for 1910 as for 1907, and the relation is in accordance with that rigorously established in the case of other short-period stars.

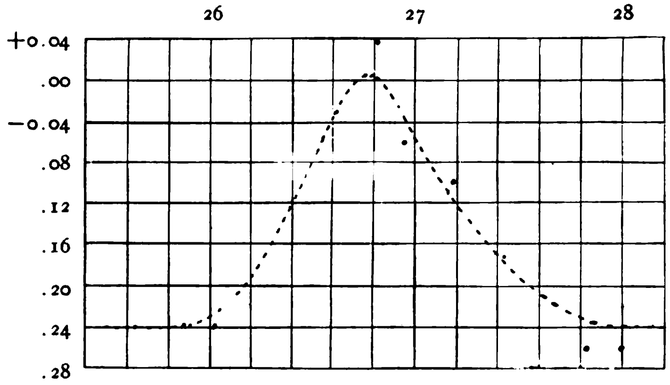

FIG. 2.—Secondary phase of *κ Pavonis*, 1910. Julian Day 2419000+

There is a temptation to compute the relations of size, brightness, inclination, and indirectly density, that would give a light-curve such as is set forth in Fig. 2. But the quantities are quite indeterminate. All we can assure ourselves of is that if eclipse be an explanation of the secondary phase of *κ Pavonis*, then the central star is larger, darker, and more massive than its satellite. Also we may conclude that the rotation and revolution of both stars are not synchronous. More than this, at this present stage, we are unable to say.

A rigorous investigation of the orbital movement of this star is urged upon those who have the necessary equipment for such an inquiry. For if it can be placed beyond the region of doubt that the binary character of *κ Pavonis* is as we have here urged, then we have moved forward somewhat toward a fuller understanding of the nature and cause of short-period variation.

LOVEDALE, SOUTH AFRICA
December 1910

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THE
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THE EFFECT OF RED AND INFRA-RED ON THE DECAY OF PHOSPHORESCENCE IN ZINC SULPHIDE

By HERBERT E. IVES AND M. LUCKIESH

The effect of long-wave radiation upon phosphorescence has been studied by nearly all investigators of phosphorescent phenomena. The action of red or infra-red is to hasten the decay of phosphorescence, a phenomenon used by Draper and others to map infra-red spectra. The manner of action differs with the phosphorescent substance. With many the incidence of long-wave energy causes a flashing-up of luminescence, after which follows a rapid decay. With some others this preliminary flash is apparently absent. Among these latter is zinc sulphide, variously known as sidot-blende or "Emanationspulver," depending on its source or method of preparation. Among recent observers who have studied this substance may be mentioned Dahms,[1] Nichols and Merritt,[2] Werner,[3] and Lenard.[4] Various phases of the behavior of this sulphide have occupied the attention of each of these investigators but they have been in general agreement until recently that the flashing-up of phosphorescence is absent. But Lenard, who distinguishes several varieties of phosphorescent zinc sulphide, mentions in his latest publication that even in the variety most susceptible to the extinguishing action of red light a very brief flash may be seen if properly looked for.

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[1] *Annalen der Physik*, **13**, 425, 1904.

[2] *Physical Review*, **25**, 362; **23**, 37; **22**, 279; etc.

[3] *Annalen der Physik*, **24**, 164, 1907; **30**, 257, 1909.

[4] *Ibid.*, **31**, 641, 1910.

This phenomenon is of some importance in developing a satisfactory theory of phosphorescence. It was originally held, by Becquerel, that the total quantity of radiation given out by a phosphorescent mass during decay is a constant. The effect of red radiation is simply to accelerate the giving up of the stored energy; consequently the flashing-up is a necessary process to maintain the constancy of the total radiation. The existence of substances not exhibiting this effect demands the discarding or the modifying of this idea. A knowledge of the true character of the behavior of these substances under the influence of long-wave energy is necessary to indicate the requisite modification of this original theory or the substitute for it.

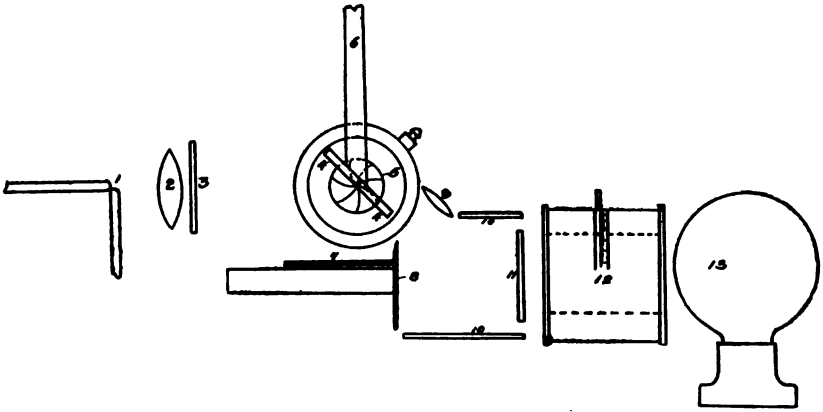

FIG. 1.—Diagram of apparatus

The experiments described in this paper show that in zinc sulphide the flashing-up phenomenon is clearly present, with the peculiarity that it becomes striking only upon the lapse of some time after the beginning of the decay of phosphorescence. The investigation started by an observation made during a test of zinc sulphide for its use in the detection of infra-red radiation in the fire-fly. At first it appeared that a flash of red light extinguished the phosphorescence only if it occurred within a few seconds after the beginning of decay. Later during the process of decay a similar brief exposure to red light caused no extinction. But further observation showed that the extinguishing effect was apparently merely delayed in starting. If given time, the extinction produced by an exposure five minutes after excitation would attain as great a value as that from a similar exposure immediately after excitation.

For the exact study of this phenomenon a new arrangement of photometric apparatus was employed, possessing some decided advantages over any heretofore published. The apparatus is shown diagrammatically in Fig. 1: 7 is the surface of zinc sulphide lying horizontally; 8, a thin metal sheet whitened with magnesium oxide and standing vertically. Excitation of the sulphide is produced by the light of a carbon arc through blue glasses (transmitting no red) at 3, the light being reflected by the platinum-faced mirror, 4. This mirror can be turned about a vertical axis, 6, so as to receive when desired the red or infra-red radiation from a 100 c.p. carbon lamp behind 5. At 5 is a "Volute" photographic shutter for regulating the time of exposure to red or infra-red. The vertical surface at 8 is illuminated by the incandescent lamp at 13 whose light passes through the Brodhun sector, 12, and through the colored glasses, 11. By changing the voltage of the lamp and by altering the opening of the sector, a very large range of illuminations is obtainable. The observer looking through the lens, 9, sees as photometric field the zinc sulphide surface, 7, and the vertical sheet, 8. Holding his hand upon the sector he is able to make photometric settings rapidly, while an assistant reads the times and the opening of the sector. By this means such phenomena as the flash-up of luminosity may be followed and measured accurately. By the addition of a chronograph and an automatic recorder this apparatus should make it possible to secure measurements just as fast as the eye and hand can make settings. For the present work sufficient speed was attained without these additions.

The first measurements made with this apparatus revealed a more interesting phenomenon than had been suspected. The initial experiments were to determine the effect of flashes of one second's duration of the light from the incandescent lamp through a ruby glass (carefully tested to insure no blue or ultra-violet transmission). The phosphorescent surface was first reduced to a standard condition according to the procedure of Nichols and Merritt by five minutes' exposure to red, followed by five minutes' rest. The duration of excitation was uniformly two minutes (except where noted otherwise) with a constant current through the arc. Under these conditions, the initial brightness of phosphorescence and the rate of decay were very satisfactorily uniform. The results are shown in Fig. 2. It appears that after a certain lapse of time, zinc sulphide does show the flashing-up phenomenon. The apparent lag in the extinguishing effect which first attracted attention appears to be merely the intermediate stage between the early condition where rapid extinction occurs and the later condition where increased brightness precedes extinction.

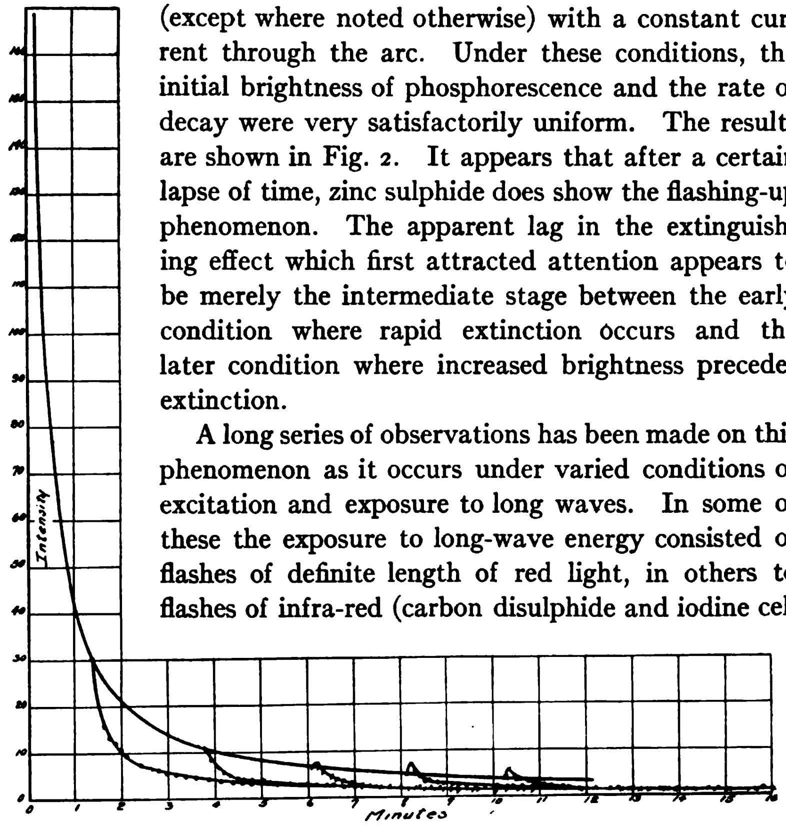

FIG. 2.—Effect of red light on the phosphorescence of zinc sulphide at different periods during decay. Full excitation 2 minutes; one second flashes of red.

A long series of observations has been made on this phenomenon as it occurs under varied conditions of excitation and exposure to long waves. In some of these the exposure to long-wave energy consisted of flashes of definite length of red light, in others to flashes of infra-red (carbon disulphide and iodine cell added to ruby glass), in still others the exposure to infra-red once started was allowed to continue. This last procedure of course is only compatible with the observation and measurement of phosphorescence if the extinguishing radiation is invisible. The particular mode of working will be noted as the various experiments are described. Those first performed were with red light while the later ones demanded infra-red, and in making the change no qualitative difference in the effect of the two was found.

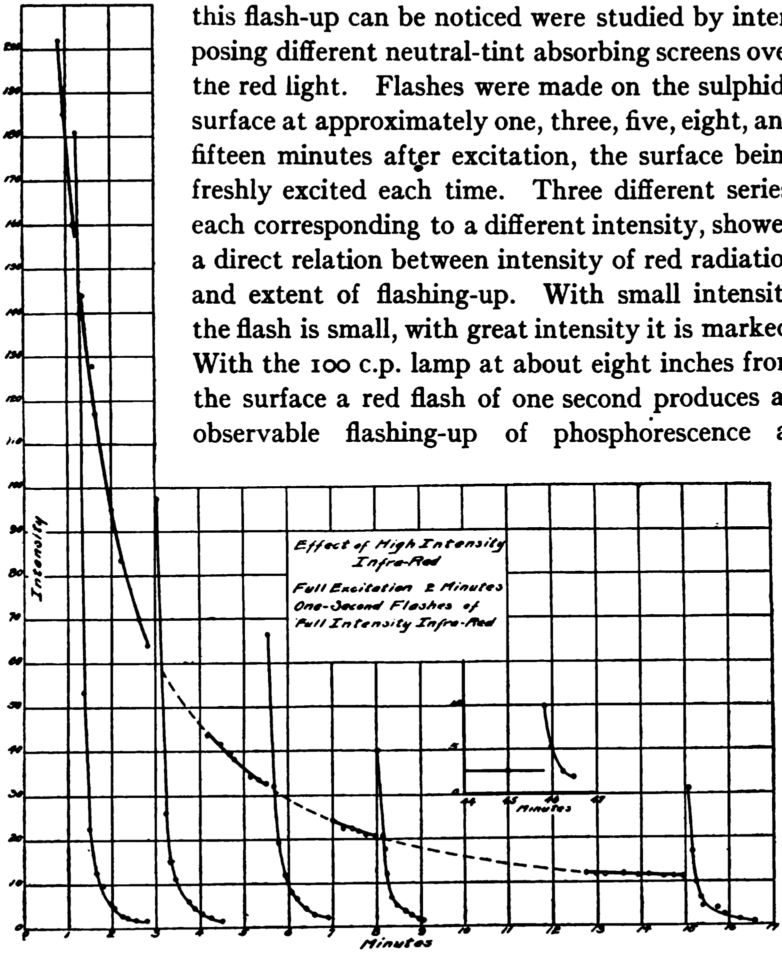

FIG. 3.—Effect of high intensity of infra-red

Effect of varied quantity of red radiation.—The amount of flashing-up and the shortest interval after the beginning of decay that this flash-up can be noticed were studied by interposing different neutral-tint absorbing screens over the red light. Flashes were made on the sulphide surface at approximately one, three, five, eight, and fifteen minutes after excitation, the surface being freshly excited each time. Three different series, each corresponding to a different intensity, showed a direct relation between intensity of red radiation and extent of flashing-up. With small intensity the flash is small, with great intensity it is marked. With the 100 c.p. lamp at about eight inches from the surface a red flash of one second produces an observable flashing-up of phosphorescence as close as thirty seconds from the beginning of decay, while at 15 minutes the flash is several hundred per cent of the brightness of phosphorescence just preceding it. With a tenth of this intensity of the red the flashing-up is much smaller, and not noticeable till a later time. Fig. 3 gives a typical decay-curve with the effects of a red flash of high intensity at various times. In the observations plotted in Fig. 4 a large range of intensities of red light were used and the flashes were all made at approximately five minutes after excitation. They show the dependence of the amount of the flashing-up on the intensity of red light. These have been plotted diagrammatically to one side.

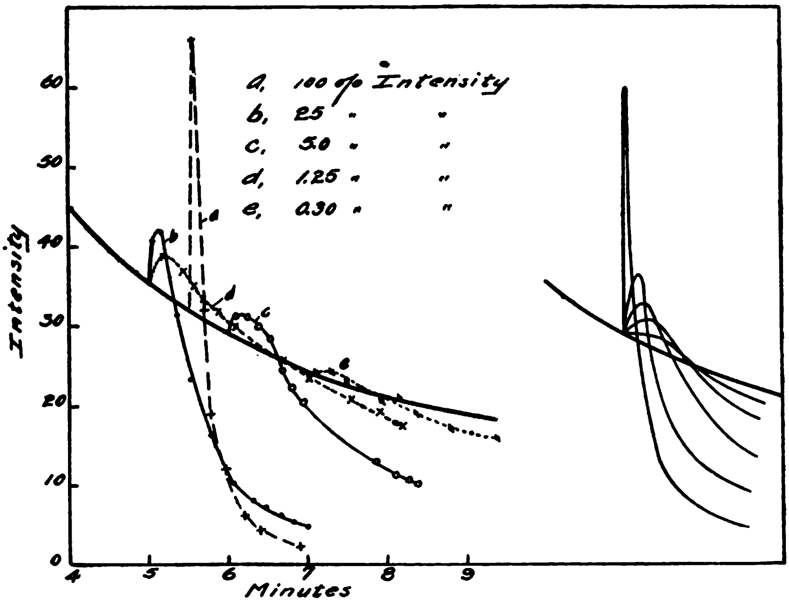

FIG. 4.—Effect on flashing-up phenomenon of varying the intensity of infra-red

Effect of weaker excitation.—Fig. 5 shows three decay-curves, corresponding to three different intensities of the exciting light. At approximately the same times after the beginning of decay the same exposure to red light has been given to each. It is at once evident that the significant factor is not the brightness to which the phosphorescence has fallen but the time that has elapsed since excitation. The first red flash on the lowest curve, for instance, occurs when the brightness of phosphorescence is already lower than it is on the highest curve at a time when the flashing-up is very marked.[1]

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[1] The indicated flash-ups on the lowest curve (*c*) were observed qualitatively, but no measured points above the curve were obtained.

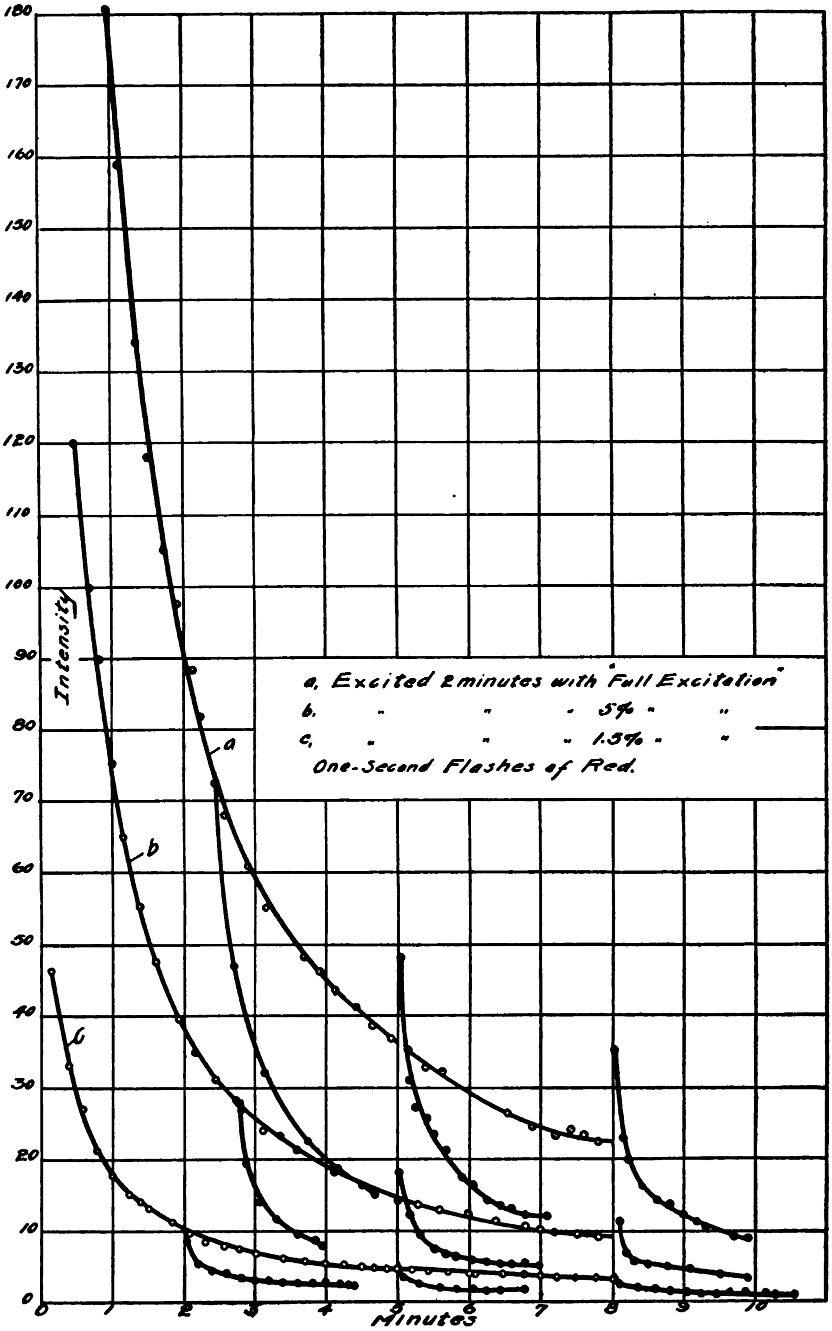

FIG. 5.—Effect of different intensities of excitation on the flashing-up phenomenon

Effect of shorter excitation.—Exactly similar results to those of Fig. 5 are obtained if lower decay-curves are produced by shortening the excitation to a few seconds. These confirm the conclusion that the occurrence of the flashing-up is determined not by the condition of phosphorescence as measured by its brightness, but by the time that has elapsed since the beginning of decay.

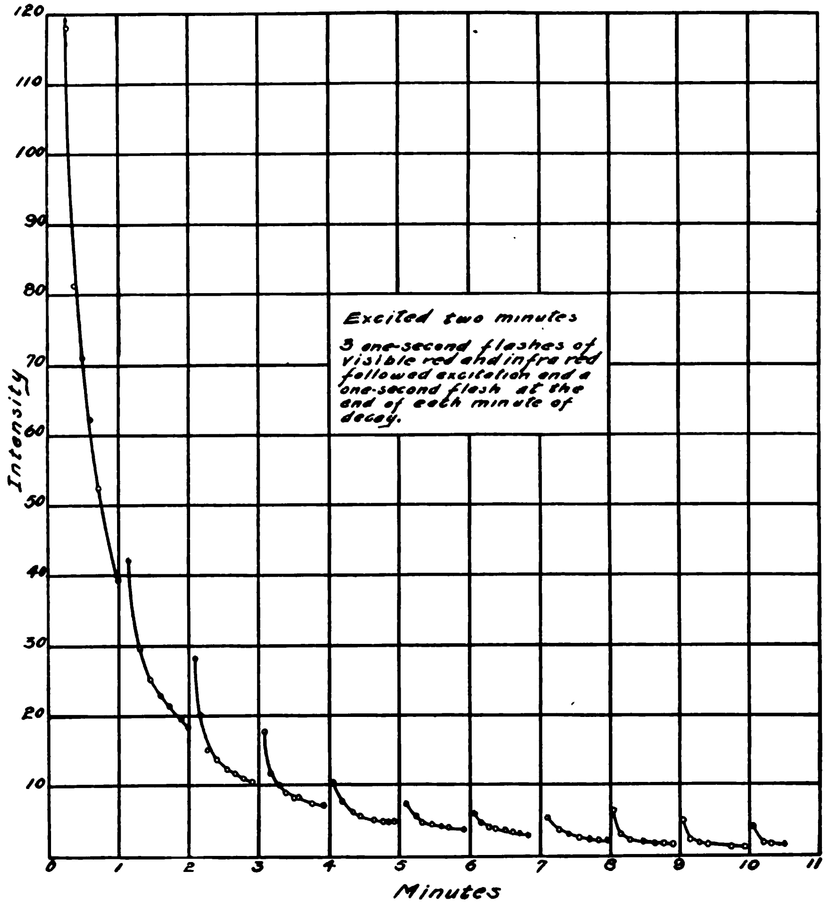

FIG. 6.—Effect of previous exposure to red light on the susceptibility to flashing-up

Effect of previous exposure to red or infra-red.—Short exposure to long-wave radiation, which in general acts to decrease the brightness of phosphorescence, does, unlike decreased or shortened excitation, hasten the condition of flashing-up. An exposure of 3 seconds to red light, immediately after excitation, brings the substance to a condition such that there is a pronounced flashing-up long before flashing-up would occur under other circumstances when flashed with a given intensity of red light. Successive exposures to red light produce strong flashes of phosphorescence apparently for an indefinite period. We have watched and measured these bursts of phosphorescence produced by successive flashes of one second each at intervals of one minute up to the number of ten, when the phenomenon was as pronounced as at the start (Fig. 6).

Observations on different samples.—The zinc sulphide used in these experiments was a Verneuil preparation furnished by Eimer & Amend. With the possibility in mind that it was of a quite different character from that employed by previous observers an effort was made to secure other specimens. Through the kindness of Professor E. L. Nichols we were provided with a piece of the sidot-blende screen used by him and Professor Merritt in their work, and with a specimen of "Emanationspulver." Some of the experiments above described were repeated with these samples. With all three specimens the flashing-up phenomenon is present, but in different degree; similarly other properties, such as color of phosphorescence and shape of the decay-curve, are different, and it will be seen that they fall into a certain uniform order with respect to all their properties.

Flashing-up phenomenon.—The flash-up is least marked in the sidot-blende screen, most marked in the "Emanationspulver," our sample of zinc sulphide lying between. Fig. 7 shows the percentage increases in brightness due to exposure to infra-red radiation, of uniform intensity in all cases. The infra-red was allowed to continue its action when once turned on, the luminosity being followed by the Brodhun sector till it reached a maximum. The sidot-blende screen, which shows the least flashing-up, also shows the most rapid decay under the infra-red.

Decay-curves.—The decay of phosphorescence is frequently plotted with time as abscissas, and $\frac{1}{\sqrt{I}}$, where I is intensity, as ordinates. A close approximation to a straight line has usually been found, this corresponding to a decay law of $I=\frac{1}{(a+bt)^m}$ where $m=2$. Nichols and Merritt find two straight lines, merging one into the other a few seconds after the beginning of the decay. Werner, in agreement with the theory of Lenard, finds two processes in the decay: one a long process which plots as a straight line in terms of $\frac{1}{\sqrt{I}}$ against time; the other a short process superposed on the long, and following an exponential law. In sulphides containing a large amount of active metal the short process lasts for a much shorter period than in sulphides containing a small quantity. Consequently the decay-curves when plotted in terms of $\frac{1}{\sqrt{I}}$ more quickly become straight lines in the former than in the latter case. Werner has also experimentally determined the constants in the two equations $I_1=\frac{1}{(a+bt)^2}$ and $I_2=ae^{-\beta t^m}$ such that the summation of these two curves represents very satisfactorily the different decay-curves obtained with different compositions of the sulphide. Upon plotting the decay-curves of our sample of zinc sulphide in

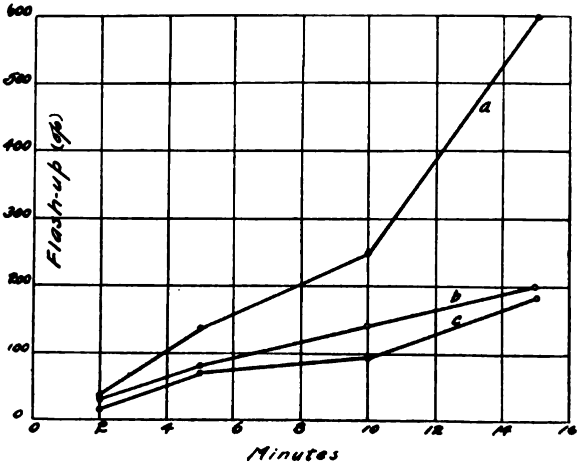

FIG. 7.—Magnitude of flashing-up for different samples. Time is that elapsed since excitation.

terms of $\frac{1}{\sqrt{I}}$ a curve concave to the time-axis was obtained, which approached nearer a straight line as time went on, although even at thirty minutes it is questionable whether perfect straightness is reached. A decay-curve of this substance—the mean of many

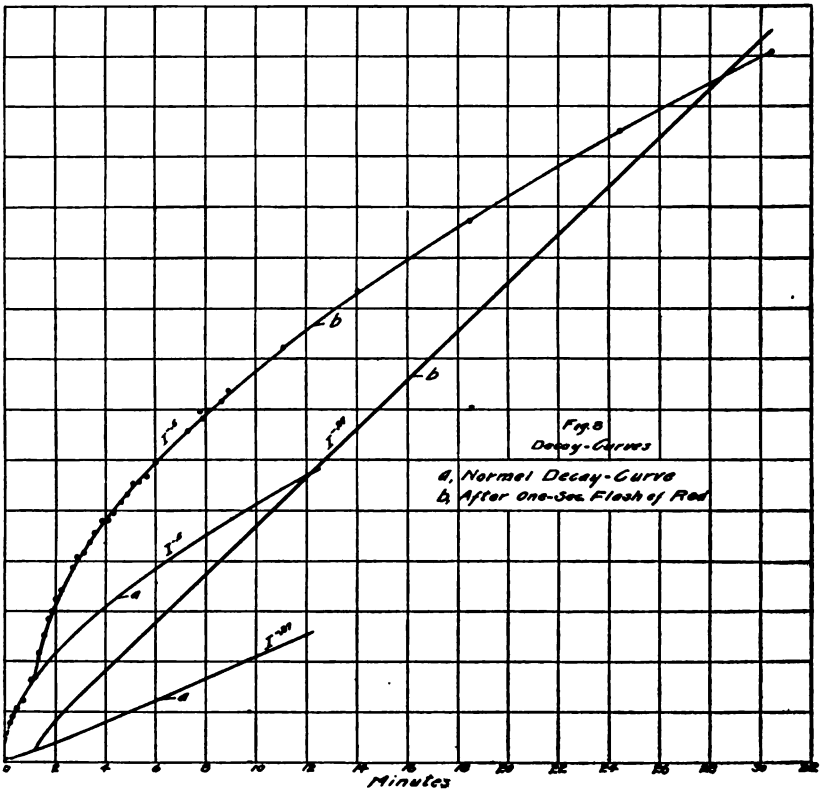

FIG. 8.—Decay-curves

observations—plotted in terms of $\frac{1}{\sqrt{I}}$ is shown in Fig. 8. There is also shown a curve obtained from the substance after one second's exposure to red light. This gives a similar concave curve, not, as has previously been supposed, the first curve with its origin shifted. It is a matter of some interest to note that all our curves plot as straight lines in terms of $\frac{1}{I^{.97}}$, a fact learned upon plotting them on logarithmic paper. Great significance is not attached to this point because, for reasons below noted, it appears more probable that we have, as Lenard and Werner have proposed, two superposed processes, following different laws. That the resultant curves plot as straight lines in terms of $\frac{1}{I^{.97}}$ is perhaps only an accident.

The decay-curves of the three different samples at our disposal show their different degrees of straightness in exactly the order of their exhibition of the flashing-up effect. The sidot-blende screen shows the greatest curvature, while the "Emanationspulver" is, within the error of observation, a straight line from one minute onward. These are shown in Fig. 9. If we accept the two processes of Lenard, and consider these three samples as sulphides of different content of active metal, exhibiting the two processes in different degree, in agreement with the work of Werner, then it appears that the flashing-up

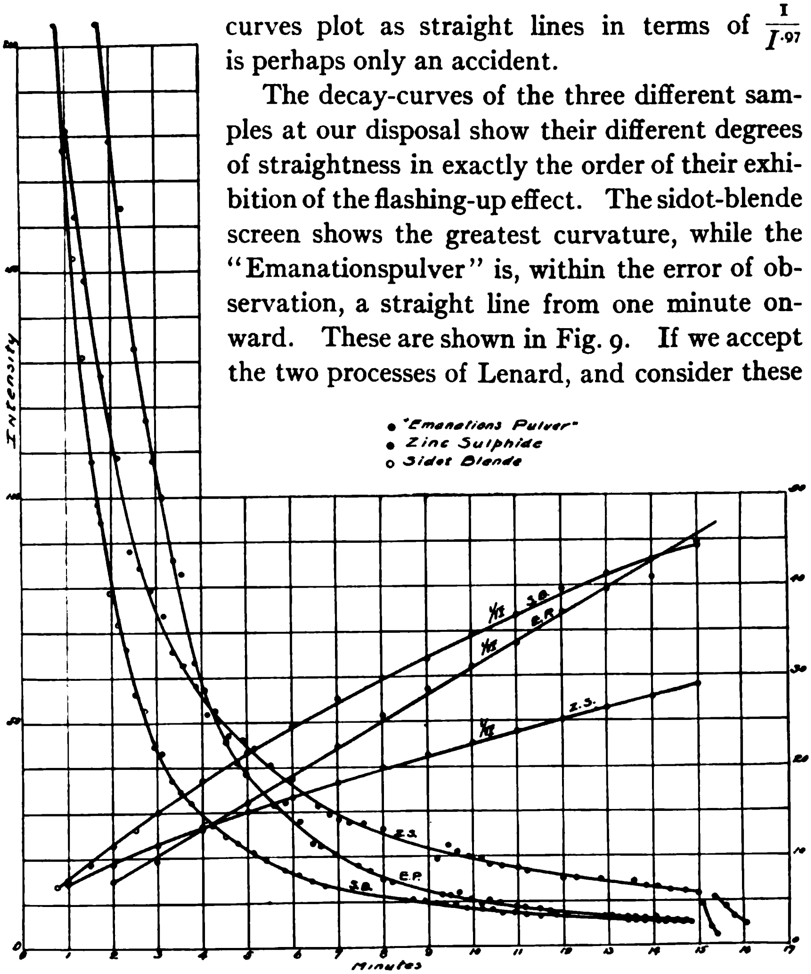

FIG. 9.—Normal decay-curves of three specimens of zinc sulphide

phenomenon is a characteristic of the long process (*Dauerprozess*) which plots as a straight line, while the rapid drop is a characteristic of the short process (*Momentanprozess*), which furnishes the initial curvature (when plotted in terms of $\frac{1}{\sqrt{I}}$ against time.)

SPECTROSCOPIC WORK

Following these photometric observations attention was turned to spectroscopic lines of investigation. The dependence of the flashing-up effect on the wave-length of the exciting light and of the extinguishing light was first investigated. Then with the assistance of photography the spectral character of the phosphorescent light was studied.

For the first problems a small Fuess quartz spectrograph was employed. Owing to the extreme obliquity of the plate with respect to the axis of the instrument it was possible to cut an opening in the side of the camera, through which the phosphorescent surface could be observed during excitation or decay and through which red radiation could fall upon the sulphide when desired. An iron arc was used in studying the characteristics of the substance under different kinds of excitation. Zinc sulphide has a maximum of response to equal energy excitation in the blue and the violet, falling off to zero in the green, and being of lower value in the ultra-violet. Study of the effects of the flashes of red upon the strip of phosphorescence caused by the iron arc spectrum disclosed no noticeable difference at different wave-lengths. No matter by what light it was excited, the phosphorescence showed, after proper lapse of time, this flashing-up of brightness.

A Nernst glower (focused on the slit by a glass lens) was used in studying the effect of different wave-lengths of extinguishing light. The phosphorescent surface was of such size that it could be moved past the slot in the camera back, and six observations could be made, at appropriate intervals after excitation. There are two maxima of extinguishing action, one at 0.7 μ, the other at 1.3 μ (for a normal spectrum), with a minimum between, at 1.1 μ. The action extends to about 1.6 μ and to about 0.6 μ. A rather interesting phenomenon was noted in a series of observations where the exposure to extinguishing light consisted of flashes of equal length. This was that the flashing-up effect is first apparent on the short-wave side of the extinguishing spectrum, afterward progressing toward the long-wave side. At first it was thought that this must be actual production of phosphorescence, i.e., excitation; but upon cutting off all the blue and blue-green light by a yellow stained film the effect persisted, and finally, upon exposing a surface thoroughly extinguished beforehand by red light and by rest in the dark, no trace of luminescence was produced, clearly showing this to be not an excitation of phosphorescence, but a flashing-up of decay.

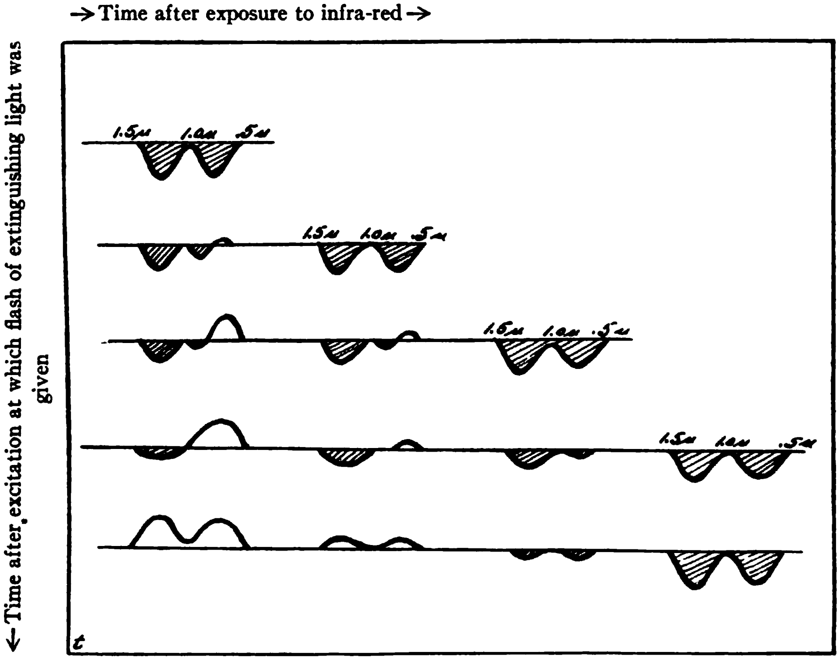
→Time after exposure to infra-red→
←Time after excitation at which flash of extinguishing light was given
1.5μ
1.0μ
.5μ
t

Fig. 10.—Progress of flash-up from green to infra-red

Fig. 10 shows diagrammatically the apparent course of the extinguishing action. From top to bottom the time after excitation at which the red flash occurs is supposed to increase, as shown; from left to right is illustrated the history of the decay after the flash of red. Elevation above the reference line indicates the exposed area to be brighter than the surrounding surface, while depression below the line indicates the exposed area to be darker. The scale of wave-length refers to the extinguishing light. These experiments were made only with our own specimen of zinc sulphide, as the quantity of the other samples did not make experiment with them feasible.

Spectral character of the phosphorescent light.—Visual observation shows the three samples of zinc sulphide to be different in the color of their phosphorescence, differing in the same order as that previously noted in respect to the extent of flashing-up and the shape of decay-curves. The "Emanationspulver" is the most yellow, the sidot-blende screen the most blue-green. This difference shows in the fluorescence, and lent itself readily to photographing with a grating spectrograph of large aperture giving a spectrum 75 mm in length from 0.7 μ to 0.4 μ. The blue and violet mercury lines from a Cooper-Hewitt lamp served as excitation, and a screen of potassium bichromate reduced the intensity of the reflected blue light so that it was not troublesome in the spectrograph. The three negatives obtained, with a narrow slit and exposures of 9 to 24 hours, show no evidence of structure and in spite of the observed difference in color appear rather nearly

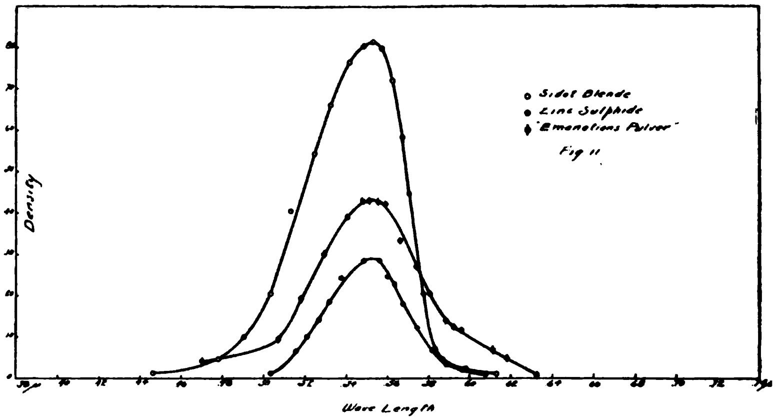

FIG. 11.—Densities of spectrograms of three specimens of zinc sulphide

alike. In Fig. 11 are plotted the densities of the negatives against wave-length. Cramer "Spectrum" plates were used. One of the most important points to notice is that the apparent color-differences are due to but slight differences in the spectra, owing to the fact that the radiation lies in the yellow-green part of the spectrum where hue changes rapidly with wave-length. As a consequence a transformation from one form to another during decay, after color is no longer observable, would be difficult to pick up without considerable dispersion.

Photography of the spectrum of the phosphorescence.—A very difficult photographic problem arises when an attempt is made to photograph the spectrum of phosphorescence, because of the excessive faintness of the phenomenon. Pierce, who as yet has done practically the only work of this kind, has not carried it beyond two minutes after excitation. For the study of the flashing-up it is necessary to photograph at least five minutes after excitation—a fact which has necessitated special apparatus and exposures of excessive length.

The most interesting problems are to determine whether any change occurs in the spectrum when the phosphorescence flashes up and whether the spectral character undergoes a change during decay into a form more susceptible to flashing-up. In the case of calcium sulphide, Stokes noted a distinct difference of color in the flash-up as compared with the unflashed phosphorescence. This would suggest the possibility of a different band of light or of only one side of the original band being responsible for this effect. Nichols and Merritt have observed the blue side of the zinc sulphide band to increase in brightness under infra-red radiation while the rest of the band decreased.

In order to search for effects of this kind at late periods in the decay a large photographic phosphoroscope was constructed. Shown in plan in Fig. 12, it consists essentially of a flat disk of metal 1 foot (30 cm) in diameter mounted on the minute shank of a strong clock. Near the rim of the wheel is a shallow groove, filled with zinc sulphide powder. Through *A* comes the exciting light (blue lines of a Cooper-Hewitt arc), to be reflected downward by a mirror, *M*, over the groove. Through *B* comes strong red light to bring the sulphide to a standard condition before excitation. At *C* is a Nernst glower, inclosed in a light-tight box. An image of the glower is formed by the lens, *D*, through the CS_2 and iodine cell, *E*, and strikes the phosphoroscope wheel after reflection from the mirror M_3. The Fuess quartz spectrograph, *F*, points directly down upon the disk, and a lens forms upon its slit an image of the phosphorescent surface and of the Nernst image when desired.

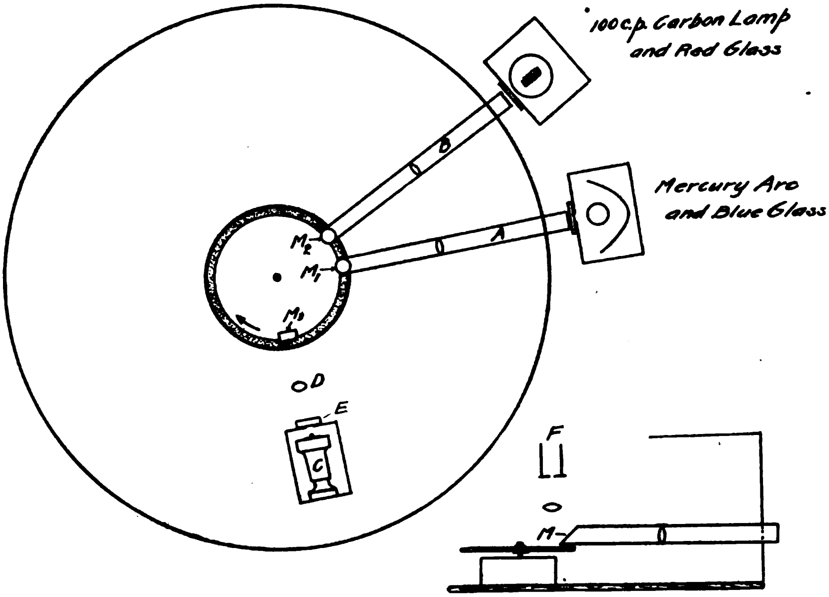

Fig. 12.—Large phosphoroscope

With this apparatus four photographs were obtained, a pair corresponding to one minute after excitation, and a pair corresponding to fifteen minutes after excitation. The exposures were from fifty hours for the shortest at one minute, to ten days for the longest at fifteen minutes. In one of each of these the extinguishing image of the Nernst glower was focused to cover half the groove.

No shift whatever in the position of the band of phosphorescence nor change in its character can be detected on comparison of the one minute and fifteen minute positions. Owing to the small dispersion, however, it is just possible that as much change as that between the sidot-blende and the "Emanationspulver" might be present (Fig. 13).

The exposures at 15 minutes under the infra-red radiation showed beyond question that the total light flashes up; that is, it is not caused by a new band or by a marked increase in one part of the yellow-green band. On the most exposed fifteen minute plate a faint additional band was present on the blue side, which could not be traced to scattered light or other false cause. (This band was too faint to reproduce in Fig. 13.) In position it corresponds to the band in the blue of very short duration which has been previously observed, notably by Nichols and Merritt. The exposure at one minute under infra-red showed no blue band nor difference in character of the main band.

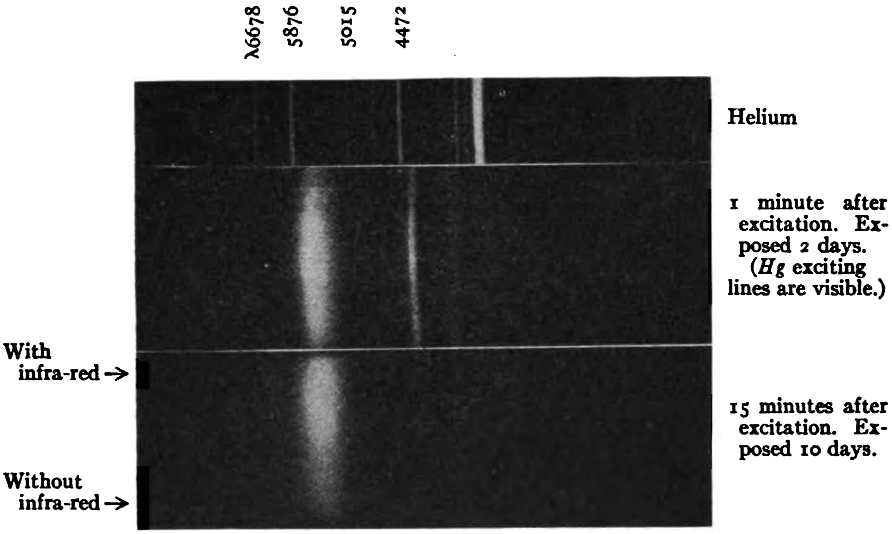
λ6678
5876
5015
4472
Helium
1 minute after excitation. Exposed 2 days. (*Hg* exciting lines are visible.)
15 minutes after excitation. Exposed 10 days.
With infra-red →
Without infra-red →

FIG. 13.—Spectra of phosphorescence (enlarged 3 times)

It is apparent from these results that the phenomenon of flashing-up will find its most probable explanation in some process affecting the total emitted light, the process varying in some manner during decay, rather than in any outburst of different spectral constitution.

THEORETICAL

The flashing-up phenomenon has not received great attention in existing theories of phosphorescence, which have had sufficient task before them to explain the more prominent phenomena of excitation and decay. What follows is a suggestion as to how the experiments just described may be explained in general accordance with the ideas of Lenard.

Lenard (*op. cit.*) classes zinc sulphide among the phosphorescent alkaline earth sulphides, which consist of an "active metal," an "alkaline earth sulphide," and a "flux." In zinc sulphide the zinc may be both the active metal and the metal of the sulphide, or other metals such as strontium, copper or manganese in minute quantities may constitute the active metal. The action of light is the photoelectric effect, which tears out the photoelectric electrons. In their oscillatory return these, by resonance, set in motion the emitting electrons, whose vibrations constitute the fluorescent and phosphorescent light. Certain of the former electrons are captured or stored up by the sulphur atoms, while others are dispersed through the surrounding atoms without becoming attached. The return of the stored-up electrons due to collisions or approaches of the metal atom and the loaded sulphur atom, will cause luminescence following the law $I=\frac{1}{(a+bt)^2}$; the return of the unattached electrons will follow the law $I=ae^{-\beta t^m}$. In accordance with this hypothesis Werner considers the sulphides with a large quantity of active metal, those which show a straight-line decay (plotted as $\frac{1}{\sqrt{I}}$ against time), as having but a small number of unattached electrons after excitation, and vice versa.

The effect of elevated temperature and of infra-red radiation is held to be a shaking-up of the atoms, by which the frequency of the collisions is increased. Red and infra-red produce the agitation by resonance, setting in motion the larger aggregates which consist of sulphur atoms and attached electrons. The size and free period of these aggregates are large; they therefore respond to long waves, just as the small photoelectric electrons respond to short-wave excitation.

It will be seen that this theory calls for no direct effect of red radiation on the unattached electrons: in other words, only that period of the decay which is wholly represented by the straight-line relation is considered. In this region an accelerated rate of return of electrons is to be expected. This will cause an initial flash followed by a rapid decay, precisely what does occur.

Let us now carry this reasoning of Lenard's further and apply it to the unattached electrons to which is due the initial curvature of the decay when plotted in the manner we have considered. The condition we imagine is that in returning to the metal atom these have to pass between the sulphur atoms, loaded and unloaded. If the sulphur atoms are set into vibration by the incidence of long-wave energy it is evident that the chance for an unattached electron to reach its goal is much reduced, for it stands a big chance of being captured by collision with the rapidly moving sulphur atoms or aggregates. We might then expect that, while the luminescence due to the return of the originally captured electrons will flash up and then decrease, that due to the unattached electrons will suffer an immediate decrease owing to their capture in a manner not productive of light. The magnitude of this latter drop in brightness may be sufficient to mask the initial flash due to the freeing of the attached electrons.

One consequence of this train of events would be that a short flash of infra-red energy would have a different effect on the subsequent decay, depending on whether it occurred during the early part of decay or at a later period. Let us assume that the effect of a given amount of long-wave energy on the aggregates composed of sulphur atoms and attached electrons is the breaking down of a certain fraction of them by collision. Let the brightness of the phosphorescence at any time be a function of the number of such aggregates existing and subject to the normal collisions at the working temperature. Then if an early flash of red both breaks down a fraction of the total aggregates and builds up new ones as indicated above, while later it breaks down the same fraction but builds up comparatively few new ones, then the brightness of the sample submitted to the earlier flash will at any time subsequent to both flashes be the greater. In short, the degree of extinction of phosphorescence produced by the same quantity of long-wave energy should increase with the increase of time after excitation.

This is actually what does take place. In a previous paper[1] it was shown by photographs of a phosphorescent zinc sulphide surface which had been given a series of exposures to red light that greater extinction occurred as time went on. This has been checked by a series of equal exposures upon a phosphorescent surface in the quartz spectrograph using the light of a Nernst glower. A photographic print from this showed clearly the continued increase of extinguishing effect with time up to about two hours, after which it became constant.[2] So marked was this effect that it was thought

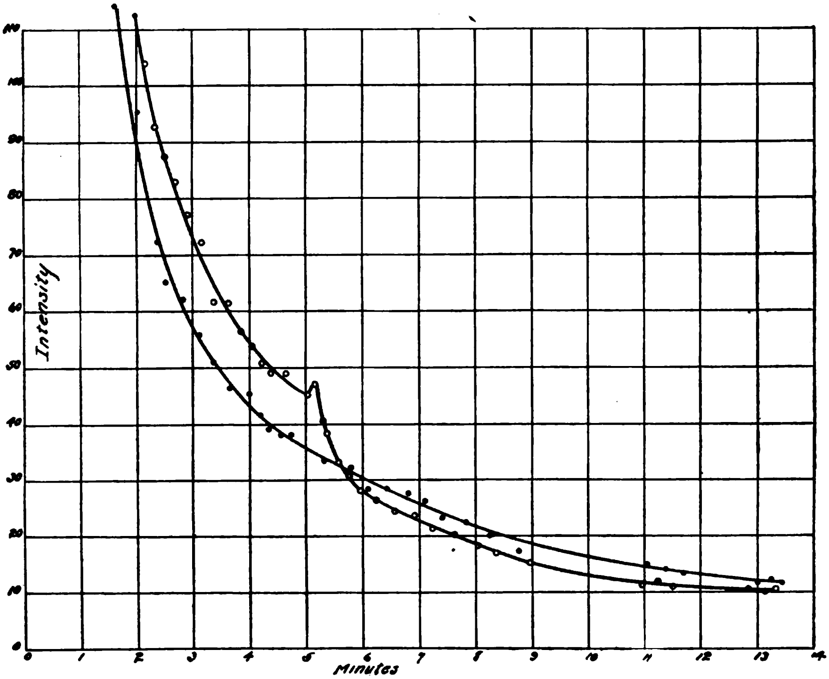

FIG. 14.—Decay produced by identical exposure to infra-red at 1 minute and 5 minutes after excitation.

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[1] "Further Studies of the Fire-Fly," *Physical Review*, 31, 637, 1910.

[2] If the exposure to the photographic plate is made too soon after the last exposure to infra-red (say within 1 or 2 hours) the appearance is as though the extinguishing effect were decreasing again. This is due to the flashing-up phenomenon, which after this time becomes very large and long continued.

well to look for it by visual observation with the photometric apparatus. The result is shown in Fig. 14. The phosphorescent surface was excited, half of it covered with a metal plate, and the other half given, after 12 seconds, a 1-second flash of infra-red. The decay of both halves was followed to five minutes, then the first half was covered and the other half given an exactly similar flash of infra-red. After this measurements of each were made as plotted in the figure. The greater effect of the same quantity of infra-red at the later point in the decay is clearly indicated.

This proposed explanation would place the three samples which were studied in the same order with respect to character of decay curve and extent of flash-up, which agrees with the facts. No attempt is made to incorporate an explanation of the slight difference in color of the samples, for at present we have no data to tell us whether their constituents are exactly the same. The difference in color may be due to a cause different from that producing the differences in the decay-curves. The latter, from the work of Werner, are known to be producible by different proportions of the typical constituents and a difference of that character has been assumed as responsible for the difference in the decay-curves of our samples. Other differences may exist, and until we have an opportunity to make up for ourselves specimens of these sulphides of known composition, we cannot follow out with profit the connection between the flashing-up phenomenon and the character of the decay-curve more clearly than has been done.

The phenomena of the progress through the spectrum of the flashing-up effect as shown in Fig. 10 may be explained in agreement with the preceding paragraphs by assuming the atoms or aggregates which respond to long-wave energy as of different sizes, but all very much larger than the unattached electrons. In passing through a group of vibrating atoms the probability of the electron being captured is less the smaller the atoms. The smaller atoms are those thrown into vibration by the shorter wave-lengths. These, then, would be the first to show a decrease of the effect due to capturing unattached electrons; that is, they would be the first to show the flashing-up.

A somewhat simpler explanation is, however, possible, and that is that we here have differences due to different quantity of extinguishing radiation. In the Nernst glower the intensity of the radiation increases rapidly toward the infra-red. The study of the effect of different quantities of long-wave energy, plotted in Fig. 4, shows that while a small quantity produces a small flash-up, the flash is shown to die down. As the experimental difficulties were large in the observation of Fig. 10 it is possible that the effect was first noticed on the short-wave side because it had not disappeared too quickly to be caught. More exact quantitative measurements are here necessary.

SUMMARY AND CONCLUSION

It has been found, contrary to general belief, that phosphorescent zinc sulphide exhibits a flashing-up of luminosity under the incidence of long-wave energy. The peculiarity of this effect is that it becomes noticeable only after the decay of phosphorescence has proceeded for some time. Its occurrence is dependent on the time elapsing since excitation, for neither less intense excitation nor shorter excitation will hasten the bringing of the phosphorescent substance to the proper condition. Short exposure to long-wave energy does, however, hasten the attainment of the proper condition for showing the flash-up. The magnitude of the flash-up varies directly as the intensity of the red or infra-red radiation.

Study of several samples indicates that there is a direct relationship between the character of the decay-curve and the magnitude of the flashing-up phenomenon. The samples whose decay plots nearest a straight line in terms of $\frac{1}{\sqrt{I}}$ against time, show the effect more than the ones whose decay plots as a curve.

Spectroscopic observation indicates no qualitative difference in the phenomenon due to differences in the wave-length of the exciting light. The flashing-up effect appears to occur the sooner, the shorter the wave-length of the extinguishing light. Spectrograms show no change in the spectrum of the phosphorescent light during decay. The spectrum of the flash is the same as before the flash, indicating that the phenomenon is not one of a separate band or part of the radiation, but a phenomenon of the total emission.

An attempt is made to develop a theory of the effect, based on the theory of Lenard. The flashing-up is ascribed to the stored-up electrons, which determine the shape of the decay-curve in the later part of decay; the quick drop in luminosity in the early part of decay is ascribed to the unattached electrons, which influence the initial direction of the decay-curve.

It is our intention to follow up this work with similar experiments on freshly prepared sulphides of known composition, to extend the work to other substances, and to study the similar phenomena caused by heat.

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THE SUN'S ENERGY-SPECTRUM AND TEMPERATURE[1]

BY C. G. ABBOT

Spectro-bolometric determinations of the solar constant of radiation are attended by a number of by-products, among them the distribution of energy in the sun's spectrum at the earth's surface, and also outside the atmosphere. In the earlier solar-constant observations of the Smithsonian Astrophysical Observatory, the form of the energy-curve outside the atmosphere was investigated rather frequently, involving independent determinations of the transmission of the apparatus. But it soon proved that the value of the solar constant was only very slightly affected by wide changes in the supposed form of the extra-atmospheric energy-curve,[2] and we have generally assumed the form as known, and determine the transmission of the optical system therefrom. However, in each year of observation several complete determinations have been made, and the conditions of observation have varied so much that it seems hardly possible that the mean value to be given below can be very far astray, notwithstanding the great difficulty of the measurements, and the considerable divergence of the several determinations.

In short, the determinations have been made as shown in Table I.

The initials of observers refer to C. G. Abbot, F. E. Fowle, L. R. Ingersoll, L. B. Aldrich, and W. S. Adams. I take this opportunity to express my obligation to the four last-named gentlemen and to Miss Graves for their aid in the work. A statement of the method usually adopted for the observation may be found in Vol. II of the *Annals of the Astrophysical Observatory* (see pp. 24 and 50 to 57). At each of a large number of wave-lengths in the solar spectrum it is required to determine: (1) the intensity at the bolometer; (2) the selective transmission of the spectroscope; (3) the selective reflection of the heliostat;

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[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] See Abbot and Fowle, *Astrophysical Journal*, **29**, 280, 1909.

TABLE I

CONDITIONS OF THE DETERMINATIONS

OBSERVERS	DATE	STATION	ALTITUDE METERS	HELIOSTAT	PRISM	ALL MIRRORS		COLLIMATION
						No.	Kind	
C.G.A...... F.E.F......	1903 Feb.–Apr.	Washington	10	One mirror Siderostat	Flint glass	4	Silvered glass	Used
C.G.A...... L.R.I......	1905 June–Oct.	Mount Wilson	1780	Two mirrors Coelostat	Flint glass	5	Silvered glass	Used
C.G.A...... L.R.I......	1906 August	Mount Wilson	1780	Two mirrors Coelostat	Flint glass	5	Silvered glass	Used
C.G.A...... L.B.A...... W.S.A......	1908 Sept.–Oct.	Mount Wilson	1780	Two mirrors Coelostat	Flint glass	5	Silvered glass	Used
C.G.A...... L.R.I......	1909 June–July	Mount Wilson	1730	Two mirrors Coelostat	Flint glass	5	Silvered glass	Used
C.G.A...... L.R.I......	1909 August	Mount Wilson	1730	Two mirrors Coelostat	U.V. crown glass	4	Silvered glass	None
C.G.A...... L.R.I......	1909 July–Aug.	Mount Wilson	1730	Two mirrors Coelostat	Quartz	4	2 Silvered glass 2 Magnalium	None
C.G.A........	1909 September	Mount Whitney	4420	None	Quartz	2	Magnalium	None
C.G.A...... F.E.F......	1910 July	Mount Wilson	1730	Two mirrors Coelostat	Quartz	4	2 Silvered glass 2 Magnalium	None
C.G.A........	1910 August	Mount Whitney	4420	None	Quartz	2	Magnalium	None

(4) the transmission of the atmosphere. A bolograph indicates the first, and measurements on a series of bolographs, taken at different zenith distances of the sun, furnish the means of computing the last. The reflection of the heliostat is determined by taking bolographs: (*a*) with the ordinary mirror or pair of mirrors; (*b*) with a substitute mirror or pair of mirrors; (*c*) with the combination of both regular and substitute mirrors. The relative transmission of the spectroscope is determined by first passing the rays through an auxiliary spectroscope, selecting certain wave-lengths and observing their intensity: (*d*) as transmitted by the auxiliary spectroscope; (*e*) as transmitted by both spectroscopes.

The observation (*d*) is made by removing the bolometer to occupy the position usually occupied by the slit of the usual spectro scope. When transmitted by both spectroscopes, the beam becomes nearly monochromatic, but its intensity should be observed by measuring the area included in a bolograph taken by moving this monochromatic image over the bolometer, not (as stated at p. 51 of the *Annals*) by observing the maximum deflection produced by the monochromatic image. Under certain circumstances, however, the areas and the maximum deflections are nearly proportional. In all the earliest observations at Washington this seemed to be the case, and also at Mount Wilson in 1905–1906. Though we registered the type (*e*) bolographs in 1905–1906 at Mount Wilson, we generally measured merely the maximum ordinates in the reductions as published at pp. 51 and 105 of *Annals*, Vol. II. In order to see what errors may have resulted, the *areas* have lately been measured for all the determinations given in the *Annals*. They prove to be so nearly proportional to the maximum deflections that no error as great as the uncertainty of measurement is found to be indicated. In the year 1908 several determinations were made with another prism of different dispersion in use in the auxiliary spectroscope, and unfortunately the areas were not even observed. As some determinations made with still another prism, in 1909, which were reduced by both methods, indicate large errors in the deflection method, it is thought best to reject entirely all the determinations (those of 1908) in which the areas are not available.

In order to make the several determinations comparable, they are first reduced from the prismatic to the normal, or wave-length scale. In order to do this the deviation of the prism is platted against the corresponding wave-length, and the tangents to the resulting curve are determined at the various prismatic deviations. In order to reduce the several determinations to comparable intensity-scales, a multiplying factor for correcting each curve has been determined, in such a manner as to make all the curves agree as well as possible between wave-lengths 0.5 μ and 1.2 μ. For the sake of brevity not all the determinations are here given separately, but all determinations made with the same arrangement of apparatus, and on the same approximate date, are combined, and the mean of several such determinations is here used. It should be remarked that the Mount Whitney determinations of 1909 depend on spectroscopic transmission factors determined at Mount Wilson in 1909, and the Mount Whitney determinations of 1910 similarly depend on Mount Wilson spectroscopic factors of 1910. Apart from this dependency, all determinations here given are wholly independent.

The accuracy of the determinations at Mount Wilson in 1905 and 1906 is prejudiced by the fact that the scale of the galvanometer had not at that time been investigated. But in the mean of so many determinations as are included in the 1905–1906 group, a great many different parts of the galvanometer scale were used, and it is probable that the scale-error is principally eliminated except in the ultra-violet, where the deflections were all near zero of the scale, and disproportionately large. This part of those curves is rejected in deriving the mean values in the following discussion. The galvanometer scale was strictly uniform in all the subsequent measurements, excepting those on Mount Whitney in 1910. The deviation from uniformity for the 1910 Mount Whitney measurements was investigated and found negligible in comparison with other sources of error.

In all the determinations the accuracy falls off rapidly in the ultra-violet spectrum. With the flint-glass prism the dispersion is so great, and the deflections and areas recorded in determining spectroscopic transmission consequently so small, that such values have not been used in fixing the mean value to be given for wave-lengths less than 0.42 μ. Only one complete determination has been made thus far with the ultra-violet prism. Although it is somewhat abnormal it is not rejected. Determinations made with the quartz prism have fair weight to about wave-length 0.37 μ. Beyond this they depend upon more and more doubtful data for the transmission of the spectroscope; and beyond wave-length 0.35 μ they depend wholly on extrapolation for this quantity.

Table II is a summary of the results. It should be considered in connection with the table given above. In the first column are wave-lengths, next mean intensities in the solar spectrum outside the atmosphere (in arbitrary units), and in the remaining columns the departures from the mean intensity in percentages for the average curves determined on the dates given at the bottom of the table.

I do not venture to decide whether the departures from the mean may be attributed in part to real changes in the distribution of energy in the solar spectrum, or are altogether the results of error in the difficult determination of spectroscopic transmission. The argument on that subject remains as yet where it was left by Abbot and Fowle.[1] It is very pleasing to note that the mean Washington, Mount Wilson, and Mount Whitney determinations agree with one another as well as the separate determinations at the three places agree each with each.

For the sake of those who may desire to compute the distribution of energy in the solar spectrum at the earth's surface, I give also the transmission of the atmosphere at various wave-lengths. The values represent the fraction of the intensity of the solar beam outside the atmosphere which would remain in a direct beam transmitted vertically to the earth's surface. Average values for cloudless (but not necessarily hazeless days) are given, as they have been found at Washington, Mount Wilson,[2] and Mount Whitney, for the various wave-lengths. To compute the

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[1] *Astrophysical Journal*, **29**, 287, 1909.

[2] For the reason stated in this Journal, **33**, 193, 1911, the Mount Wilson transmission coefficients published in Vol. II of the *Annals of the Astrophysical Observatory* are about 1.4 per cent too low.

TABLE II

SOLAR ENERGY-SPECTRUM DETERMINATIONS

WAVE-LENGTH	MEAN INTENSITY	INDIVIDUAL GROUPS OF DETERMINATIONS. PERCENTAGE DEVIATIONS FROM MEAN*											PERCENTAGE PROBABLE ERROR
		1	2	3	4	5	6	7	8	9	10	Average	
0.30.........	(440)											—	—
0.325.........	1285								−31.	+67.	+86.	61.	30.
0.35.........	2700				+51.	−30.	−17.	−22.	−12.	+16.	+10.	23.	7.3
0.375.........	3464				+29.	−11.	− 4.4	− 9.4	− 4.6	+ 3.0	− 1.9	9.0	2.7
0.39.........	3620			− 0.6	+15.	−12.	− 2.2	− 6.1	− 0.6	+10.	− 1.9	6.0	1.7
0.42.........	5261	+ 0.0	+6.9	−17.	− 4.7	− 3.5	+ 4.4	+ 2.2	+ 3.4	+ 7.5	+ 0.2	5.0	1.3
0.43.........	5340	+ 2.4	+6.4	−18.	−10.	− 4.5	+ 3.4	+ 1.2	+ 5.1	+11.	+ 3.1	6.5	1.7
0.45.........	6047	− 2.7	+1.1	−15.	− 8.7	+ 0.0	+ 8.4	+ 5.4	+ 5.3	+ 7.6	− 1.1	5.5	1.4
0.47.........	6253	− 4.7	−2.2	−16.	−12.	+ 4.9	+ 7.8	+ 5.8	+ 1.9	+12.	+ 2.3	7.0	1.8
0.50.........	6064	+ 0.3	−7.6	−17.	−13.	+ 3.5	+ 9.9	+ 7.1	+ 5.2	+12.	+ 0.2	7.6	1.9
0.55.........	5627	− 7.2	−7.8	−15.	−12.	+ 5.0	+ 8.5	+ 7.2	+ 9.4	+11.	+ 0.9	8.4	2.1
0.60.........	5047	−11.	−5.3	−14.	−10.	+ 7.4	+ 7.9	+ 7.3	+ 8.0	+ 9.4	+ 1.1	8.1	2.1
0.70.........	3650	− 4.4	−0.1	− 2.1	0	+ 1.2	+ 1.0	− 1.6	+ 1.9	+ 4.4	− 0.5	1.7	0.4
0.80.........	2672	+ 4.2	+4.8	+ 6.7	+ 5.4	− 4.3	− 4.2	− 5.9		− 4.1	− 1.9	4.6	1.2
1.00.........	1664	+ 4.4	+1.6	+ 6.4	− 1.0	− 1.3	− 4.6	− 1.1		− 3.7	− 0.4	2.7	0.7
1.30.........	897	− 7.9	+1.7	+ 7.0	+ 2.8	− 2.5	− 0.1	+ 1.2	− 3.2	+ 1.1	+ 1.1	2.9	0.7
1.60.........	526	+ 0.8	+7.6	+ 9.9	+ 6.5	− 2.1	+ 0.4	− 2.1		+ 2.1	−15.	5.2	1.4
2.00.........	245	+ 9.4	+0.4	+ 2.9	0	+ 3.3	+10.	+13.		−11.	−29.	8.8	2.4
2.50.........	43					+ 4.7	+ 9.3	− 7.0	(−65.)	+70.	− 9.3	14.	4.8
3.00.........	(12)									(+50.)	(−100.)	75:	45.

* 1, 2, 3: Flint glass prism: Washington, 1903; Mount Wilson, 1905–1906; Mount Wilson, 1909. June 29, July 7.
4: U. V. glass prism: Mount Wilson, 1909, August 11, 13.
5, 6, 7, 8: Quartz prism: Mount Wilson, 1909, July 8, 9; July 22, 26, 27; August 9, 10; 1910, July 31.
9, 10: Quartz prism: Mount Whitney, 1909, September 3; 1910, August 12, 13, 14.

transmission for other than zero zenith distances, the coefficients here given must be raised to a power equal to the secant of the zenith distance. This does not hold closely for zenith distances above 75°.

TABLE III

Wave-Length	Mean Coefficients of Atmospheric Transmission		
	Washington 1902–1907	Mt. Wilson 1909–1910	Mt. Whitney 1909–1910
0.30	.....	.....	(0.510)
0.325	.....	0.(550)	0.584
0.35	.....	0.612	0.660
0.375	.....	0.662	0.738
0.39	0.445	0.694	0.763
0.42	0.586	0.764	0.806
0.43	0.600	0.778	0.822
0.45	0.640	0.800	0.851
0.47	0.671	0.827	0.880
0.50	0.705	0.858	0.900
0.55	0.739	0.876	0.918
0.60	0.760	0.890	0.934
0.70	0.839	0.942	0.956
0.80	0.865	0.964	0.972
1.00	0.901	0.973	0.980
1.30	0.916	0.972	0.980
1.60	0.930	0.975	0.978
2.00	0.909	0.957	0.940
2.50	0.870	(0.900)	0.930
3.00	.....	.....	0.910

THE EFFECTIVE TEMPERATURE OF THE SUN'S RADIATING LAYER

It has been stated very pertinently[1] that we cannot express solar temperatures either on the thermodynamic scale or on that of the gas thermometer, because no experiments have been made to fix these scales in the range of temperatures prevailing on the sun. All that we know, which is available, is that certain radiation formulae, in which the temperature enters, agree very well with observations on the radiation of the perfect radiator, or so-called "black body," when the gas-thermometer temperatures are introduced. But this verification of these formulae extends only to about 1800° of the absolute Centigrade scale, whereas the solar temperatures seem to be of the order of 6000° or more. We can only compare the solar radiation with that of the perfect radiator, as computed by these formulae, and thus determine the temperature which a "black body" would be assumed to have when its radiation would approximate to that of the sun.

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[1] *Astrophysical Journal*, 19, 35, 1904.

There is another kind of uncertainty in the discussion of the solar temperature. We are not sure that the source of radiation is at the same temperature at the center of the sun's visible disk that it is at the limb, nor that it is the same at a given place on the disk for rays of different wave-lengths. I am of the opinion that the scattering of rays by the molecules of gas in the outer parts of the sun is what limits our depth of view, and that there is no cloudy photosphere. If this is so, we look deeper, and hence to hotter layers, when we view the center of the sun's disk at right angles, than we do when we view the limb; and as molecular scattering is far greater for violet than for red rays, the violet rays which we receive must come from more superficial, and hence cooler, layers, on the whole, than the red ones.[1]

The effect of these two influences on the form of the sun's energy-spectrum-curve may be considerable, and may tend to mislead us greatly as to the probable solar temperature. The first of the two influences considered is the less important, for it leads to no great distortion of the energy-curve, but merely to altering it to correspond approximately to a lower temperature than that which prevails at the deepest layers seen at the center of the visible disk. Much more misleading is the second influence suggested, for it tends to distort the curve so as not to correspond to any "black-body" temperature at all. Considering merely the radiation from the center of the sun's disk, rays of successive wave-lengths (passing from the infra-red toward the ultra-violet) are propagated to us from layers of lower and lower average temperature. Whereas the total radiation of a "black body" varies as the *fourth power* of its temperature, the radiation of the shorter wave-lengths varies as higher powers of the temperature than the fourth, even to the tenth, and still higher powers, as the wave-length decreases beyond that of maximum radiation. Hence, it will be expected: (1) that the solar energy-curve will rise less rapidly from the infra-red side toward the maximum than would the energy-spectrum-curve of a "black body" of the temperature at which the infra-red rays are prevailingly emitted; (2) that the maximum of solar energy will be shifted toward longer wave-lengths than it would occupy if the distortion did not exist; (3) that in the ultra-violet the solar energy-spectrum will fall off far more rapidly than that of a "black body" generally corresponding in spectral distribution to the solar curve for greater wave-lengths; and (4) that any estimate we may make as to the effective radiating temperature of the sun, based on a simple consideration of the form of its spectral energy-curve, will be too low.

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[1] This idea I have elaborated in a book entitled *The Sun*, now in press.

There is another influence tending powerfully to alter the form of the sun's energy-spectrum-curve, namely the selective absorption in the Fraunhofer lines, by virtue of which the rays which are emitted from that average depth below the solar surface at which the continuous spectrum originates, are cut off by absorption in these wave-lengths, and in their places we see only the rays emitted from relatively superficial, and therefore cooler layers. The Fraunhofer lines become so excessively numerous in the violet and ultra-violet solar spectrum that their presence profoundly diminishes the intensity of the sun's energy-spectrum-curve in that spectral region, and thus conspires to magnify the spectral distortion and the underestimation of solar temperatures we have already discussed.

It seems to me, in consideration of these tendencies, that the process to which Goldhammer has given preference,[1] and which is akin essentially to that of Wilsing and Scheiner,[2] is ill-adapted to give a trustworthy estimate of solar temperatures. It gives[3] for the sun temperatures of the order of only 4000°, when applied in the infra-red spectral region, where, as I believe, the temperature is really the highest of all. The diagram shows a comparison between the form of the solar energy-spectrum-curve and those of "black-body" curves at 6200° and 7000° absolute Centigrade, respectively. The solar curve is repeated to match each curve

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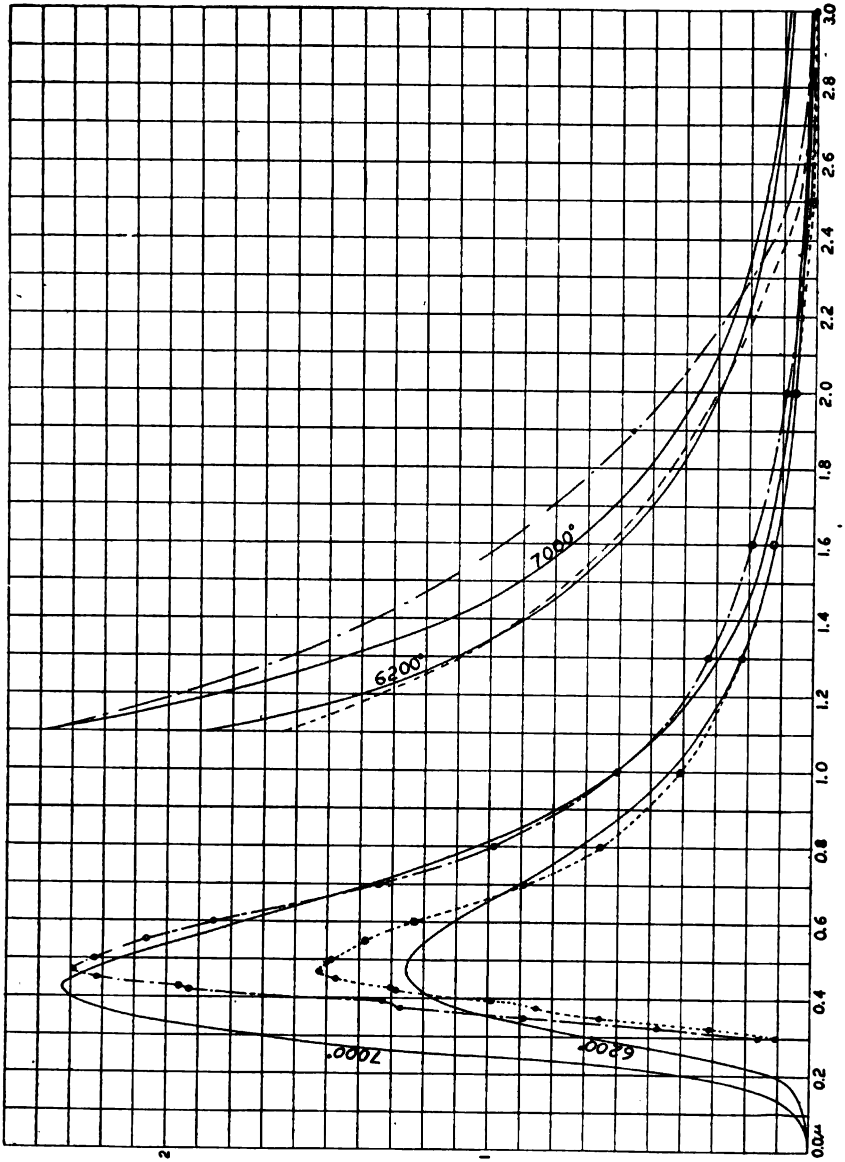
[1] *Annalen der Physik* (4), **25**, 905–920, 1908.

[2] *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, Bd. XIX, No. 56, 1909.

[3] *Astrophysical Journal*, **29**, 282, 1909. See also my book *The Sun* above mentioned.



Energy-spectrum of sun compared with black body as well as possible in the region $\lambda = 0.5\ \mu$ to $\lambda = 1.2\ \mu$. The longer wave-lengths are repeated on an enlarged scale.

If we take the Wien displacement formula, $\lambda_{max} T = 2930$, and the wave-length of maximum energy as $0.470\ \mu$, the effective solar temperature comes out 6230° absolute Centigrade.

If we take the Stefan formula for total radiation: $E = 76.8 \times 10^{-12}\ T^4$; the solar radius as 696,000 km; the mean radius of the earth's orbit as 149,560,000 km; and the solar constant as 1.922 calories per square centimeter per minute; the effective solar temperature comes out 5830° absolute Centigrade. This last method of computing the solar temperature would perhaps be the best if we could only form some independent estimate of the emissive power of the sun's effective radiating layers. As we are inclined to think that the methods based on the form of the sun's spectrum-energy-curve must necessarily give too low temperature values, and as we find that the method based on total radiation gives still lower ones, it appears that the emissive power of the sun must fall decidedly below that of the perfect radiator.

In view of all this we can only conclude that the sun's effective radiating layer is roughly comparable with a "black body" at 6000° absolute Centigrade, but that the prevailing solar radiating temperatures greatly exceed 6000°, and may even exceed 7000° absolute Centigrade.

SUMMARY

Spectro-bolometric observations extending from the year 1903 to 1910 at Washington, Mount Wilson, and Mount Whitney (altitudes 10, 1750, and 4420 meters, respectively) made with various optical systems, including flint glass, ultra-violet crown glass, and quartz prisms, with mirrors sometimes of silvered glass, sometimes of magnalium, have been used to determine the distribution of energy in the solar spectrum outside the atmosphere. The results do not seem to depend on the altitude of the observing station, but, especially in the ultra-violet, seem to depend somewhat on the character of the spectroscope employed. It is thought that in the ultra-violet spectrum beyond wave-length $0.40\ \mu$ little weight should be given to the results obtained thus far with glass prism spectroscopes.[1]

[1] Better results are expected this summer.

In the ultra-violet region the results here given are obtained with a quartz-magnalium spectroscope.

For the adopted mean normal spectrum-energy-curve the maximum of energy falls at wave-length 0.470 μ.

The intensities at various wave-lengths are as follows:

Wave-Length	μ 0.30	μ 0.35	μ 0.40	μ 0.45	μ 0.47	μ 0.50	μ 0.60	μ 0.80	μ 1.0	μ 1.3	μ 1.6	μ 2.0	μ 2.5	μ 3.0
Intensity....	440	2700	4345	6047	6253	6064	5047	2672	1664	897	526	245	43	12

For additional wave-lengths see the preceding table.

The mean coefficients of atmospheric transmission for vertical rays are given as determined at Washington, Mount Wilson, and Mount Whitney. Coefficients for various wave-lengths are as follows:

Wave-Length		μ 0.35	μ 0.40	μ 0.45	μ 0.50	μ 0.60	μ 0.70	μ 0.80	μ 0.90	μ 1.00	μ 1.50	μ 2.00
Transmission.....	Washington	.....	0.543	0.640	0.705	0.760	0.839	0.865	0.886	0.901	0.922	0.909
	Mt. Wilson	0.612	0.724	0.800	0.858	0.890	0.942	0.964	0.968	0.973	0.974	0.957
Coefficient.......	Mt. Whitney	0.660	0.783	0.815	0.900	0.934	0.956	0.972	0.979	0.980	0.980	0.940

These are smooth-curve values, and do not take account of powerful selective absorption in water-vapor bands. For additional wave-lengths see preceding table.

The form of the spectrum-energy-curve at the earth's surface (not allowing for water-vapor or oxygen bands) may be obtained by multiplying the extra-atmospheric intensities by the corresponding coefficients of atmospheric transmission raised to powers whose exponents are the solar zenith distances.

The temperature of the sun is discussed, and attention is drawn to certain influences which probably tend to distort the sun's spectral energy-curve, and thereby to incline us to set too low values for the probable temperatures prevailing in the sun's radiating layers. The conclusion is drawn from the observations cited that the sun's emission approximates roughly to that of a "black-body" at 6000° absolute Centigrade, but that on account of influences probably active, we ought to assign a solar radiating temperature of the order of 7000° absolute Centigrade.

ASTROPHYSICAL OBSERVATORY
SMITHSONIAN INSTITUTION
May 1911

THE ZEEMAN EFFECT FOR VANADIUM[1]

BY HAROLD D. BABCOCK

On account of the importance of vanadium in the study of magnetic fields in sun-spots, the following material is presented now. It is preliminary to a fuller discussion of the subject which must be deferred for a period.

The apparatus employed and the manner of taking the photographs have been described in a preceding paper.[2] The field-strengths employed for different plates ranged from 17,500 to 19,800 gausses. For nearly all of the plates lumps of pure vanadium were used as spark terminals, but two photographs were taken with pieces of ferro-vanadium supposed to contain ten per cent of the metal. This substance in the spark yields the vanadium lines in abundance, as well as the stronger iron lines.

In Table I are given the results obtained from all the plates taken up to this time. From λ 5500 to λ 6600 the number of plates upon which the results depend is relatively greater than for the remainder of the spectrum. The wave-lengths employed are those of Hasselberg[3] and of Shaw,[4] except for a very few lines given only by Exner and Haschek.[5] The wave-lengths taken from Shaw's paper are on the Rowland system.

The second column of the table gives the total number of components into which the line appears separated when viewed at right angles to the magnetic field. The number 1, which is occasionally found in this column, indicates that the line is undivided by the field. The letter C, with or without a number, is used here as an abbreviation for complex, and signifies that higher resolving power is required for the complete analysis of the components. If a number accompanies the letter it indicates how many groups of components were observed. Measures given for such lines refer to the average separations of the groups of components unless otherwise noted in the remarks.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 55.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 52; *Astrophysical Journal*, 33, 217, 1911.

[3] B. Hasselberg, "Die Spectra der Metalle im electrischen Flammenbogen," *Kongl. Svenska Vetenkaps-Akademiens Handlingar*, Bandet 32, No. 2.

[4] H. Shaw, "The Arc Spectrum of Vanadium," *Astrophysical Journal*, 30, 127, 1909.

[5] *Wellenlängen Tabellen der Funkenspectren der Elemente*, Leipzig and Vienna, 1902.

The third and fourth columns show the separations in Ångströms for the *n*- and *p*-components, respectively, for a field-strength of 20,000 gausses, while the fifth and sixth columns contain for the same field the corresponding values of $\Delta\lambda \cdot 10^8/\lambda^2$.

In some cases it was impossible to measure both members of a pair of components on account of blending with neighboring lines. For such the component clearly shown was referred to the normal position of the line, and the measured difference doubled.

In the "Remarks" column numbers are used to express the relative intensities of the components of a group in case they differ. The relative position of the numbers corresponds to that of the components when viewed with longer wave-lengths on the right. Thus *n* 2:1 means that of the two *n*-components of a line the one on the violet side is twice as intense as the other. No attempt is made to compare the intensities of *n*-components with those of *p*-components. In this column E means that the line is given in Lockyer's list of enhanced lines; −r means not resolved; w with a subscript means widened, the magnitude of the subscript indicating the amount of widening; w_3 represents the greatest widening observed.

As regards types of separation, vanadium is simpler than chromium. It has very few cases of asymmetry and comparatively few lines of more than four components. The largest number of components observed for one line is 12, for λ 6119.74. The one line having ten components is λ 6081.66. There are twenty lines having six components, two each of five, seven, and eight, and eight undivided lines. In addition to these there are 46 quadruple, 349 triple lines, and some whose classification is uncertain.

The average value of $\Delta\lambda \cdot 10^8/\lambda^2$ for all the triplets is 2.20, the extremes being 0.71 and 6.01. For those triplets which lie

TABLE I

SEPARATION OF VANADIUM LINES IN MAGNETIC FIELD

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
3865.02	3?	0.279	.....	1.87	....	*p* not observed
3867.00	3	0.392	.....	2.62	....	E
3867.77	3	0.357	.....	2.39	....	
3871.23	3	0.326	.....	2.18	....	
3873.80	1?	.....	.....	....	....	
3875.22	3	0.331	.....	2.20	....	
3876.05	6?	0.440	.....	2.93	....	*p*-components uncertain
3876.21	4	0.314	0.173	2.09	1.15	
3878.90	3	0.304	.....	2.02	....	E
3879.82	3	.....	.....	....	....	
3884.04	3	.....	.....	....	....	
3885.00	4	0.240	0.243	1.59	1.61	E
3886.72	3	0.178	.....	1.18	....	
3890.33	4	0.294	0.244	1.94	1.61	
3893.03	3	0.420	.....	2.77	....	
3896.29	1	.....	.....	....	......	
3897.22	3	0.310	.....	2.04	..	
3898.15	3	0.210	.....	1.38	....	
3899.30	3	0.282	.....	1.85	....	E
3900.33	3	.....	.....	....	......	Too weak to measure
3901.30	3	0.320	.....	2.10	....	*p*-component not sharp
3902.40	3	0.364	.....	2.39	....	
3903.42	3	0.381	.....	2.50	....	E
3906.89	3	0.412	.....	2.69	....	
3910.01	3 3	0.442 0.253	....	2.89 1.66	....	Double line −r
3910.95	4	0.315	0.116	2.06	0.76	
3912.36	4	0.301 0.000	....	1.97 0.00	....	N 1:2:1; 1 *p*-component
3913.03	3	0.379	.....	2.47	....	
3914.44	3	0.239	.....	1.56	....	E
3916.55	3	0.250	.....	1.63	....	E; *p*-component w_1
3920.15	3	0.408	.....	2.66	....	*n*-components of this line and the following blended and measured together
3920.65	4	0.408	0.161	2.66	1.05	
3922.05	6	0.484 0.186	0.160	3.15 1.21	1.04	
3922.58	3	0.376	.....	2.44	....	
3924.84	3	0.338	.....	2.19	....	
3930.19	3	0.295	.....	1.91	....	
3931.50	3	0.272	.....	1.76	....	
3934.16	3	0.398	.....	2.51	....	Measured normal and red component
3935.28	3	0.185	.....	1.20	....	
3936.42	3	0.579	.....	3.74	....	*p*-component w_3; very weak
3937.68	3	0.429	.....	2.77	....	
3938.35	3	0.292	.....	1.88	....	

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
3939.48	3	0.444	.....	2.86	....	*p*-component w_2 diffuse
3942.16	3	0.446	.....	2.87	....	*p*-component w_2 diffuse
3943.77	3	0.426	.....	2.74	....	
3950.37	3	0.281	.....	1.80	....	
3952.09	3	0.346	.....	2.22	....	E; fringed to red
3963.77	3	0.272	.....	1.73	....	Very difficult
3968.24	1	.....	.....	....	....	
3973.79	3	0.253	.....	1.60	....	E
3979.30	3	0.345	.....	2.18	....	
3979.59	3	0.272	.....	1.72	....	
3980.66	3	0.263	.....	1.66	....	
3984.75	4	.....	0.240	....	1.52	*n*-components too weak to measure
3985.90	3	0.296	.....	1.86	....	E
3988.97	4	.....	0.210	....	1.32	*n*-components too weak to measure
3990.71	3?	0.265	.....	1.66	....	
3992.95	3	0.307	.....	1.93	....	
3997.30	3	0.298	.....	1.86	....	E
3998.87	3	0.321	.....	2.01	....	
3999.30	3	0.296	.....	1.85	....	E
4003.10	6	0.438 0.074	0.367	2.73 0.46	2.29	
4005.86	3	0.361	.....	2.25	....	E
4017.00	3	0.276	.....	1.71	....	E
4023.50	3	0.310	.....	1.92	....	E
4031.37	4	0.395	.....	2.43	....	2 *p*-components too weak to measure
4031.98	3	0.330	.....	2.03	....	
4035.77	3	0.227	.....	1.39	....	E
4036.93	4	0.373	0.242	2.29	1.48	*n*-components; w_2 not sharp
4042.78	3	0.288	.....	1.76	....	*p*-component w_2
4048.77	3	0.341	.....	2.08	....	Red component wider than violet
4051.11	3	0.383	.....	2.33	....	Measured normal and violet component
4051.48	3	0.391	.....	2.38	....	Measured normal and red component
4053.80	3	0.302	.....	1.84	....	E; not sharp
4057.21	3	0.303	.....	1.84	....	
4064.09	3	0.202	.....	1.22	....	
4065.20	3	0.282	.....	1.71	....	E; *p*-component w_1
4067.90	3	0.366	.....	2.21	....	Difficult; *p*-component w_2
4071.67	3	0.318	.....	1.92	....	*p*-component w_1
4090.70	3	0.352	.....	2.10	....	
4092.09	4	0.270	0.283	1.61	1.69	*n* 1:2
4092.54	3	0.460	.....	2.75	....	Components shifted 0.048 Å to red; w_2
4092.83	3	0.445	.....	2.66	....	
4093.65	4	0.429	0.414	2.56	2.49	

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
4094.42	3	0.374	.....	2.23	....	Difficult; components w_3
4095.64	3	0.331	.....	1.97	....	
4097.09	4?	.....	0.191	....	1.14	*n*-components not measurable
4098.54	4?	.....	0.396	....	2.36	*n*-components not measurable
4099.93	3	0.454	.....	2.70	....	
4102.32	3	0.275	.....	1.63	....	
4104.55	3	0.419	.....	2.49	....	
4104.92	3	0.435	.....	2.58	....	Measured normal and violet component
4105.32	3	0.466	.....	2.77	....	
4107.64	4?	.....	0.442	....	2.62	*n*-components not measurable
4108.36	3	0.236	.....	1.40	....	
4109.94	6	0.791 0.340	0.227	4.68 2.01	1.34	
4111.92	3	0.467	.....	2.76	....	
4112.47	3	0.344	.....	2.04	....	Measured normal and red component
4113.65	3	0.362	.....	2.14	....	w_1
4115.32	3	0.490	.....	2.89	....	
4116.64	5	1.026 0.511	....	6.20 3.02	....	See description in text
4118.34	3	0.362	.....	2.14	....	w_1; measured normal and violet component
4118.73	4	0.425	0.182	2.51	1.07	Diffuse; measured normal and red component
4119.58	4	0.396	0.242	2.33	1.43	Components w_2; not sharp
4120.69	6	0.365 0.149	0.334	2.15 0.88	1.97	Weak; not sharp
4123.65	6	0.810 0.345	0.245	4.76 2.03	1.44	Outer *n*-components lightly fringed outside
4124.23	4	0.354	0.182	2.08	1.07	
4128.25	3	0.475	.....	2.79	.....	
4129.00	3	0.149	. ....	0.87	....	Not sharp
4132.13	3	0.470	.....	2.75		
4133.92	3	0.483	.....	2.83	....	
4134.61	3	0.471	.....	2.75	....	
4136.25	4	0.346	0.165	2.02	0.96	
4139.39	3	0.468	.....	2.73	....	
4149.02	3	0.358	.....	2.08	....	
4150.84	3	0.334	.....	1.94	....	
4159.84	1	.....	.....	....	....	w_3; not sharp
4171.45	4	0.419	0.152	2.41	0.87	
4174.18	3	0.384	.....	2.20	....	
4177.25	3	0.460	.....	2.64	....	Very difficult; w_2
4178.50	3	0.306	...	1.75	....	E; w_2; not sharp
4179.53	4	.....	0.529	....	3.03	*n*-components not measurable; *p*-components fringed inside

TABLE I—*Continued*

λ	Components	Δλ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
4182.23	3	0.310	.....	1.77	....	
4182.74	1	.....	.....	....	....	w_3; not sharp
4183.59	3	0.360	.....	2.06	....	E
4191.70	4	$\left\{\begin{matrix}0.516\\0.000\end{matrix}\right\}$	....	$\left\{\begin{matrix}2.94\\0.00\end{matrix}\right\}$	....	*n* 3:2:3; 1 *p*-component
4197.77	3	0.340	.....	1.93	....	
4202.52	3	0.395	.....	2.24	....	E; w_1
4204.67	4	0.524	.....	2.96	....	Weak, not sharp, w_2
4205.23	3	0.332	.....	1.88	....	E
4209.98	4	0.480	.....	2.71	....	*p*-components not measurable; w_3; not sharp
4224.30	3	0.501	.....	2.81	....	
4225.40	3	0.263	.....	1.47	....	E
4226.78	3	0.540	.....	3.02	....	Weak; measured normal and violet component
4227.90	3	0.334	.....	1.87	....	
4232.20	3	0.451	.....	2.52	....	E
4232.62	3	0.451	.....	2.52	....	
4233.09	3	0.415	.....	2.32	....	
4234.12	3	0.360	.....	2.01	....	
4234.70	4	0.328	0.185	1.83	1.03	Not sharp
4235.90	3	0.128	.....	0.71	....	
4243.10	3	0.326	.....	1.81	....	E; not sharp
4247.46	3	0.468	.....	2.60	....	*p*-component w_2
4254.55	3	0.435	.....	2.40	....	
4257.53	3	0.516	.....	2.85	....	
4259.46	3	0.416	.....	2.29	....	
4262.32	3	0.533	.....	2.94	....	
4265.28	3	0.395	.....	2.17	....	
4267.50	3	0.526	.....	2.89	....	
4268.78	3	0.414	.....	2.27	....	
4269.92	3?	0.670	.....	3.68	....	*p*-component w_3
4270.49	3	0.447	.....	2.45	....	
4271.71	3	0.384	.....	2.10	....	
4277.12	3	0.330	.....	1.80	....	
4284.19	3	0.238	.....	1.30	....	
4286.57	3	0.520	.....	2.83	....	Difficult
4287.97	3	0.385	.....	2.10	....	
4289.89	3	0.572	.....	3.11	....	*p*-component diffuse
4291.97	3	0.381	.....	2.07	....	
4296.28	3	0.354	.....	1.92	....	
4297.86	3	0.329	.....	1.78	....	
4298.17	3	0.298	.....	1.61	....	*p*-component w_2
4302.30	3	0.329	.....	1.78	....	
4305.63	3	0.411	.....	2.22	....	
4306.35	3	0.523	.....	2.82	....	
4307.33	3	0.521	.....	2.81	....	w_2; diffuse
4309.95	3	0.517	.....	2.78	....	
4330.18	3	0.143	.....	0.76	....	
4332.98	3	0.354	.....	1.88	....	
4341.15	3	0.430	.....	2.28	....	

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
4343.00	3	0.325	.....	1.72	....	
4346.59	3	0.305	.....	1.61	....	*p*-component diffuse; difficult
4353.02	3	0.469	.....	2.48	....	
4355.09	3	0.393	.....	2.07	....	
4356.10	3	0.613	.....	3.23	....	*p*-component w_3
4363.69	3	0.560	.....	2.94	....	
4364.37	3	0.191	.....	1.00	....	
4368.25	3	0.560	.....	2.94	....	*p*-component w_1
4368.76	3	0.357	.....	1.87	....	
4373.40	3	0.398	.....	2.08	....	
4373.99	3	0.396	.....	2.07	....	
4375.47	3	0.386	.....	2.02	....	*n*-component w_2
4379.38	3	0.449	.....	2.34	....	Very strong, w_3
4380.69	3	0.457	.....	2.38	....	Measured normal and violet component
4384.87	3	0.429	.....	2.23	....	
4390.13	3	0.405	.....	2.10	....	Very strong, w_3
4392.24	3	0.636	.....	3.30	......	
4394.01	3	0.348	.....	1.80	....	w_2
4395.40	3C	0.396	.....	2.05	....	
4399.63	3	0.310	.....	1.60	....	Difficult; *p*-component w_2
4400.74	5	0.785 0.000	0.409	4.05 0.00	2.11	*n* 1:3:1
4403.86	3	0.453	.....	2.34	....	
4406.30	3	0.395	.....	2.04	....	*p*-component diffuse
4406.80	4?	0.537	.....	2.76	....	2 strong *p*-components –r
4407.85	4	0.573	0.228	3.09	1.17	w_2; not well resolved
4408.36	4	.....	0.329	....	1.69	*n*-components blended with near-by lines; very strong, w_2
4408.67	6	0.676 0.309	0.471	3.48 1.59	2.42	*n* 1:2:2:1; *n*-components shifted 0.09 Å to red; w_2
4412.30	3	1.050	.....	5.40	....	*p*-components w_3; not sharp
4416.63	6	1.110 0.223	0.439	5.69 1.14	2.25	*n* 2:1:1:2
4420.08	3	0.639	.....	3.27	....	
4421.73	3C	0.894	.....	4.57	....	
4423.32	3	0.408	.....	2.09	....	Probably double line; components blended; difficult
4423.41	3?	....	.....	....	....	See preceding line
4424.74	3	0.374	.....	1.91	....	Very weak
4425.86	1?	.....	.....	....	....	
4426.17	3	0.729	.....	3.72	....	*p*-component w_2
4428.68	3	0.677	.....	3.45	....	
4429.95	3	0.655	.....	3.34	....	*p*-component w_2; not sharp

TABLE I—*Continued*

λ	Components	Δλ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
4430.68	3	0.900	.....	4.58	....	
4434.80	3	0.425	.....	2.16	....	Diffuse
4436.31	6	{1.029 0.682}	0.189	{5.23 3.47}	0.96	*n* 1:3:3:1
4438.02	3?	0.594	.....	3.02	....	*p*-component w_3
4441.88	4	0.641	0.206	3.25	1.04	w_1
4443.52	3	0.367	.....	1.86	....	Measured normal and red component
4444.40	4	0.768	0.282	3.89	1.43	w_1
4449.77	3	0.355	.....	1.79	....	
4451.09	3	0.367	.....	1.85	....	
4452.19	3	0.399	.....	2.01	....	
4457.65	8?	{0.440 0.197}	....	{2.22 0.99}	....	*p*-component w_3, sharp, probably four parts -r
4457.97	3	0.365	.....	1.84	....	Measured normal and red component
4459.93	3	0.374	.....	1.88		*p*-component w_2
4460.46	3	0.459	.....	2.31	....	w_2
4462.56	3	0.373	.....	1.87	....	
4468.19	3	0.524	.....	2.63	....	w_2, diffuse
4469.88	3	0.369	.....	1.85	....	
4474.21	3	0.427	.....	2.13	....	
4474.89	3	0.391	.....	1.95	....	
4480.20	4	0.405	0.240	2.02	1.20	Weak and diffuse
4489.06	3	0.375	.....	1.86	....	w_2
4490.95	3	0.476	.....	2.36	....	
4496.26	3	0.393	.....	1.94	....	
4502.12	3	0.425	.....	2.10	....	
4509.49	4?	.....	0.450	....	2.21	*n*-components not measurable
4513.79	3	0.470	.....	2.31	......	
4514.36	3	0.336	.....	1.65	....	
4520.67	4?	.....	0.247	....	1.21	*n*-components not measurable
4524.38	3	0.440	.....	2.15	....	*p*-component w_2; slightly shifted to violet
4525.31	5	{0.785 0.377}	.....	{3.84 1.84}	....	*p*-component w_3, diffuse. Also *Fe* line
4528.16	3	0.458	.....	2.23	....	Measured normal and violet component
4528.66	3	0.460	.....	2.24	....	E
4529.47	3	0.392	.....	1.91	....	Difficult
4529.76	3	0.288	.....	1.40	....	
4530.97	3	0.436	.....	2.12	....	
4537.84	3	0.458	.....	2.22	....	
4540.18	3?	0.595	.....	2.89	....	w_2, diffuse
4545.57	3	0.425	.....	2.06	....	
4549.81	3	0.395	.....	1.91	....	
4552.05	3	0.409	.....	1.97	....	
4553.25	3	0.385	.....	1.86	....	
4560.90	3	0.399	.....	1.92	......	

TABLE I—*Continued*

λ	Components	Δλ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
4564.76	3	0.378	.....	1.81	....	E, w_1
4570.60	4	0.465	0.252	2.23	1.21	
4571.96	3	0.366	.....	1.75	....	
4577.36	3	0.282	.....	1.35	....	
4578.92	3	0.307	.....	1.46	....	
4579.38	4	0.478	0.286	2.28	1.36	
4580.57	3	0.360	.....	1.72	....	
4583.96	4	0.461	0.383	2.19	1.82	Diffuse
4586.54	3	0.412	.....	1.96	....	
4590.70	3	0.470	.....	2.23	....	
4591.39	3	0.406	.....	1.93	....	
4594.27	3	0.442	.....	2.09	....	
4600.34	3	0.317	.....	1.50	....	E, diffuse
4606.33	4?	.....	0.425	....	2.00	*n*-components not measurable
4609.84	4	0.463	0.220	2.18	1.04	
4611.10	4?	.....	0.362	....	1.70	*n*-components not measurable
4619.85	3	0.392	.....	1.84	....	*n*-components of this line and the following blended and measured together
4619.97	4	0.392	0.267	1.84	1.25	
4624.62	4	0.583	0.346	2.73	1.62	w_2
4626.67	4	0.545	0.535	2.55	2.50	
4635.35	4	0.508	0.247	2.37	1.15	Diffuse
4640.25	4	0.144	0.311	0.67	1.44	
4640.92	4	0.635	0.181	2.95	0.84	
4644.64	3	0.540	.....	2.50	....	
4646.59	3	0.408	.....	1.89	....	
4670.66	3	0.487	.....	2.23	....	
4687.10	3	0.616	.....	2.80	....	*p*-component w_2
4699.52	3	0.463	.....	2.10	....	
4705.26	3	0.487	.....	2.20	....	
4706.34	3	0.553	.....	2.50	....	
4706.75	3	0.443	.....	2.00	....	
4707.62	3	0.443	.....	2.00	....	
4710.74	3	0.436	.....	1.96	....	
4714.28	3	0.400	.....	1.80	....	
4716.08	3	0.391	.....	1.76	....	
4717.85	3	0.345	.....	1.55	....	
4721.70	3	0.431	.....	1.93	....	
4723.06	3	0.494	.....	2.22	....	
4729.73	3	0.764	.....	3.41	....	
4730.57	3	0.436	.....	1.95	....	
4742.79	3	0.389	.....	1.73	....	
4746.81	4?	.....	0.684	....	3.04	*n*-components very diffuse
4748.70	4?	.....	0.427	....	1.89	*n*-components very diffuse
4751.16	4	0.549	0.361	2.43	1.60	Diffuse

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
4751.75	3	0.683	.....	3.02	....	Diffuse; measured normal and red component
4754.13	4	0.578	0.258	2.56	1.14	Diffuse
4757.68	1	.....	.....	....	....	
4766.80	3	0.296	.....	1.30	....	
4776.54	3	0.380	.....	1.67	....	Violet component blended with that of the following
4776.70	3	0.532	.....	2.33	....	
4786.70	3	0.454	.....	1.98	....	
4797.07	3	0.490	.....	2.13	....	
4807.70	3	0.512	.....	2.22	....	
4827.62	4	0.598	0.228	2.57	0.99	
4831.80	4	0.520	0.350	2.23	1.50	Components w_2
4832.59	7	.....	0.541	....	2.32	Probably 5 *n*-components −r
4851.65	3	0.224	.....	0.95	....	
4864.93	3	0.405	.....	1.71	....	
4875.66	3	0.486	.....	2.04	....	
4881.75	3	0.526	.....	2.21	....	
4900.84	3	0.486	.....	2.02	....	
4904.59	3	0.500	.....	2.08	....	
4925.83	3	0.741	.....	3.05	....	
4932.24	3	0.725	.....	2.98	....	
5002.54	3	0.450	.....	1.80	....	
5014.83	3	0.436	.....	1.74	....	
5064.32	3	0.489	.....	1.91	....	
5105.37	3	0.431	.....	1.65	....	
5128.71	3	0.597	.....	2.27	....	
5138.58	3	0.566	.....	2.14	....	
5139.74	3	0.427	.....	1.62	....	
5148.95	3	0.532	.....	2.01	....	
5159.56	3	0.393	.....	1.48	....	w_2; diffuse
5170.15	3	.....	0.512	....	1.92	1 *n*-component, 2 *p*-components
5177.03	3	0.530	.....	1.98	....	
5192.22	3	0.317	.....	1.18	....	
5193.18	3	0.735	.....	2.73	....	
5193.82	3	0.495	.....	1.84	....	Measured normal and red component
5195.01	3	0.664	.....	2.46	....	
5195.58	3	0.612	.....	2.27	....	
5216.80	3	0.780	.....	2.87	....	
5225.97	3	0.781	.....	2.86	....	
5233.91	3	0.763	.....	2.78	....	
5234.31	3	0.458	.....	1.67	....	
5241.06	3	0.552	.....	2.01	....	
5385.39	3	0.633	.....	2.18	....	
5402.17	3	0.524	.....	1.80	....	
5415.51	3	0.579	.....	1.98	....	

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
5434.43	3	0.631	.....	2.14	....	
5487.48	3	0.712	.....	2.36	....	
5488.18	3	0.650	.....	2.16	....	
5490.22	3	0.576	.....	1.91	....	
5505.13	6	0.942 0.372	0.324	3.11 1.23	1.07	*n* 1:3:3:1; poor
5507.97	3	0.452	.....	1.49	....	
5511.41	3	0.942	.....	3.10	....	
5546.18	3	1.083	.....	3.52	....	Measured normal and violet component
5547.31	3	1.032	.....	3.35	....	
5559.00	6	0.977 0.368	0.258	3.16 1.19	0.83	*n* 1:3:3:1
5561.92	4	0.864	.....	2.79	....	*p*-components too weak to measure
5584.75	3	0.898	.....	2.88	....	
5585.00	3	0.696	....	2.23	....	
5586.26	3	0.642	.....	2.06	....	
5588.71	3	0.778	.....	2.49	....	
5592.67	3	0.901	.....	2.88	....	
5601.63	3	0.648	.....	2.06	....	
5605.20	6	1.086 0.347	0.349	3.46 1.10	1.11	*n* 1:3:3:1
5624.80	3	0.824	.....	2.60	....	
5625.16	3	0.692	.....	2.19	....	
5626.27	1	.....	.....	....	....	
5627.86	3	0.863	.....	2.72	....	
5635.76	3	0.532	.....	1.67	....	
5646.36	6	1.080 0.350	0.363	3.39 1.10	1.14	*n* 3:1:1:3
5657.11	3?	0.453	.....	1.42	....	Measured normal and violet component; doubtful
5657.67	3	0.926	.....	2.89	....	
5668.61	3	0.916	.....	2.85	....	
5671.10	3C	0.400	.....	1.24	....	Strong, w_3; fringed
5683.47	4	0.696	.....	2.16	....	*p*-components too weak to measure
5698.74	3	0.593	.....	1.83	....	Strong, w_2
5703.83	3	0.571	.....	1.75	....	
5707.26	3	0.347	.....	1.06	....	Strong, w_2
5709.25	3	0.582	.....	1.78	....	
5716.49	3	0.706	.....	2.16	....	
5725.90	3	0.656	.....	2.00	....	
5727.25	3	0.700	.....	2.13	....	
5727.90	7	0.973 0.523 0.000	0.687	2.97 1.60 0.00	2.09	
5733.34	3	0.574	.....	1.75	....	
5734.26	3	0.598	.....	1.82	....	
5737.28	4	0.766	0.478	2.33	1.45	w_3

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
5743.67	4	0.820	0.496	2.48	1.50	w$_2$
5747.98	3	0.684	.....	2.07	....	
5749.13	3	0.556	.....	1.68	....	
5750.90	3	0.839	.....	2.54	....	
5752.99	6	1.229 0.320	0.412	3.71 0.97	1.24	*n* 1:3:3:1
5761.70	6	1.810	0.820 0.264	5.45	2.47 0.80	
5772.66	3	0.724	.....	2.17	....	*n*-components lightly fringed outside
5776.95	4	0.576	0.779	1.72	2.33	
5783.76	3	0.477	.....	1.42	....	
5784.64	3	0.718	.....	2.14	....	
5786.42	3	0.630	.....	1.88	....	
5788.85	4	0.896	0.842	2.67	2.51	
5807.40	3	0.541	.....	1.60	....	w$_2$
5817.33	4	0.487	0.500	1.44	1.48	
5830.97	3	0.732	.....	2.15	....	Strong, w$_2$, not sharp
5846.56	3	0.796	.....	2.33	....	Not sharp, w$_2$
5850.60	4	0.976	0.356	2.85	1.04	
5924.80	3	0.775	.....	2.21	....	
5979.18	3	0.688	.....	1.92	....	
5981.03	3	0.682	.....	1.91	....	w$_2$
6002.60	3	0.782	.....	2.17	....	
6002.83	3	1.370	.....	3.80	....	
6008.97	4	0.839	0.955	2.32	2.65	
6016.38	3	0.698	.....	1.93	....	
6018.16	4?	.....	0.506	....	1.40	*n*-components not measurable
6022.02	3	0.708	.....	1.95	....	
6025.66	3	0.626	.....	1.72	....	
6039.95	4	0.992	0.341	2.72	0.93	
6054.78	3	0.615	.....	1.68	....	
6058.36	6	1.762 0.585	0.577	4.80 1.59	1.57	*n* 1:3:3:1
6063.56	3	0.521	.....	1.42	....	
6067.48	3	0.646	.....	1.76	....	
6081.66	10	1.372 1.007 0.621	0.552	3.71 2.72 1.68	1.49	*n* 2:3:2:2:3:2; inner *p*-components too weak to measure
6090.43	3C	0.857	.....	2.31	....	
6107.20	3	0.707	.....	1.90	....	
6111.85	4	0.878	0.895	2.35	2.40	
6119.74	12	0.719	0.388 0.135	1.92	1.04 0.36	*n* 1:2:3:4:4:3:2:1
6128.56	5	1.498 0.000	0.617	3.99 0.00	1.64	
6135.56	6	1.325 0.359	0.500	3.52 0.95	1.33	*n* 1:3:3:1

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
6150.35	3	0.846	.....	2.24	....	
6170.54	3	0.808	.....	2.12	....	
6189.57	3	0.719	.....	1.88	....	w_2
6190.67	4?	.....	0.840	....	2.19	*n*-components not measurable; *p*-components fringed inside
6199.38	3	1.054	.....	2.74	....	
6214.06	4	1.078	0.329	2.79	0.85	
6216.55	3	1.057	.....	2.74	....	
6218.55	3	0.959	.....	2.48	....	
6221.54	4?	.....	0.390	....	1.01	*n*-components uncertain
6224.71	4	1.072	0.393	2.77	1.01	
6230.98	3	1.018	.....	2.62	....	*n*-components fringed outside
6233.39	4	1.048	0.502	2.70	1.29	
6236.52	3	0.744	.....	1.91	....	
6240.35	8	1.545 1.092 0.618	0.762	3.97 2.80 1.59	1.96	
6243.02	4	1.314	0.557	3.37	1.43	
6243.29	3	1.151	....	2.95	....	
6245.43	4	1.076	1.421	2.76	3.64	
6252.02	3	1.133	.....	2.90	....	
6257.12	3	1.188	.....	3.03	....	
6258.79	3	2.354	.....	6.01	....	
6261.42	6	2.310 0.571	0.820	5.89 1.46	2.09	*n* 3:2:2:3
6266.54	3C	1.688	.....	4.30	....	
6269.05	3	1.406	.....	3.58	....	
6274.85	6	1.821 0.925	0.457	4.62 2.35	1.16	*n* 1:3:3:1
6282.57	3	0.858	.....	2.17	....	
6285.38	3	1.156	.....	2.93	....	
6293.03	3	1.114	.....	2.81	....	
6296.70	3	1.131	.....	2.85	....	
6309.98	3	1.004	.....	2.52	....	
6311.74	3	0.790	.....	1.98	....	
6321.47	3	1.152	.....	2.88	....	w_2
6324.93	3	0.828	.....	2.07	....	w_2
6327.05	3	0.834	.....	2.08	....	
6339.32	3	0.781	.....	1.94	....	
6349.68	3	0.701	.....	1.74	....	
6355.83	3	0.673	.....	1.66	....	
6357.53	3	0.589	.....	1.45	....	
6359.05	3	0.833	.....	2.06	....	
6361.51	3	0.788	.....	1.95	....	
6374.74	3	0.487	.....	1.20	....	
6379.58	3	0.734	.....	1.80	....	
6393.56	3	0.598	.....	1.46	....	
6430.75	3	0.990	.....	2.39	....	
6431.90	3	0.958	.....	2.32	....	

TABLE I—*Continued*

λ	Components	$\Delta\lambda$ for 20,000 Gausses		$\frac{\Delta\lambda}{\lambda^2}\cdot 10^8$ for 20,000 Gausses		Remarks
		n	*p*	*n*	*p*	
6433.44	3	0.808	.....	1.95	....	
6438.35	3	0.863	.....	2.08	....	
6452.56	3	1.153	.....	2.77	....	
6488.33	3	0.877	.....	2.08	....	
6504.40	6	1.746 1.040	0.418	4.14 2.46	0.99	*n* 1:3:3:1
6531.66	3	1.262	.....	2.96	....	
6550.33	3	0.876	.....	2.04	....	
6558.28	3	0.920	.....	2.14	....	
6566.13	3	2.125	.....	4.93	....	
6606.20	6	..... 1.104	0.398	.... 2.53	0.91	Outer *n*-components too weak to measure
6625.05	3	1.125	.....	2.56	....	

between λ 3850 and λ 4800 the average is 2.17, which is the same as the corresponding quantity for chromium.[1] As in the case of the latter metal, so for vanadium, the values of $\Delta\lambda\cdot 10^8/\lambda^2$ for the triple lines occur in two main groups, of which one is much larger than the other. The first column of Table II shows a series of intervals in the values of $\Delta\lambda\cdot 10^8/\lambda^2$, while the second and third columns contain the numbers of values occurring in the intervals. The enhanced lines are given here separately in order to show that their grouping is not peculiar.

The average of all the values lying between 1.40 and 2.50 is 1.99 ± 0.011, while the average for the second group, between the limits 2.50 and 3.20, is 2.78 ± 0.012, the appended quantities being probable errors. For the enhanced lines the average is 1.95. Although the mean for the enhanced lines should be compared with the mean of those not enhanced, rather than with the general average, it does not appear that the enhanced lines are distinctly different from the others in their magnetic separation. The same conclusion was reached in the case of chromium.

For 46 quadruple lines the average value of $\Delta\lambda\cdot 10^8/\lambda^2$ for the *n*-components is 2.38, for the *p*-components 1.45.

There are a few peculiar lines in Table I to which attention should be called. The first is λ 4116.64, which has four *n*-components and only one *p*-component. The outer *n*-components show a very large separation and are shifted (by a field of 20,000 gausses) 0.033 Å to the red, while the inner pair are shifted 0.041 Å to the violet, the mean position of all four coinciding with the normal. λλ 3912.36 and 4191.70 each have three *n*-components and only one *p*-component. λ 5170.15 shows only one *n*-component, but two *p*-components—if other components exist they are very weak. λ 5761.70 is an interesting sextuple line having two widely separated *n*-components and four *p*-components. λ 5776.95 is a quadruple line of which the *p*-components are more widely separated than the *n*-components, in the ratio 4:3, while λ 4640.25 of the same type has its *p*-components twice as far apart as its *n*-components.

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[1] *Op. cit.*

TABLE II

DISTRIBUTION OF TRIPLET SEPARATIONS

INTERVALS IN $\frac{\Delta\lambda}{\lambda^2} \cdot 10^8$	NUMBER OF VALUES	
	Not Enhanced	Enhanced
0.60–0.80	2	..
0.80–1.00	2	..
1.00–1.20	4	..
1.20–1.40	7	1
1.40–1.60	11	3
1.60–1.80	37	5
1.80–2.00	59	9
2.00–2.20	67	2
2.20–2.40	38	4
2.40–2.60	17	2
2.60–2.80	27	1
2.80–3.00	28	..
3.00–3.20	6	..
3.20–3.40	5	..
3.40–3.60	4	..
3.60–3.80	2	..
3.80–6.20	5	..

For lines having more than three components various ratios of separation occur. These are collected in Table III. Ciphers in this table indicate central components coinciding with the normal position of the line.

The order of the ratios in any column corresponds to the frequency with which they occur, the most common ratio being at the top.

TABLE III

RATIOS OF SEPARATION

Total Number of Components						
4	5	6	7	8	10	12
2:1	0:2:5	1:1:3	0:3:4:6	4:5:7:10	?:1:1:2:3	1:3:6:?:?:?
3:1	0:1:2	2:3:8				
1:1		2:3:7				
5:3		1:3:7				
3:2		1:2:2				
3:4		1:2:4				
1:2		1:2:5				
		1:4:6				
		1:5:6				
		2:3:4				
		2:5:8				

My thanks are due to Mr. King for supplying two of the photographs used in this work, and to Miss Griffin for assistance in the reduction of the values.

MOUNT WILSON SOLAR OBSERVATORY
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THE INFLUENCE OF A MAGNETIC FIELD UPON THE SPARK SPECTRA OF IRON AND TITANIUM—SUMMARY OF RESULTS[1]

BY ARTHUR S. KING

The purpose of this paper is to give an outline of a detailed study of the Zeeman effect for iron and titanium carried out in the Pasadena Laboratory of the Solar Observatory, which is being published in full under the above title by the Carnegie Institution of Washington in Vol. II of the *Papers of the Mount Wilson Solar Observatory*.

The range of wave-length covered is from λ 3660 to about λ 6700, slightly beyond this for titanium. Between these limits, 662 lines in the spark spectrum of iron and 458 in that of titanium have been treated, at least to the extent of describing their type of separation by the magnetic field.

The spectra of both iron and titanium have been investigated previously to some extent. Measurements for portions of the iron spectrum have been published by H. M. Reese,[2] N. A. Kent,[3] W. Hartmann,[4] and Mme. H. B. van Bilderbeek-van Meurs.[5] These measurements are confined for the most part to the blue and violet regions and cover only the stronger lines. For titanium, extensive lists have been published by J. E. Purvis[6] and by the author.[7] The former list is mainly for the ultra-violet, including the stronger lines. The present investigation includes about 50 per cent more lines than the previous one by me, the data being compiled from a much more extensive set of plates, taken with higher dispersion and stronger field. The gain in all points is so great that these measures may be taken as superseding the former list.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 56.

[2] "An Investigation of the Zeeman Effect," *Astrophysical Journal*, **12**, 120, 1900.

[3] "Notes on the Zeeman Effect," *Astrophysical Journal*, **13**, 288, 1901.

[4] "Das Zeeman Phaenomen im sichtbaren Spectrum von Kupfer, Eisen, Gold, und Chrom," Dissertation, Halle, 1907.

[5] "Magnetische Splitsing van het Ultraviolette IJzerspectrum," Dissertation Amsterdam, 1909.

[6] "The Influence of a Strong Magnetic Field on the Spark Spectra of Titanium, Chromium, and Manganese," *Proceedings Cambridge Philosophical Society*, **14** (1), 41, 1906.

[7] "The Zeeman Effect for Titanium," *Contributions from the Mount Wilson Solar Observatory*, No. 39; *Astrophysical Journal*, **30**, 1, 1909.

The weaker lines in the two spectra have been photographed as far as they were obtainable, an object which has added to the labor and altered to some extent the experimental methods which would be used for the stronger lines alone. The tables for titanium contain all but the weakest of those lines given in the regular lists of arc and spark lines. As much cannot be claimed for iron, however, as numerous lines, fairly strong in the arc, are not brought out by the spark in the magnetic field even with an exposure of many hours. This is especially true of those lines of diffuse appearance, which are particularly numerous in the iron spectrum.

The publication summarizes the theory and the results of former investigations on the Zeeman effect so far as they apply directly to the present research. The apparatus and experimental methods are described in detail. The spark was given by a 5 K.W. transformer, the terminals being held between the poles of a Du Bois electro-magnet. The light was usually taken at right angles to the lines of magnetic force and passed through a Nicol prism placed above the slit of the vertical Littrow spectrograph. By rotating the Nicol through 90°, the components given by light vibrating parallel to the lines of force and in a plane perpendicular to the force-lines were successively photographed. A variety of field-strengths were used, sometimes slightly over 20,000 gausses, but all values for the tables were reduced to standard fields of 16,000 gausses for the iron spectrum and 17,500 gausses for that of titanium. A system of checking the field-strengths for different plates, somewhat difficult in the case of iron, was carried out, so that considerable confidence can be placed in the uniformity of the field for all parts of the spectrum.

The Littrow spectrograph was used with objectives of either 13 ft. (4 m) or 30 ft. (9.1 m) focal length, the third order of the plane grating generally being used at the shorter focus and the second order at the longer. Scales of about 1.3 and 0.95 Ångström units per mm, respectively, were obtained for photographs with the two arrangements, but dispersions as high as 0.62 and 0.44 Å per mm were occasionally employed with higher orders of the grating.

The photographic processes were adapted as nearly as possible to the somewhat conflicting requirements of speed, moderately fine grain, contrast sufficient to give sharply defined lines and still not such as to give too large differences in intensity between the stronger and weaker lines. For the most part, the Seed "27" plate was used for the blue end of the spectrum, and for wave-lengths greater than 4600 the same plate was sensitized by a three-dye bath. Regions of the spectrum containing a wide range in the intensity of lines often required special treatment both as to instrumental and photographic methods.

In the measurement of the plates, I am indebted to Miss Wickham and to Miss Griffin for much assistance.

In making extracts from the tables, there is no criterion which makes a certain class or type of lines of special importance for a summary of this sort, so the method adopted is to take the descriptions of lines for various short stretches of wave-length bodily from the complete tables. These regions are selected to represent as well as possible the types of separation occurring throughout the two spectra. Each region contains a number of strong lines and includes some complex types of separation. The change in average separation with the wave-length may be noted by comparing the general magnitude of the measurements in successive regions.

EXPLANATION OF THE TABLES

The letters *n* and *p* in the tables are used to denote the Zeeman components given by light vibrations in a plane at right angles to the lines of magnetic force and parallel to these respectively. The wave-lengths are on the Rowland system. The second column gives a rough value for the intensity of lines in the spark spectrum, taken partly from the tables of Exner and Haschek and partly estimated on the same scale from my plates. Weak lines are graded 1, but there is a considerable variation in the strength of lines which are given this value.

The third column gives the best judgment of the type of separation that can be made from the photographs. The description given must be considered in connection with the measured separation and widening of components given in the columns for $\Delta\lambda$ of the n- and p-components, and is usually made clear by these. Frequently a supplementary remark is needed in the case of complex lines. The interrogation point is freely used to indicate that the line is probably of the character given, though not clearly shown to be so on the plates. The reason for doubt is usually given in the columns for $\Delta\lambda$. Thus "triple?" means that the p-component is slightly widened and may not be simple, though the widening may be explained by the strength of the component or by the fact that the no-field line is slightly diffuse. "Quadruple?" signifies that the two n-components are fairly sharp, but the p-component is probably double. The doubtful sextuplet is very common and as a rule has its two n-components each widened so that there are probably two pairs, while the p-component is either distinctly double or unresolved and considerably widened. The questioned septuplet has almost the same appearance, but shows a central maximum to the widened p-component, indicating three constituents. Lines whose n-components shade off either outward or toward the center are given in the "Remarks" column as "fringed." The fringes indicate very close, unresolved components. If the structure of the line is very complex, an interrogation point is used without any attempt to give the number of components.

Although the character of separation is thus doubtful in many cases, the large number of plates from which the material was taken gave an opportunity to study each line under various conditions of intensity and degree of separation so that the classification as to character is probably as accurate as can be made without very much higher field-strength combined with as large dispersion as was here used.

In the fourth column, lines are weighted 3, 2, or 1 according as the quality of the components for measurement is good, fair, or poor. Weight 3 indicates that the components are sharply defined, the probable error of measurement being in the third decimal place, often not greater than two or three thousandths of an Ångström. Weight 2 is given to lines which do not permit such close measurement, but which as a rule have a probable error not greater than 10 per cent. Lines weighted 1 are either very faint, much disturbed by blends, or of such complex structure that the components are very diffuse. The error of measurement for such lines may be large, but the figures given show whether the line is to be classed as having small, medium, or large separation. When both the *n*- and the *p*-components are measurable, the weight for each is given, separated by a comma. In case only the *p*-component is measured, a dash before the comma indicates the omission of the weight for the *n*-component.

The fifth and sixth columns give the separation in Ångström units of the *n*- and *p*-components respectively. A single value denotes a pair of components. When there is more than one pair for the same polarization, the largest separation is given first, but the innermost pair is designated as "Pair I." When an odd number of *n*- or *p*-components is present, any outer components that may appear are measured from the central component, the latter being denoted by 0.000. The relative intensities of multiple components are indicated by figures in parentheses after the value of the separation.

The widening of individual components, which often denotes a blend of two or more, is indicated by "w" with subscript 1, 2, or 3, according to the degree of widening. The letter "w" in the *p*-component column shows that the unresolved component may not be single. If a measurement followed by "w" is given in the *n*-component column, a more or less diffuse pair is present, which might be resolved by higher field into two or more pairs.

The letters "n.m." indicate that a separation exists but is not measurable, generally by reason of faintness. In such cases it is usually possible to tell the character of the separation with fair certainty and the line is included on this account. When the components are hazy as well as faint "n.m.w." is used.

The seventh and eighth columns give the values of $\Delta\lambda/\lambda^2$, the positions in the column corresponding to those of the values of $\Delta\lambda$. When λ is expressed in Ångström units, the values given for $\Delta\lambda/\lambda^2$ are to be multiplied by 10^{-8}.

TABLE I

MEASUREMENTS OF ZEEMAN EFFECT FOR IRON

λ	Intensity	Character of Separation	Weight	Δλ		Δλ/λ²		Remarks
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
3722.071......	1	Triple	1	0.266	.....	1.920	.....	Faint in spark
3722.729......	4	12 comps.	2, 2	Pair IV, 0.415 (2)	Pair II, 0.195 (5)	2.994	1.407	
				Pair III, 0.311 (3)	Pair I, n.m. (1)	2.244	?	
				Pair II, 0.211 (3)	.....	1.522	.....	
				Pair I, 0.103 (2)	.....	0.743	.....	
3724.526......	2	Triple	3	0.256	.....	1.845	.....	
3727.778......	5	Septuple?	2	0.318 W_1	W_2	2.288	.....	*n*-comps. have inner fringes. Probably 3 *p*-comps.
3730.534......	1	Triple?	1	0.341 W_1	W_1	2.451	.....	Blend with faint lines
3731.093......	1	Triple	1	0.176	.....	1.264	.....	
3732.545......	2	Triple	3	0.399	.....	2.865	.....	
3733.469......	3	Quintuple	3, 3	0.157 (2)	0.322	1.127	2.311	
				0.000 (3)	.....	0.000	.....	
				0.158 (2)	.....	1.134	.....	
3735.014......	10	Triple	2	0.310	.....	2.222	.....	
3735.485......	1	Quadruple?	2, 2	0.472 W_1	0.166	3.384	1.190	
3737.281......	7	Septuple?	2	0.254 W_1	W_2	1.818	.....	*n*-comps. fringed. Probably 3 *p*-comps.
3738.454......	2	Triple	3	0.207	.....	1.481	.....	
3743.508......	6	Octuple	3, 3	0.208 (4)	0.107 (2)	1.484	0.763	Comps. of faint line to red (computed 3743.615) are superposed on central and outer red *n*-comps., giving apparent dissymmetry. Compare 3788.046
				0.104 (2)	0.000 (3)	0.742	0.000	
				0.000 (2)	0.106 (2)	0.000	0.756	
				0.112 (2)		0.799	.....	
				0.213 (5)	.....	1.520	.....	
3744.251......	1	Septuple?	1, 1	0.401 W_2	0.283 (1)	2.861	2.018	Probably 4 *n*-comps. Apparently 3 *p*-comps., but faint. Compare 3747
					0.000 (1)	.....	0.000	
					0.415 (1)	.....	2.960	

3745.717......	5	Septuple?	2	0.228 w_1	w_2	1.624	... :	*n*-comps. fringed. Probably 3 *p*-comps.
3746.058......	4	Unaffected	..	.....	.....	.....	.....	
3747.065......	1	Septuple?	1, 1	0.413 w_2	0.306 (1)	2.941	2.180	Apparently same type as 3744. Comps. more distinct
					0.000 (2)	.....	0.000	
					0.341 (1)	.....	2.429	
3748.408......	4	9 comps.?	2	Pair III, 0.316 (4)	w_3	2.250	.....	Probably 3 *p*-comps. almost resolved
				Pair II, 0.226 (3)	.....	1.609	.....	
				Pair I, 0.101 (1)	.....	0.719	.....	
3749.049......	1	Quadruple?	2	0.240 w_1	w_2	1.708	.....	
3749.631......	10	Triple	2	0.289	.....	2.055	.....	
3753.732......	2	Triple	3	0.395	.....	2.803	.....	
3756.213......	1	Triple?	2	0.300	w_1	2.126	.....	
3757.081......	1	Triple?	1	0.197	w_1	1.396	.....	
3757.597......	1	Triple	1	0.388	.....	2.747	.....	Faint in spark
3758.375......	8	Triple	3	0.269	.....	1.905	.....	
3760.196......	2	Triple	3	0.235	.....	1.662	.....	
3760.679......	1	Quintuple?	3	0.146 (1)	w_2	1.032	.....	*n*-comp. appears as unsymmetrical triplet. No evidence of blend
				0.000 (1)	.....	0.000	.....	
				0.179 (1)	.....	1.265	.....	
3763.945......	6	Triple	3	0.218	.....	1.538	.....	
3765.689......	3	Triple	3	0.228	.....	1.608	.....	
3767.341......	5	Unaffected	..	.....	.....	.....	.....	
3768.173......	1	Triple	2	0.600	.....	4.225	.....	Faint in spark
3770.446......	1	Triple	2	0.191	.....	1.343	.....	
3773.803......	1	Unaffected	..	.....	.....	.....	.....	
3774.971......	1	Sextuple	2, 1	Pair II, 0.545 (1)	0.240	3.824	1.684	*p*-comps. faint
				Pair I, 0.274 (1)	.....	1.922		
3776.600......	1	Triple?	2	0.229	w_1	1.605	.....	
3777.593......	1	Quadruple?	..	n.m.	n.m.	.....	.....	Very faint. Wide separation of *p*-comp.
3778.652......	1	Triple	1	0.344	.....	2.408	.....	
3779.569......	1	Triple?	1	0.277	.....	1.938	.....	
3781.330......	1	?	..	n.m.	w_2	.....	.....	Many comps. *n*-comps. diffuse. Not resolved
3786.092......	2	Triple	3	0.220	.....	1.535	.....	

TABLE I—*Continued*

λ	INTENSITY	CHARACTER OF SEPARATION	WEIGHT	$\Delta\lambda$		$\Delta\lambda/\lambda^2$		REMARKS
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
3786.314......	2	Triple?	..	n.m.w_2	w_2	.....	.....	*n*-comps. diffuse
3786.820......	2	Unaffected	..	.....	.....	.....	.....	
3788.046......	4	Octuple	3, 3	0.214 (4)	0.111 (2)	1.491	0.744	Magnetic duplicate of 3743.508
				0.108 (2)	0.000 (3)	0.753	0.000	
				0.000 (1)	0.109 (2)	0.000	0.760	
				0.111 (2)	.....	0.774	.....	
				0.219 (4)	.....	1.526	.....	
4233.772......	2	9 comps.	2, 2	Pair III, 0.780 (1)	0.140 (2)	4.351	0.780	
				Pair II, 0.550 (2)	0.000 (3)	3.068	0.000	
				Pair I, 0.265 (5)	0.140 (2)	1.478	0.780	
4236.112......	4	Triple	3	0.452	.....	2.519	.....	
4238.970......	1	Triple	2	0.320	.....	1.781	.....	
4240.014......	1	Triple	2	0.440	.....	2.447	.....	
4245.422......	1	Triple	3	0.493	.....	2.736	.....	
4246.251......	1	Triple	1	0.273	.....	1.514	.....	
4247.591......	1	Triple	3	0.356	.....	1.973	.....	
4248.384......	1	Triple	2	0.377	.....	2.089	.....	
4250.287......	8	Septuple?	2	0.382 w_1	w_3	2.115	.....	*n*-comps. fringed. Probably 3 *p*-comps.
4250.945......	9	12 comps.?	2, 2	0.246 w_3	0.211 w_1	1.361	1.168	Probably 8 *n*-, 4 *p*-comps.
4260.640......	10	Triple	3	0.423	.....	2.330	.....	
4267.122......	1	Triple	1	0.300	.....	1.648	.....	
4267.985......	1	Triple	1	0.528	.....	2.899	.....	
4268.915......	1	Triple?	1	0.462	w_1	2.535	.....	
4271.325......	9	Triple	3	0.394	.....	2.160	.....	
4271.934......	10	Triple	3	0.341	.....	1.868	.....	
4282.565......	1	Septuple?	2	0.310 w_2	w_3	1.691	.....	*n*-comps. fringed. Probably 3 *p*-comps.
4285.605......	1	Triple	3	0.315	.....	1.715	.....	
4294.301......	5	Sextuple?	2, 2	0.319 w_2	0.138	1.730	0.748	*n*-comps. almost resolved

4298.195......	1	Triple?	2	0.457	w_1	2.474	.....	
4299.410......	5	Triple	3	0.406	.....	2.197	.....	
4302.353......	1	Triple	2	0.316	.....	1.707	.....	Enhanced line
4303.337......	1	Sextuple?	2, 2	0.415 w_2	0.265	2.241	1.431	Enhanced line
4305.614......	1	Quadruple?	1	0.328 w_1	w_2	1.769	.....	
4308.081......	15	Triple	3	0.320	.....	1.724	.....	
4309.541......	1	Triple	2	0.325	.....	1.750	.....	
4315.262......	3	Sextuple?	3, 3	0.517 w_1	0.090	2.777	0.483	
4325.939......	15	Triple	3	0.245	.....	1.309	.....	
4327.274......	1	Triple	2	0.313	.....	1.672	.....	
4328.080......	1	Triple	2	0.246	.....	1.313	.....	
4337.216......	2	Sextuple?	2, 3	0.264 w_2	0.154	1.404	0.819	
4346.725......	1	Triple	1	0.292	w_2?	1.545	.....	Blend with air lines
4351.930......	1	Septuple?	1	0.311 w_2	w_3	1.642	.....	Probably 3 *p*-comps. May be *Cr*, but given by Lockyer as enhanced line *Fe*
4352.908......	2	Septuple?	2, 3	0.416 w_2	0.075 (1) 0.000 (2) 0.075 (1)	2.195	0.396 0.000 0.396	*n*-comps. fringed
4367.749......	1	Triple	2	0.311	.....	1.630	.....	
4369.941......	2	Triple	3	0.282	.....	1.477	.....	
4376.107......	2	Triple	3	0.424	.....	2.214	.....	
4383.720......	20	Triple	3	0.332	.....	1.727	.....	
4385.548......	1	Quadruple	2, 2	0.367	0.391	1.910	2.032	Enhanced line
4388.057......	1	Triple	..	n.m.	.....	.....	.....	*n*-comps. blended with adjacent lines
4388.571......	1	Triple	2	0.432	.....	2.243	.....	
4391.123......	1	Triple	..	n.m.	.....	.....	.....	Faint
4404.927......	15	Triple	3	0.334	.....	1.720	.....	
4407.871......	1	Triple?	2	0.631	w_1	3.247	.....	
4408.582......	1	Triple?	2	0.488	w_1	2.511	.....	
4415.293......	10	Septuple?	2	0.338 w_2	w_2	1.734	.....	*n*-comps. have inner fringes. Probably 3 *p*-comps.
4422.741......	1	Sextuple	2, 3	Pair II, 0.432 (1) Pair I, 0.154 (1)	0.280	2.208 0.787	1.431	

TABLE I—*Continued*

λ	Intensity	Character of Separation	Weight	Δλ		Δλ/λ²		Remarks
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
3786.314......	2	Triple?	..	n.m.w_2	w_2	.....	.....	*n*-comps. diffuse
3786.820......	2	Unaffected	..	.....	.....	.....	.....	
3788.046......	4	Octuple	3, 3	0.214 (4)	0.111 (2)	1.491	0.744	Magnetic duplicate of 3743.508
				0.108 (2)	0.000 (3)	0.753	0.000	
				0.000 (1)	0.109 (2)	0.000	0.760	
				0.111 (2)	.....	0.774	.....	
				0.219 (4)	.....	1.526	.....	
4233.772......	2	9 comps.	2, 2	Pair III, 0.780 (1)	0.140 (2)	4.351	0.780	
				Pair II, 0.550 (2)	0.000 (3)	3.068	0.000	
				Pair I, 0.265 (5)	0.140 (2)	1.478	0.780	
4236.112......	4	Triple	3	0.452	.....	2.519	.....	
4238.970......	1	Triple	2	0.320	.....	1.781	.....	
4240.014......	1	Triple	2	0.440	.....	2.447	.....	
4245.422......	1	Triple	3	0.493	.....	2.736	.....	
4246.251......	1	Triple	1	0.273	.....	1.514	.....	
4247.591......	1	Triple	3	0.356	.....	1.973	.....	
4248.384......	1	Triple	2	0.377	.....	2.089	.....	
4250.287......	8	Septuple?	2	0.382 w_1	w_3	2.115	.....	*n*-comps. fringed. Probably 3 *p*-comps.
4250.945......	9	12 comps.?	2, 2	0.246 w_3	0.211 w_1	1.361	1.168	Probably 8 *n*-, 4 *p*-comps.
4260.640......	10	Triple	3	0.423	.....	2.330	.....	
4267.122......	1	Triple	1	0.300	.....	1.648	.....	
4267.985......	1	Triple	1	0.528	.....	2.899	.....	
4268.915......	1	Triple?	1	0.462	w_1	2.535	.....	
4271.325......	9	Triple	3	0.394	.....	2.160	.....	
4271.934......	10	Triple	3	0.341	.....	1.868	.....	
4282.565......	1	Septuple?	2	0.310 w_2	w_3	1.691	.....	*n*-comps. fringed. Probably 3 *p*-comps.
4285.605......	1	Triple	3	0.315	.....	1.715	.....	
4294.301......	5	Sextuple?	2, 2	0.319 w_2	0.138	1.730	0.748	*n*-comps. almost resolved

4298.195......	1	Triple?	2	0.457	w_1	2.474	.....	
4299.410......	5	Triple	3	0.406	.....	2.197	.....	
4302.353......	1	Triple	2	0.316	.....	1.707	.....	Enhanced line
4303.337......	1	Sextuple?	2, 2	0.415 w_2	0.265	2.241	1.431	Enhanced line
4305.614......	1	Quadruple?	1	0.328 w_1	w_2	1.769	.....	
4308.081......	15	Triple	3	0.320	.....	1.724	.....	
4309.541......	1	Triple	2	0.325	.....	1.750	.....	
4315.262......	3	Sextuple?	3, 3	0.517 w_1	0.090	2.777	0.483	
4325.939......	15	Triple	3	0.245	.....	1.309	.....	
4327.274......	1	Triple	2	0.313	.....	1.672	.....	
4328.080......	1	Triple	2	0.246	.....	1.313	.....	
4337.216......	2	Sextuple?	2, 3	0.264 w_2	0.154	1.404	0.819	
4346.725......	1	Triple	1	0.292	w_2?	1.545	.....	Blend with air lines
4351.930......	1	Septuple?	1	0.311 w_2	w_3	1.642	.....	Probably 3 *p*-comps. May be *Cr*, but given by Lockyer as enhanced line *Fe*
4352.908......	2	Septuple?	2, 3	0.416 w_2	0.075 (1)	2.195	0.396	*n*-comps. fringed
					0.000 (2)	.....	0.000	
					0.075 (1)	.....	0.396	
4367.749......	1	Triple	2	0.311	.....	1.630	.....	
4369.941......	2	Triple	3	0.282	.....	1.477	.....	
4376.107......	2	Triple	3	0.424	.....	2.214	.....	
4383.720......	20	Triple	3	0.332	.....	1.727	.....	
4385.548......	1	Quadruple	2, 2	0.367	0.391	1.910	2.032	Enhanced line
4388.057......	1	Triple	..	n.m.	.....	.....	.....	*n*-comps. blended with adjacent lines
4388.571......	1	Triple	2	0.432	.....	2.243	.....	
4391.123......	1	Triple	..	n.m.	.....	.....	.....	Faint
4404.927......	15	Triple	3	0.334	.....	1.720	.....	
4407.871......	1	Triple?	2	0.631	w_1	3.247	.....	
4408.582......	1	Triple?	2	0.488	w_1	2.511	.....	
4415.293......	10	Septuple?	2	0.338 w_2	w_2	1.734	.....	*n*-comps. have inner fringes. Probably 3 *p*-comps.
4422.741......	1	Sextuple	2, 3	Pair II, 0.432 (1)	0.280	2.208	1.431	
				Pair I, 0.154 (1)	.....	0.787	.....	

TABLE I—*Continued*

λ	Intensity	Character of Separation	Weight	$\Delta\lambda$		$\Delta\lambda/\lambda^2$		Remarks
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
5283.802......	3	Triple	3	0.623	.....	2.232	.....	
5302.480......	2	Triple	3	0.632	.....	2.272	.....	
5316.790......	4	Triple?	2	0.455 W_1	W_2	1.610	.....	Enhanced line. Diffuse comps. may be due to character of line
5324.373......	5	Triple	3	0.648	.....	2.286	.....	Red comp. slightly stronger than violet
5328.236......	7	Septuple?	2	0.470 W_1	W_3	1.656	.....	*n*-comps. fringed. Probably 3 *p*-comps.
5328.696......	3	Sextuple?	2, 2	0.488 W_2	0.275	1.718	0.968	
5340.121......	2	Triple	2	0.664	.....	2.329	.....	
5341.213......	3	?	1, 2	0.486 W_3	0.429	1.703	1.502	*n*-comps. blurred, probably at least 6
5353.571......	1	Triple	..	n.m.	.....	.....	.....	Very faint
5365.069......	1	Sextuple?	1	0.354 W_2	W_2	1.229	.....	Comps. very diffuse
5365.596......	1	Triple	1	0.471	.....	1.636	.....	Blend with preceding line
5367.669......	1	Triple?	2	0.414	W_1	1.437	.....	Diffuse
5370.166......	2	Triple?	2	0.456	W_1	1.581	.....	Diffuse
5371.734......	6	9 comps.?	2	0.413 W_2	W_2	1.431	.....	*n*-comps. fringed. Probably 6 *n*-, 3 *p*-comps.
5383.578......	4	Triple?	3	0.480 W_1	W_1	1.656	.....	
5393.375......	3	Triple	2	0.673	.....	2.314	.....	
5397.344......	6	Sextuple?	2, 3	0.630 W_2	0.222	2.163	0.762	
5404.357......	5	Triple?	2	0.467 W_1	W_1	1.599	.....	Diffuse
5405.989......	6	9 comps.?	2, 2	Pair II, 0.461 (1)	0.117 (2)	1.577	0.400	Probably third pair *n*-comps. outside
				Pair I, 0.222 (4)	0.000 (3)	0.760	0.000	
					0.121 (2)	.....	0.414	
5411.124......	3	Triple?	2	0.435 W_1	W_1	1.486	.....	Diffuse
5415.416......	5	Triple?	2	0.510 W_1	W_1	1.739	.....	Diffuse
5424.290......	5	Triple?	2	0.498 W_1	W_1	1.693	.....	Diffuse

5429.911......	6	10 comps.?	2, 3	0.607 w_2	0.300	2.059	1.017	3 or possibly 4 pairs *n*-comps. Probably weak inner pair *p*-comps.
5434.740......	5	Unaffected	..	.....	.....	.....	.....	
5445.259......	2	Triple?	2	0.415	w_1	1.399	.....	Diffuse
5447.130......	5	12 comps.	2, 3	Pair IV, 0.874 (1)	Pair II, 0.477 (6)	2.946	1.507	*n*-comps. barely resolved
				Pair III, 0.701 (2)	Pair I, 0.226 (1)	2.363	0.762	
				Pair II, 0.477 (2)	.....	1.608	.....	
				Pair I, 0.219 (1)	.....	0.738	.....	
5455.834......	4	Quintuple	2, 3	0.347 (1)	0.680	1.163	2.283	Central line of *n*-triplet displaced 0.009 Å toward red from normal
				0.000 (2)	.....	0.000	.....	
				0.345 (1)	.....	1.160	.....	
6136.829......	5	Triple	3	0.515	.....	1.367	.....	
6137.915......	5	Triple	3	0.654	.....	1.736	.....	
6141.938......	1	Triple?	2	1.017	w_1	2.696	.....	Faint
6148.040......	1	Sextuple?	1	0.649 w_2	w_1	1.717	.....	Very diffuse
6149.458......	1	Quadruple	1, 1	0.812	0.740	2.148	1.950	Enhanced line
6157.945......	1	Triple?	2	0.719	w_1	1.896	.....	
6165.577......	1	Sextuple?	..	n.m.w_2	w_2	.....	.....	*n*-comps. diffuse
6170.730......	1	Sextuple?	2	0.725 w_2	w_3	1.904	.....	*p*-comp. almost resolved
6173.553......	1	Triple	2	1.590	.....	4.171	.....	Difficult. Blend with air band
6180.420......	1	Triple?	..	n.m.	w_1	.....	.....	Faint
6188.210......	1	Triple	..	n.m.	.....	.....	.....	Faint
6191.779......	5	Triple	3	0.541	.....	1.411	.....	
6200.527......	1	Sextuple?	1	1.026 w_2	w_3	2.669	.....	Comps. diffuse
6213.644......	1	Sextuple	2, 3	Pair II, 1.603 (1)	0.572	4.151	1.481	
				Pair I, 0.798 (1)	.....	2.067	.....	
6215.360......	1	Triple	2	0.583	.....	1.509	.....	
6219.494......	1	Sextuple?	2, 2	0.991 w_2	0.385	2.562	0.995	
6230.943......	6	Triple	3	0.767	.....	1.976	.....	
6232.856......	1	Sextuple?	2	1.205 w_1	w_2	3.102	.....	
6238.598......	1	Sextuple?	1, 1	1.042 w_1	0.512	2.677	1.315	Enhanced line
6246.535......	2	Sextuple?	2, 2	0.958 w_1	0.278	2.456	0.713	

TABLE I—*Continued*

λ	Intensity	Character of Separation	Weight	Δλ		Δλ/λ²		Remarks
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
6247.774......	1	Septuple?	1	0.670 w_2	w_3	1.716	.....	Enhanced line. *n*-comps. very diffuse. Probably 3 *p*-comps.
6252.773......	3	Triple	3	0.582	.....	1.488	.....	
6254.456......	1	Triple	3	0.952	.....	2.434	.....	
6256.572......	1	Sextuple?	–, 2	n.m.w_3	0.696	.....	1.778	*n*-comps. very diffuse
6265.348......	1	Sextuple?	2, 2	0.969 w_2	0.278	2.469	0.708	
6270.442......	1	Triple?	..	n.m.	w_1	.....	.....	Very faint
6291.184......	1	Sextuple?	1	1.014	w_2	2.562	.....	
6298.007......	1	Sextuple?	..	n.m.w_2	w_3	.....	.....	
6301.718......	3	Sextuple?	2, 2	1.063 w_2	0.392	2.676	0.987	*n*-comps. probably double
6302.709......	1	Triple	2	1.618	.....	4.073	.....	
6315.517......	1	Sextuple?	1	0.906 w_1	w_2	2.271	.....	Faint. *p*-comp. almost resolved. Enhanced line
6318.239......	3	Triple	2	0.452	.....	1.132	.....	
6322.907......	1	Triple	2	0.965	.....	2.414	.....	
6331.067......	1	Sextuple?	1	0.820 w_2	w_1	2.046	.....	Very faint
6335.554......	3	Septuple?	2	0.665 w_2	w_3	1.656	.....	Probably 3 *p*-comps.
6337.048......	3	Sextuple	2, 2	Pair II, 1.641 (1)	0.632	4.086	1.574	
				Pair I, 0.946 (1)	.....	2.356	.....	
6339.096......	1	Triple?	..	n.m.	.....	.....	.....	Very faint
6344.371......	1	Sextuple?	1	0.771 w_1	n.m.	1.916	.....	Very faint. *p*-comp. double
6355.246......	1	Triple	1	0.730	.....	1.808	.....	Very faint
6358.898......	1	Sextuple?	1	0.746 w_2	w_1	1.845	.....	Very faint
6380.958......	1	Triple	1	0.464	.....	1.140	.....	Enhanced line
6393.820......	8	Triple	2	0.593	.....	1.450	.....	
6400.217......	9	Septuple?	2	0.802 w_1	w_2	1.958	.....	*n*-comps. slightly fringed. Probably 3 close *p*-comps.

6408.233......	1	Septuple?	–, 2	n.m.	0.343 (1) 0.000 (2) 0.349 (1)	....	0.837 0.000 0.852	Apparently 4 weak *n*-comps. about equally spaced
6411.865......	5	Septuple?	2	0.686 w_2	w_3	1.668	.....	*n*-comps. fringed, probably 3 *p*-comps.
6417.133......	1	?	1	1.018 w_2	w_3	2.472	.....	*n*- and *p*-comps. very diffuse. Enhanced line
6420.169......	1	Sextuple?	1	0.743 w_2	w_2	1.802	.....	*n*-comps. diffuse. Enhanced line
6421.570......	2	Triple	3	0.993	.....	2.408	.....	
6431.066......	3	Sextuple?	2	0.775 w_1	w_1	1.873	.....	

TABLE II

MEASUREMENTS OF ZEEMAN EFFECT FOR TITANIUM

λ	Intensity	Character of Separation	Weight	$\Delta\lambda$		$\Delta\lambda/\lambda^2$		Remarks
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
4278.390......	2	Triple	3	0.304	.....	1.661	.....	
4281.530......	2	Octuple	3, 3	0.444 (4)	0.222 (2)	2.422	1.211	
				0.218 (2)	0.000 (3)	1.189	0.000	
				0.000 (1)	0.224 (2)	0.000	1.222	
				0.218 (2)	.....	0.189	.....	
				0.448 (4)	.....	2.443	.....	
4282.860......	3	Triple	3	0.244	.....	1.330	.....	
4285.164......	5	Quadruple?	3	0.566	w_1	3.083	.....	
4286.168......	4	Sextuple?	2, 3	0.400 w_2	0.166	2.177	0.904	
4287.566......	4	Sextuple?	2, 3	0.421 w_2	0.146	2.290	0.794	
4288.038......	2	Septuple?	2	0.516 w_2	w_3	2.806	.....	*n*-comps. have inner fringes. Probably 3 *p*-comps.
4289.237......	4	12 comps.	2, 3	Pair IV, 0.586 (1)	Pair II, 0.286 (6)	3.186	1.566	
				Pair III, 0.443 (2)	Pair I, 0.140 (1)	2.408	0.761	
				Pair II, 0.306 (2)	.....	1.663	.....	
				Pair I, 0.144 (1)	.....	0.783	.....	
4290.377......	10	?	2	0.284 w_2	w_3	1.543	.....	*n*-comps. strongly fringed 3 or more *p*-comps. Enhanced line
4291.114......	2	Quintuple	3, 3	0.221 (1)	0.445	1.200	2.417	
				0.000 (2)	.....	0.000	.....	
				0.220 (1)	.....	1.195	.....	
4291.375......	2	Triple	1	0.230	.....	1.249	.....	Difficult blend with 4291.114
4294.204......	10	Triple	3	0.361	.....	1.958	.....	Enhanced line
4295.914......	4	Unaffected	..	.....	.....	.....	.....	
4298.828......	4	Septuple	2, 3	Pair II, 0.292 (1)	0.060 (2)	1.580	0.325	*p*-comps. distinctly unsymmetrical
				Pair I, 0.145 (5)	0.000 (3)	0.784	0.000	
					0.086 (2)	.....	0.465	

4299.410......	3	Quadruple?	1	0.430	w_2	2.327	.....	
4299.803......	2	Triple?	1	0.356	w_1	1.925	.....	
4300.211......	8	?	2	0.367 w_1	w_2	1.985	.....	n-comps. fringed. 3 or more p-comps. Enhanced line
4300.732......	2	Septuple?	2	0.265 w_1	w_2	1.432	.....	n-comps. fringed, probably 3 p-comps.
4301.158......	3	Triple?	3	0.350	w_1	1.892	.....	
4302.085......	5	Sextuple	3, 3	Pair II, 0.585 (1)	0.216	3.161	1.167	Enhanced line
				Pair I, 0.151 (3)	.....	0.816	.....	
4306.078......	8	Septuple?	2	0.367 w_1	w_2	1.979	.....	n-comps. fringed, probably 3 p-comps.
4308.081......	8	Octuple	2, 2	Pair III, 0.588 (1)	0.236	3.168	1.272	Blend with iron impurity line probably gives apparent dissymmetry in intensity of n-comps. Enhanced line
				Pair II, 0.442 (2)	.....	2.382	.....	
				Pair I, 0.291 (3)	.....	1.568	.....	
4311.880......	1	Sextuple?	2	0.147	w_1	0.791	.....	Outer pair n-comps. not measurable. Possibly 3 p-comps. Blend with faint lines
4313.034......	8	Sextuple?	2, 2	0.449 w_2	0.159	2.414	0.855	Enhanced line
4314.964......	3	Triple	3	0.424	.....	2.277	.....	
4315.138......	5	Quadruple	3, 3	0.392	0.349	2.105	1.874	Enhanced line
4316.962......	2	Triple	3	0.207	.....	1.111	.....	Enhanced line
4318.817......	3	Triple	3	0.337	.....	1.807	.....	
4321.119......	3	Sextuple	3, 3	Pair II, 0.785 (2)	0.261	4.204	1.398	Enhanced line
				Pair I, 0.257 (1)	.....	1.376	.....	
4321.813......	3	Triple	3	0.310	.....	1.660	.....	
4323.531......	1	Triple	2	0.464	.....	2.482	.....	
4325.306......	3	Triple	3	0.301	.....	1.609	.....	
4326.520......	2	Triple	3	0.403	.....	2.152	.....	
4330.405......	3	Sextuple?	2	0.415 w_1	w_2	2.213	.....	Enhanced line
4330.866......	3	?	2	0.654 w_2	w_3	3.487	.....	Enhanced line. n-comps. have inner fringes
4338.084......	10	Triple	3	0.247	.....	1.312	.....	Enhanced line

TABLE II—*Continued*

λ	Intensity	Character of Separation	Weight	$\Delta\lambda$		$\Delta\lambda/\lambda^2$		Remarks
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
4468.663......	15	Triple	3	0.340	.....	1.702	.....	Enhanced line
4469.316......	1	Triple	2	0.426	.....	2.133	.....	
4471.017......	2	10 comps.?	2, 2	Pair III, 0.724 (1)	0.458	3.622	2.292	Trace of inner pair *p*-comps.
				Pair II, 0.386 (1)	.....	1.931	.....	
				Pair I, 0.126 (1)	.....	0.630	.....	
4471.408......	2	9 comps.	2, 2	Pair III, 0.826 (1)	0.113 (2)	4.132	0.565	
				Pair II, 0.613 (2)	0.000 (3)	3.066	0.000	
				Pair I, 0.364 (4)	0.116 (2)	1.821	0.580	
4475.026......	2	Septuple?	2	0.509 W_1	W_3	2.542	.....	Probably 4 *n*-, 3 *p*-comps.
4479.879......	2	Triple	3	0.829	.....	4.130	.....	
4480.752......	1	Triple	2	0.611	.....	3.043	.....	
4481.438......	3	Triple	3	0.548	.....	2.729	.....	
4482.904......	2	Sextuple?	2	0.498 W_1	W_2	2.478	.....	
4488.493......	6	Triple	3	0.355	.....	1.762	.....	Enhanced line
4489.262......	3	9 comps.	2, 2	Pair III, 0.858 (1)	0.111 (2)	4.258	0.551	Compare λ 4471.408
				Pair II, 0.597 (2)	0.000 (3)	2.963	0.000	
				Pair I, 0.382 (4)	0.110 (2)	1.896	0.546	
4495.182......	1	Triple	1	0.353	.....	1.747	.....	
4496.318......	3	Triple	3	0.493	.....	2.439	.....	
4497.842......	1	Triple	2	0.534	.....	2.639	.....	
4501.445......	15	Triple	3	0.298	.....	1.471	.....	Enhanced line
4512.906......	4	Triple	3	0.501	.....	2.460	.....	
4518.198......	4	Quadruple?	3	0.498	W_1	2.440	.....	
4518.866......	1	Triple	2	0.220	.....	1.077	.....	
4522.974......	4	Sextuple?	3	0.502 W_1	W_2	2.454	.....	
4527.490......	4	Octuple	3, 3	0.324 (7)	0.164 (1)	1.581	0.800	
				0.162 (2)	0.000 (2)	0.790	0.000	
				0.000 (1)	0.165 (1)	0.000	0.805	
				0.166 (4)	.....	0.810	.....	
				0.338 (7)	.....	1.649	.....	
4529.656......	2	Sextuple?	2, 2	0.358 W_2	0.278 W_1	1.745	1.355	Enhanced line

4533.419......	5	Triple	3	0.469	.....	2.282	.....	
4534.139......	6	Triple?	2	0.360 w_1	w_1	1.751	.....	*n*-comps. slightly fringed. Enhanced line
4534.953......	4	Triple	3	0.449	.....	2.183	.....	
4535.741......	3	Triple	3	0.424	.....	2.061	.....	
4536.094......	3	Triple	2	0.323	.....	1.570	.....	
4536.222......	3	Unaffected?	..	.....	.....	.....	.....	No resolution. Blend with 36.094 may conceal slight widening of *n*-comp.
4537.389......	1	Triple	1	0.355	.....	1.725	.....	
4544.190......	1	Quadruple?	1	0.308	w_2	1.492	.....	
4544.864......	3	Octuple	3, 3	0.334 (7)	0.171 (1)	1.617	0.828	Comps. in all respects similar to 4427.490
				0.168 (2)	0.000 (2)	0.813	0.000	
				0.000 (1)	0.166 (1)	0.000	0.804	
				0.170 (4)	.....	0.823	.....	
				0.332 (7)	.....	1.607	.....	
4548.938......	3	Septuple?	2	0.560 w_1	w_2	2.706	.....	*n* comps. have inner fringes. Probably 3 *p*-comps.
4549.808......	20	Triple	3	0.440	.....	2.125	.....	Enhanced line
5016.340......	7	Sextuple?	2, 3	0.543	0.214	2.158	0.851	
5020.208......	8	Octuple?	2, 3	0.507 w_3	0.276	2.012	1.095	Probably 3 pairs *n*-comps.
5023.052......	8	12 comps.?	2, 3	0.466 w_3	0.370 w_3	1.847	1.466	Probably 4 pairs *n*-comps. *p*-comps. have inner fringes.
5025.027......	7	10 comps.	2, 2	Pair III, 0.684 (1)	Pair II, 0.546 (6)	2.709	2.162	
				Pair II, 0.416 (2)	Pair I, 0.269 (1)	1.647	1.065	
				Pair I, 0.133 (4)	.....	0.527	.....	
5025.749......	5	Triple	3	0.471	.....	1.865	.....	
5036.089......	10	Triple	3	0.455	.....	1.794	.....	
5036.645......	8	Triple	3	0.436	.....	1.718	.....	
5038.579......	8	Triple	3	0.340	.....	1.339	.....	
5040.138......	8	Triple	3	0.404	.....	1.590	.....	
5053.056......	3	Triple?	2	0.449	w_1	1.759	.....	

TABLE II—*Continued*

λ	Intensity	Character of Separation	Weight	Δλ		Δλ/λ²		Remarks
				n-Components	*p*-Components	*n*-Comp.	*p*-Comp.	
5062.285	3	Triple?	2	0.412	w_1	1.608	.....	
5064.244	1	Triple	..	n.m.	.....	.....	.....	Very faint
5064.836	8	Triple	3	0.463	.....	1.805	.....	
5066.174	1	Sextuple?	–,1	n.m.w_3	0.407	.....	1.586	
5069.592	1	Triple	2	0.235	.....	0.914	.....	
5071.666	4	Sextuple?	1, 1	0.470 w_2	0.275	1.827	1.069	
5072.479	6	Triple	3	0.502	.....	1.951	.....	Enhanced line
5087.239	4	Triple	3	0.329	.....	1.271	.....	
5113.617	5	Triple	3	0.431	.....	1.648	.....	
5120.592	7	Triple	3	0.434	.....	1.655	.....	
5129.336	8	Triple	3	0.478	.....	1.812	.....	Enhanced line
5145.636	6	Triple	3	0.493	.....	1.862	.....	
5147.652	5	Septuple?	2	0.805 w_2	w_3	3.038	.....	Probably more than 7 comps. *p*-comps. almost resolved
5152.361	5	Quadruple?	3	0.671 w_1	w_1	2.528	.....	
5154.244	4	Sextuple?	2	0.666 w_2	w_2	2.507	.....	Enhanced line
5173.917	10	Triple	2	0.292	.....	1.091	.....	
5186.073	8	Triple	3	0.385	.....	1.432	.....	Enhanced line
5188.863	12	Triple	3	0.512	.....	1.902	.....	Enhanced line
5193.139	10	Triple	3	0.468	.....	1.735	.....	
5201.260	3	Triple?	2	0.648 w_1	w_1	1.396	.....	
5206.215	4	Triple	2	0.470	.....	1.734	.....	
5210.555	10	Triple	3	0.547	.....	2.014	.....	
5210.875	4	Sextuple?	2	0.326 (2)	w_2	1.191	.....	Unsymmetrical. Probably 4 *n*-comps., 2 violet blended, 2 red separated. *p*-comp. displaced 0.062 to violet from normal. All comps. measured from normal
				normal	.....	.....	.....	
				0.264 (1)	.....	0.969	.....	
				0.400 (1)	.....	1.468	.....	
5222.849	3	Unaffected	..	.....	.....	.....	.....	

5866.675......	10	Triple	3	0.674	.....	1.958	.....	
5880.490......	3	Triple	3	0.830	.....	2.401	.....	
5899.518......	7	Triple	3	0.656	.....	1.885	.....	
5903.555......	2	Triple	2	0.876	.....	2.513	.....	Red comp. strongest?
5918.773......	3	Triple	3	0.882	.....	2.518	.....	
5922.334......	5	Triple	2	0.312	.....	0.890	.....	
5938.035......	2	Sextuple?	2, 1	0.840 w_2	0.226	2.382	0.641	p-comp. scarcely resolved
5941.985......	5	Sextuple	2, 2	Pair II, 0.905 (2)	0.547	2.563	1.549	
				Pair I, 0.295 (3)	.....	0.836	.....	
5953.386......	8	Triple	3	0.637	.....	1.798	.....	
5966.055......	7	Triple	3	0.590	.....	1.617	.....	
5978.768......	7	Triple	3	0.533	.....	1.491	.....	
5999.920......	2	Quadruple?	2	0.823 w_1	w_2	2.286	.....	
6064.853......	2	Triple	3	1.159	.....	3.151	.....	
6085.470......	4	Sextuple?	2, 2	1.001 w_2	0.307	2.703	0.829	
6091.395......	6	Triple	3	0.679	.....	1.830	.....	
6093.030......	1	Triple	2	0.755	.....	2.034	.....	
6098.870......	2	Triple?	2	0.563 w_1	w_1	1.514	.....	
6121.215......	1	Triple	2	0.622	.....	1.660	.....	
6126.435......	5	Septuple?	2	0.706 w_1	w_3	1.881	.....	n-comps. fringed. Probably 3 p-comps.
6146.445......	1	Triple	2	0.476	.....	1.260	.....	
6149.950......	1	Triple	2	0.698	.....	1.845	.....	
6215.630......	3	Triple	3	0.697	.....	1.804	.....	
6220.700......	2	Triple	2	0.618	.....	1.597	.....	
6221.552......	1	Triple	2	0.511	.....	1.320	.....	
6258.322......	10	Triple	2	c.661	.....	1.688	.....	Red n-comps. of first line blended with violet comp. of second
6258.927......	12	Triple	2	0.714	.....	1.823	.....	
6261.316......	10	Triple	3	0.556	.....	1.418	.....	
6303.985......	5	Octuple?	2, 2	0.565 w_3	0.554	1.422	1.394	n-comps. very wide and diffuse. Probably at least 3 pairs
6312.456......	5	Octuple?	2, 2	0.766 w_3	0.463 w_1	1.923	1.162	n-comps. wide, not so diffuse as 6304. Probably 3 pairs. Possibly 4 p-comps.

EXPLANATION OF THE PLATES

Enlargements of selected portions of the iron and titanium spectra are reproduced in Plates X–XIII. In Plate X, the spectra of the spark without magnetic field is placed above and below, while the three magnetic-field spectra are those given when the light is observed across the lines of force with a Nicol in two positions 90° apart, and without a Nicol, the first two giving the *p*- and *n*-components respectively, while the last gives the two superposed.

For the other plates, the same plan is followed, but only the spectra showing the *p*- and *n*-components are presented. The four spectra in Plate XII show two portions of the titanium spectrum, the range of intensity among the lines being so great that one photograph can do justice only to a certain set.

The number of lines for each type of separation, including both the clear and the questioned cases as given in the complete tables, is shown in Table III.

The publication discusses in detail the characteristics of each type.

RELATION OF SEPARATIONS TO THE NORMAL INTERVAL

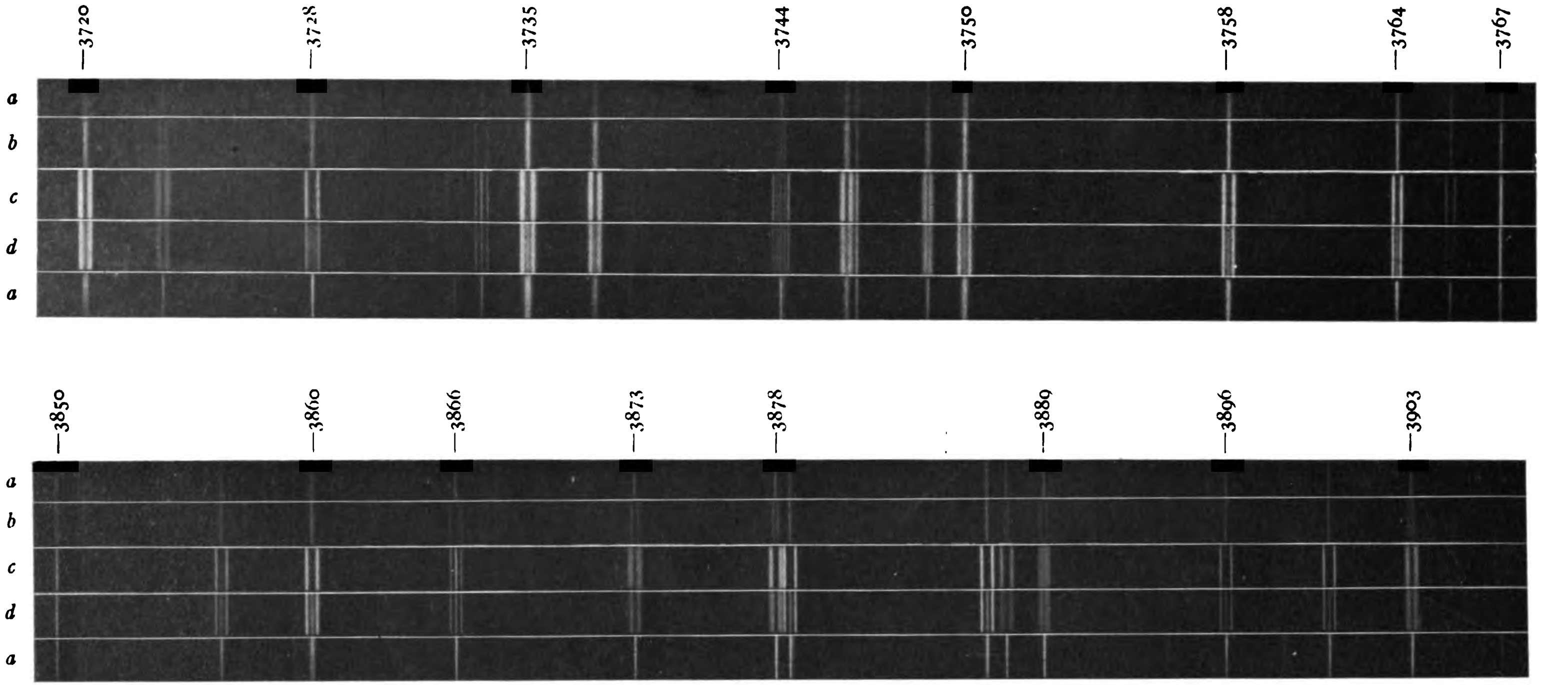
The study of how generally the separations observed show a simple relation to a fundamental interval has been gone into with some detail. The relation

$$a=\frac{\Delta\lambda}{\lambda^2}=\frac{e}{m}\cdot\frac{H}{4\pi v},$$

where e/m is the ratio of charge to mass of the electron, H the field-strength, and v the velocity of light, gives values for a of 0.753 and 0.812 for the field-strengths corresponding to the measurements for iron and titanium, respectively. The "normal triplet" should then show values of $\Delta\lambda/\lambda^2$ for the distance between the side components of about 1.500 and 1.600, respectively.

Various types of separation are considered in turn with regard to how nearly the measured separations can be expressed in terms of the normal interval a. In the iron spectrum, 272 clear triplets were measured, i.e., triplets whose components show no widening which would indicate that the line is of higher type. Of these, 230

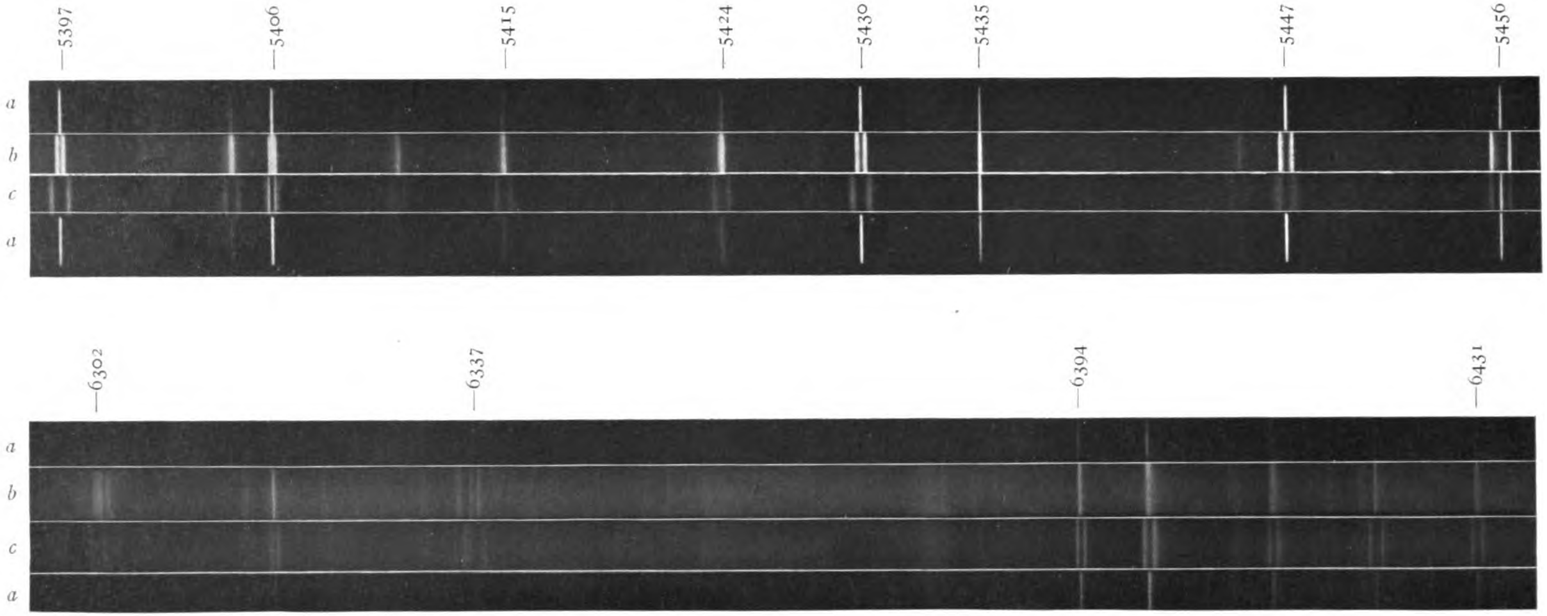
PLATE X



Effect of Magnetic Field on the Spectrum of Iron

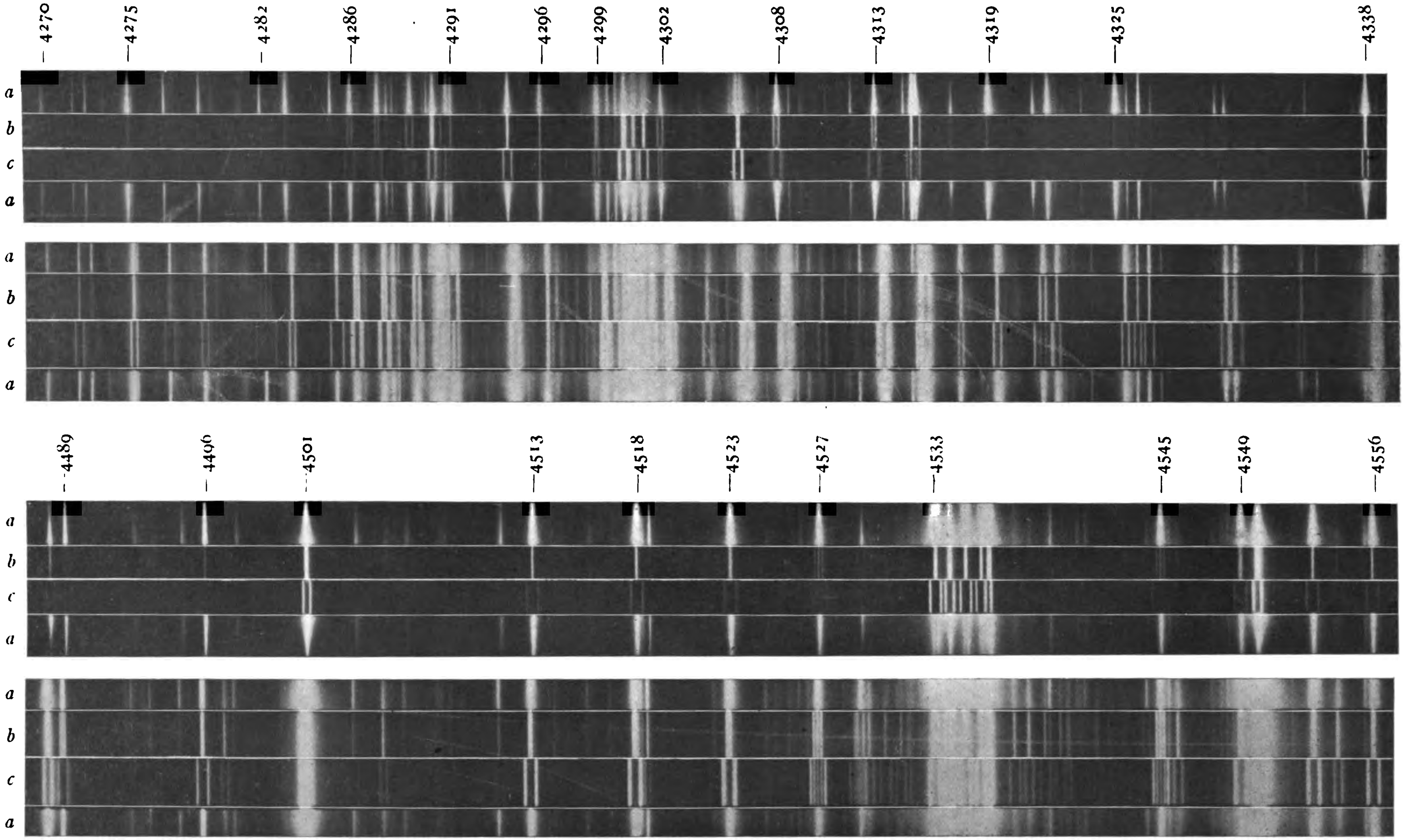
a. Spark without field. *b*. *p*-Components. *c*. *n*-Components. *d*. Spectrum without Nicol, *n*- and *p*-Components superposed

PLATE XI



Effect of Magnetic Field on the Spectrum of Iron

a. Spark without field. *b*. *p*-Components. *c*. *n*-Components

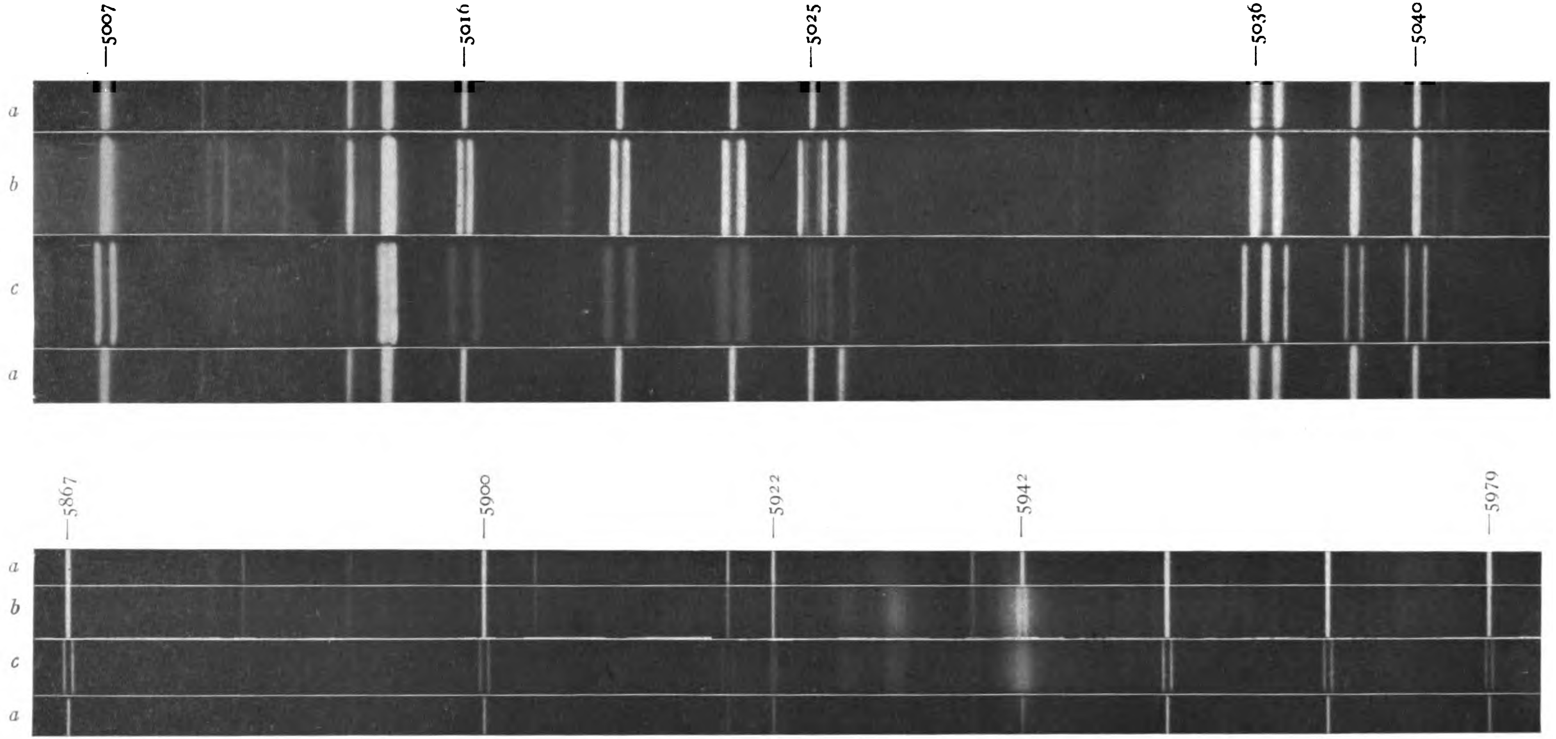


Effect of Magnetic Field on the Spectrum of Titanium

Experimental conditions varied to bring out both strong and weak lines

a. Spark without field. *b*. *p*-Components. *c*. *n*-Components

PLATE XIII



Effect of Magnetic Field on the Spectrum of Titanium

a. Spark without field. *b*. *p*-Components. *c*. *n*-Components

gave separations which could be expressed readily as multiples of $a/2$, 54 showing a value of $\Delta\lambda/\lambda^2$ close to $2a$, and 72 close to $3a$. The titanium table contains 242 clear triplets, 145 of which gave multiples of $a/2$, 73 of these being normal triplets. Many of the remaining triplets allow their separations to be expressed as even multiples of $a/4$, but, with the fields employed, this brings the unit close to the possible error of measurement, except for lines of high weight.

TABLE III

SUMMARY OF TYPES OF SEPARATION

Separation	Iron	Titanium
Unaffected	9	4
Triple	393	291
Quadruple	49	28
Quintuple	7	5
Sextuple	118	77
Septuple	37	12
Octuple	6	11
9 components	9	3
10 components	7	7
11 components	2	2
12 components	4	2
13 components	2	0
Unclassified	19	16
Total	662	458

The lines appearing as quadruplets (in many cases really sextuple owing to two pairs of *n*-components being blended) in the majority of cases allow their separations to be expressed as multiples of a or $a/2$, the most common ratios of *n*- and *p*-separations being $2a:a$ and $3a:a$.

The lines of more complex type which are fully resolved, ranging from quintuplets to 12-component lines, are tabulated in a form to show how the separations may be expressed in terms of the normal interval and also to give the ratio, usually very simple, between the successive separations. Fifty-seven complex lines are thus treated, and in almost every case the ease with which the separations can be expressed in terms of a leaves no doubt as to the generality of a relation of this sort. As an example, the 12-component lines λλ 3722.729, 3872.639, and 5447.130 of iron, and 4289.237 of titanium, some of which are given in Tables I and II in this abstract, present in each case the following arrangement of the six pairs of components:

$\pm 2a$	n	4
$3a/2$	n	3
a	n, p	2
$a/2$	n, p	1

The ratio separations for successive pairs is thus 1:2:3:4. Not all lines show so simple a ratio, and probably a much stronger field, permitting the use of small fractional parts of a, would cause some revision of the relations here given for a few lines. The results in general, however, are fully in accordance with the rule given by Runge[1] that for complex lines the distances of the various components from the center may be expressed as multiples of aliquot parts of a.

This treatment of complex separations has brought to notice many cases of "magnetic duplicates," lines showing exactly the same structure and with the same intervals between components. The knowledge of the structure of these lines may be of service in the search for series relations, but as yet nothing definite in this direction has been developed.

DISSYMMETRY

The number of lines showing clear dissymmetry in spacing or intensity of components is not large, fourteen such lines being listed for the two spectra.

Measurements were made for a number of sharp iron triplets to test the dissymmetry predicted by Voigt and investigated by Zeeman and others, according to which the violet component of a triplet should have a slightly greater spacing from the central component, while the red component should be slightly stronger than the violet. Of 26 lines favorable for this purpose, all but 3 show a larger spacing for the violet component, while most of the lines show a difference considerably greater than the probable error of measurement. Numerous triplets also appear for which the red component is very slightly stronger than the violet. A detailed study must wait until a much greater field-strength is available, but this preliminary examination indicates that the effect is real for many triplets.

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[1] *Physikalische Zeitschrift*, **8**, 232, 1907.

LAW OF CHANGE OF THE AVERAGE SEPARATION OF THE *n*-COMPONENTS WITH THE WAVE-LENGTH

The extensive range of wave-length studied for iron and titanium, together with the large number of lines measured, has furnished material for a statistical study of the change of the average separation with the wave-length. The method of treatment has been to obtain the mean value of $\Delta\lambda/\lambda^2$ for the *n*-components of lines for each 500 Å from λ 3700 to λ 6700. When there are two or more pairs of *n*-components the mean of the separations is taken. The averages thus obtained are presented in Table IV. The means for the six groups of 500 Å are given first, then the means for the three groups of 1000 Å. These latter are the means for the whole number of lines included in the range, not the average of the means for the 500-groups.

TABLE IV

MEANS OF $\Delta\lambda/\lambda^2$ (*n*-COMPONENTS) FOR SUCCESSIVE REGIONS OF WAVE-LENGTH

RANGE OF λ	IRON		TITANIUM	
	No. Lines	Mean $\Delta\lambda/\lambda^2$	No. Lines	Mean $\Delta\lambda/\lambda^2$
3700–4200....	267	2.003	80	1.909
4200–4700....	101	2.051	152	2.027
4700–5200....	74	2.125	81	1.684
5200–5700....	62	1.932	47	1.819
5700–6200....	37	1.837	34	1.942
6200–6700....	41	2.131	28	1.764
3700–4700....	368	2.016	232	1.986
4700–5700....	136	2.037	128	1.734
5700–6700....	78	1.989	62	1.862

The close agreement of the means shows that there is an approximate constancy of the values of $\Delta\lambda/\lambda^2$ for different parts of the spectrum. Taking the successive means of the 500-groups, the average value for iron is 2.013, for titanium 1.858. The largest deviation from the mean for any group is 8.7 per cent for iron and 9.4 per cent for titanium. For neither element is there any systematic change of the means for successive groups.

The means for the groups of 1000 Å show a still closer agreement, the largest deviation from the mean of these groups being only 1.2 per cent for iron and 6.8 per cent for titanium.

The mean values for titanium run smaller than those for iron, although the titanium measurements correspond to the greater field-strength. When a number of spectra are examined in this way, it may prove that this difference is connected with certain properties of the elements concerned.

This constancy of the mean values of $\Delta\lambda/\lambda^2$ appears to be based on the general relation of the separations to the fundamental interval a, and to result from the fact that the great majority of the separations for the n-components range from the values of $2a$ to $3a$. Among triplet lines, the greater number show a separation larger than $2a$. The exceptionally large and small values for triplets, together with the mean separations of the complex lines, combine to form a fairly definite mean which holds for the whole range of spectrum examined.

Since $\Delta\lambda/\lambda^2$ is thus shown to be nearly constant, it may be said that *for the spectra of iron and titanium and probably for spectra in general, the mean separation of the n-components varies as the square of the wave-length.* A similar rule must hold for the p-components, since complex lines of the same structure in different parts of the spectrum show the same relation to the interval a.

A result obtained by a different method, but doubtless based on the same general relation, was given by Mr. Hale[1] in a comparison of sun-spot doublets with Zeeman separations on laboratory plates. He observed that the mean $\Delta\lambda$ divided by the square of the mean λ agreed exactly for two regions of the iron spectrum.

THE EFFECT OF THE MAGNETIC FIELD UPON ENHANCED LINES

In my former paper on the titanium spectrum, as well as in the recent papers of Mr. Babcock[2] on the Zeeman effect for chromium and vanadium, the conclusion was reached that the enhanced lines as a class are not affected differently from the non-enhanced lines of these elements. The larger amount of material in the present publication has verified this conclusion in general, but it also presents evidence that the stronger enhanced lines of titanium show a tendency toward one type. The question is of interest because the enhanced lines form a fairly well defined class, requiring certain physical conditions of the light-source for their production, and a modification of these conditions affects the intensity and appearance of all such lines in the same way though in different degrees.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 30; *Astrophysical Journal*, **28**, 315, 1908.

[2] *Contributions from the Mount Wilson Solar Observatory*, Nos. 52 and 55; *Astrophysical Journal*, **33**, 217, 1911.

Among the lines measured for magnetic separation in this publication, the ratio of enhanced to non-enhanced lines is about 1:14 for iron and 1:5 for titanium. There appears to be no undue proportion of any one type of separation among the enhanced lines considered as a whole. None of the lines unaffected by the magnetic field are enhanced, and only one quintuplet occurs in this class. The more common types are all well represented among them. However, one noteworthy peculiarity is shown by the enhanced lines of titanium in the region from λ 3600 to λ 4600, rich in this kind of lines. Twenty-two lines occur here which show a high degree of enhancement, and are, as a rule, much stronger in the spark than any of the lines characteristic of the arc. Seventeen of these lines are clear triplets, while in the whole titanium spectrum measured, only a little over half of the lines are clear triplets. The remaining five, which are with one exception the weakest in the list, are of more complex character. As regards magnitude of separation, none of these 17 lines are normal triplets, and they appear to be less closely related to the interval *a* than a like number of triplets taken at random. These data are all presented in tabular form in the complete publication.

COMPARISON OF THE RESULTS FOR THE ZEEMAN EFFECT AND FOR PRESSURE DISPLACEMENT

This subject was treated for the spectra of iron, titanium, and chromium in a former paper.[1] The material for the first two elements is now presented with revised measurements and much more extensive data for titanium, for which I am indebted to Mr. Gale, photographs having been made by him in this laboratory for the titanium arc under pressure. This extension gives 173 iron and 122 titanium lines available for a comparison of magnetic separation and pressure displacement.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 46; *Astrophysical Journal*, 31, 433, 1910.

The method of treatment of the material in tables is much the same as before, and similar conclusions are reached. The ratios of separation to displacement for different lines show no approach to constancy. In many cases the magnitudes of the two effects are very different, the extreme cases being lines unaffected by the magnetic field which show large pressure displacement. Taking the means of separation and displacement for large numbers of lines in the same region of the spectrum, however, it is found that these means are in general of the same order of magnitude when they are classified broadly as small, medium, and large. This shows that the lines as a rule show some degree of similarity in their response to the two displacing influences. Both effects are observed to increase with the wave-length, although it is not yet established that the rate of increase for the pressure-effect is the same as that for the Zeeman phenomenon, i.e., as the square of the wave-length.

Mount Wilson Solar Observatory
April 10, 1911

BODE'S LAW AND SPIRAL STRUCTURE IN NEBULAE

By WILLIAM SUTHERLAND

By the law of Titius, known as Bode's, with the mean radius of the earth's orbit as unit, the mean orbital radius of the planet which is nth from *Mercury* is

$$r=0.4+0.15\times 2^n. \tag{1}$$

For *Ceres*, as representative of the asteroids, $n=4$. As far as *Uranus*, for which $n=7$, the maximum difference between the observed values of r and those calculated by (1) is 5 per cent. In the case of *Neptune*, with $n=8$, the observed value of r is 30, while that calculated is 38.8, the difference rising to 29 per cent. We can write (1) in the form

$$r=r_1+r_2, \tag{2}$$

$$r_1=a=0.4, \tag{3}$$

$$r_2=be^{\theta/a}=0.15\times 2^n, \tag{4}$$

thus expressing that r can be obtained by adding to the radius vector of a circle of radius a the radius vector at angle $na\log_e 2$ of the logarithmic (equiangular) spiral (4). If we note that the circle is a limiting form of a spiral when a is infinite and θ is finite, we get the interesting mathematical result that the planetary distances are derived from two equiangular spirals, in one of which the constant angle between radius vector and tangent is $\pi/2$, and in the other the tangent of that angle is a. The law of planetary distances introduces us to a type of spiral simply related to the simple logarithmic spiral. So we are led to the physical hypothesis that the solar system in its evolution passed through the stage of a spiral nebula. Let us take the simplest case when $r_1=0$ and we consider the nebula whose spiral is (4).

The mean radius between $\theta=m\beta$ and $(m+1)\beta$ is

$$\bar{r}=\frac{1}{\beta}\int_{m\beta}^{(m+1)\beta} be^{\theta/a}d\theta=be^{m\beta/a}(e^{\beta/a}-1)a/\beta. \tag{5}$$

Thus in the planetary distances we can regard r_2 as derived from the breaking up of a logarithmic spiral nebula into equal angular parts, that is, into parts which originally subtended the constant angle β at the origin. Equating coefficients in the second term of the right-hand side of (1) and (5) we get, when $m=n$, $e^{m\beta/a}=2^n$, $b(e^{\beta/a}-1)a/\beta=0.15$ $\therefore$ $\beta/a=\log_e 2=0.693$, and $b=0.15\times 0.693=0.104$. The breaking up of the original nebula into equal angular portions is suggestive of the existence of stationary vibrations in it. Let us consider the origin of these. Take to begin with a swarm of meteorites which has reached this sort of steady state, that it is no longer expanding. Gravitation is just sufficient to make the swarm cohere at its maximum size. It has lost all those meteorites which can escape from it. Gravitational collapse has just begun. If the velocities of the meteorites relative to the center of mass of the swarm are uniformly distributed in direction, the swarm will move on with uniform velocity of translation, without any prevailing rotation, and collapsing uniformly. These uniform conditions may not prevail throughout the swarm. Local condensations may be taking place, but on the whole there is no prevailing structure except that due to the inward motion of the parts collapsing. This is the simplest type of nebula.

But when the gravitationally cohering swarm has a prevailing rotation as well as translation, then that rotation prevents collapse in directions at right angles to its axis. In this case collapse goes on parallel to the axis and the swarm takes the form of a disk or lamina. We may start then with a spherical swarm as type of a uniform nebula and pass to the swarm typified by an oblate spheroid as representative of one in which a prevailing rotation destroys the symmetry with respect to the center and replaces it with symmetry in regard to an axis of rotation. When the oblateness of the spheroid is great, we reach the type of the disk nebula, which includes the great majority of the spiral nebulae. Now to the generalized swarm we can apply the principles of the kinetic theory of perfect gases. But when gravitational cohesion is taken into account, we pass to a case analogous to the kinetic theory of imperfect gases, in which cohesional force is producing measurable effects recognized in such an equation as that of Van der Waals. In this way we reach the conclusion that in a swarm of meteorites gravitational cohesion can produce effects like those of liquidity and solidity in the general kinetic theory of matter. In a swarm of meteorites whose elements range from separate molecule to particle of dust and from particle of dust to star we have a large-scale rigidity and a large-scale bulk modulus (reciprocal of compressibility) connected with the gravitational energy and the kinetic energy of the elements of the swarm. In "The Electric Origin of Rigidity and Consequences" (*Phil. Mag.* [6], 7, 417, 1904) it is shown that at the absolute zero of temperature the rigidity of a solid is equal to the electrostatic energy per unit volume of its molecular electrization. At higher temperatures molecular kinetic energy plays a part. It seems to me then that in cosmical physics we must freely supplement the theory of three bodies with a theory of cosmic elasticity, with the aid of which to study the changes observed in nebulae regarded as solid and liquid bodies of which meteorites are the molecules. Stationary vibrations in a disk nebula are a dynamical necessity of this theory. The simplest type is that in which the disk vibrates in n equal segments with the radii as nodes. As to the causes which give n a definite numerical value I refrain from speculating at present. The separation into n vibrating segments determines the ultimate condensation into n planets in a manner which observation may elucidate.

Let us now trace the probable course of the development of spiral structure in a disk nebula. The most probable motion of a meteorite of mass m at distance r from the center is that in a circle with velocity v such that mv^2/r is equal to the gravitational attraction of the whole swarm toward the center. Near a circle of radius r we shall have two circular streams traveling in opposite directions with this velocity v. One stream is more massive than the other, whence the prevailing rotation of the whole disk, though we see that a disk could exist without a prevailing rotation, if it had equal opposite circular streams that failed to destroy one another by collisions. Where one stream is more massive than the oppositely directed one, we may regard it as containing more meteorites of the average size. Suppose two average meteorites moving in opposite directions collide so as to form a mass of velocity 0. This at once proceeds to fall toward the center, describing a radius. We may designate this radial path as a spiral of zero spirality, and the circular path as one of infinite spirality. We see that the radially falling body will get circular motion imparted to it by collision, and thus we arrive at an average path between circular and radial, namely a spiral of finite spirality. The meteorites of the collapsing swarm move in toward the center on spiral paths. If these are uniformly distributed round the center, they may give little or no evidence of spirality, they appear as simple disk nebulae of unknown spirality which may have any value between zero and infinity, a ring nebula perhaps making its infinite spirality known.

We have now to consider the remarkable fact that so many of the typical spiral nebulae consist of two spirals. Theoretically we can see how there is a tendency for tracks of greater meteoritic density to form. In the first place, gravitation helps any denser region to become more dense. In the second, collisions are more frequent in the denser region, making its spirality smaller, so that it cuts the less dense tracks of larger spirality, robbing them of meteorites by reason of the more frequent collisions and the stronger gravitation. In these ways the dense region generally tends to become more dense. This is only a particular case of the universal tendency under gravitational attraction. We see that there is a process by which the spiral tracks coalesce into fewer ones of higher meteoritic concentration. But we should expect the number of spirals to be determined by the initial departures from uniformity of the distribution of the meteorites in the swarm. It is probable that, as the central body grows by the inflow of meteorites along the spirals, it creates dynamical conditions favorable to the reduction of the spirals to two main ones symmetrically placed with regard to itself. In some photographs of spiral nebulae there are signs of a spiral breaking into branches, which are probably remnants of spirals coalescing into one. Two main spirals represent the simplest symmetrical arrangement by which momentum in the spiral paths can be transmitted to the rotating central body. No doubt it is a very complex problem to find the definite dynamical reason for the existence of the spirals so frequently as symmetrical pairs.

So far we have reached the conception of a disk nebula rotating as a whole round its center, while there is a streaming motion of its constituent parts inward toward the center along spiral paths which generally reduce to two. With a scale large enough in space and time we may regard it as a solid body rotating, undergoing deformation, and flowing like a liquid along the spirals, just as ordinary solids flow under suitable stresses, while at the same time the whole elastic structure is vibrating. In studying the photographic records of nebulae it will be necessary to bear in mind these general modes of motion. An important dynamical point is this: Does the disk nebula, while it concentrates into two spirals, still vibrate as a disk divided into equal segments by radiating nodal lines? We are led to suppose that it does, to account for the derivation of each planet being from an arc of the spiral subtending a constant angle at the center. So the evolution of a planet must now be discussed. The vibrational movement of each internodal segment of a spiral will be damped down by internal friction, that is to say, by collisions among meteorites. So frictional or viscous damping on the cosmic scale means coalescence of meteorites. Collisions aided by gravitation favor aggregation in each vibrating segment. But as the meteorites gather into larger and larger masses, into incipient planets, they move in a region where the meteorites are less dense. The planetary nucleus tends to move independently of its segment of the spiral, to take to a path of its own of larger spirality, cutting periodically the region near the middle of its segment of the old spiral and growing at its expense, while describing an orbit that becomes more and more nearly a circle round the center. Meanwhile other meteorites have continued to stream along the old spiral tracks into the center. Sun and planet grow simultaneously, the sun being indeed only the large central planet. If the solar system consisted at one stage of two main spirals, then the corresponding internodal segments of each must have built up together a single planet. According to this scheme of evolution it is probable that the moon and the other satellites originated as small subsidiary or branch spirals such as we see in the photographs of spiral nebulae.

A very remarkable fact about the planets is that *Mercury*, with the smallest mass, namely 0.074 of the earth's, comes next to the sun, which has the vast central mass, while *Mars*, with the next smallest, namely 0.134, comes almost next to *Jupiter*, the largest, of mass 343, the asteroids of very small mass coming between. The special enrichment of the sun and *Jupiter* and *Saturn* meant the impoverishment of the planets near them, *Mercury*, *Mars*, the asteroids, *Uranus* and *Neptune*. At first sight it appears as if these large contrasts in the masses of neighbor planets make it very improbable that the planets originate in internodal segments. One might naturally expect nodal segmentation to give rise to planets of nearly equal masses. But further consideration presents the matter in a new light. Let E be a modulus of cosmic elasticity of the matter of a nebula, ρ its density cosmically measured, then the velocity of propagation of a disturbance through the nebula is $(E/\rho)^{\frac{1}{2}}$. If E is proportional to ρ, this velocity is independent of the density in any part of the nebula, so that the disk nebula is divided up by its nodes into segments without reference to the distribution of density in the nebula. A similar case occurs with the velocity of sound in any given gas, which is independent of its pressure. If the earth's atmosphere were at a uniform temperature, its nodes of vibration would be distributed irrespective of the variation of the density of the air with altitude. When E is proportional to ρ, we may infer that the energy giving rise to the modulus E at any point in the nebula is due to the mutual gravitational energy of unit volume of the nebula at that point and the rest of the nebula, which at the stage we are considering will be nearly the mass of the central sun. In that case the gravitational energy of unit volume will be proportional to its density, and also inversely proportional to the distance of the point from the center of the sun. Parts of the nebula at the same distance from the center have E proportional to ρ. The velocity of propagation of a disturbance round a circle of radius r is constant, however the density varies along the circle. It follows then that radial nodal lines will be distributed at equal angular intervals without reference to the distribution of density in the nebula after most of its matter has gathered at the center. The contrast in the masses of neighbor planets causes no difficulty in the theory that the planets originate through the cosmical music of the nebula, "the harmony of the spheres." In the speculations of the Greeks this harmony was a harmonical spacing. To account for the contrasts a marked want of uniformity in the distribution of the meteorites in the swarm must be postulated, gravitation accentuating this by increasing the store of the denser regions at the expense of the less dense.

At the present time, thanks to recent advances in astronomical photography, we can investigate the spirality of typical spiral nebulae in a fairly definite manner, though concerning their linear dimensions we have little accurate knowledge. We recognize that the size of a nebula has a vast range if we proceed from the rings of *Saturn* to the great nebula in *Andromeda* having an angular length of 2°5. But just as the broad elastic laws of a grain of sand are the same as those of a mountain of quartz, and those of a speck of jelly the same within the right limits as those of an ingot of steel, the laws of cosmic elasticity must produce similar results in *Saturn's* rings to those in the largest nebulae. So no difficulty arises in comparing the primitive solar nebula with the vaster ones of the heavens except the practical one of measuring the linear dimensions of these remote bodies. But in the measurement of the spirality of nebulae we can obtain at once satisfactory results from the photographs. The chief uncertainty in these arises from the necessity of assuming that the lines of luminosity in a nebula show the lines of greater density. With this assumption we can obtain the spirality of typical nebulae from their photographs. As the most favorable example we may take Ritchey's photograph of the spiral nebula *Messier 51 Canum Venaticorum*, reproduced in *Nature*, **85**, 142, from this Journal, Plate IV, **32**, 1910. The following are the values of the radius vector in mm for values of θ increasing by $\pi/2$.

θ	0	$\pi/2$	$2\pi/2$	$3\pi/2$	2π
r in one spiral	7	12	18	25	
r in the other	7	11	19	28	60
r mean	7	11.5	18.5	26.5	60
r calculated	6.2	10.7	18.6	32.4	56

The calculated values of r are obtained from the equation

$$\log_{10} r = 0.79 + 0.24(2\theta/\pi) \text{ or } \log_e r = 1.819 + \theta/2.84, \quad (6)$$

constructed as the best logarithmic spiral to represent the relation between the mean r and θ. The data are hardly definite enough to warrant the attempt to treat r as r_1+r_2. The spirals of this nebula can be fairly well represented by logarithmic spirals of spirality $a=2.84$ radians. Two other photographs suitable for similar measurements are those of Nebula *M 100 Comae* by Roberts and of Nebula *M 101 Ursae Majoris* by Ritchey, reproduced in Lowell's *Evolution of Worlds*, the results being subjoined.

	Nebula *M 100 Comae*				Nebula *M 101 Ursae Majoris*			
θ	0	$\pi/2$	$2\pi/2$	$3\pi/2$	0	$\pi/2$	$2\pi/2$	$3\pi/2$
r in spiral 1	4	5	8	11	4	5	6	9
r in spiral 2	..	5	9	..	4	5.5	8	..
r mean	4	5	8.5	11	4	5.2	7	9
r calculated	4	5.6	8	11.2	4	5.2	6.9	9.1

The equations used in calculating r to represent the mean r were

$$\log_e r = 1.38+\theta/4.55 \text{ and } 1.38+\theta/5.69. \tag{7}$$

Here again we find that the spirals can be regarded as approximately logarithmic. The spiralities of these three nebulae, measured by a in radians, are 2.84, 4.55, and 5.69, that for the ring nebulae being ∞, with 0 for the case of a uniformly collapsing spherical nebula. For the case of the primitive solar nebula we did not obtain a, the spirality, but only $\beta/a=0.693$, β being the unknown angle subtended at the center by the segment of the spiral from which each planet was derived. As the visible range of θ in the three nebulae considered above does not exceed 2π, and as there are 8 planets, we may take the order of magnitude of β to be given by $2\pi/8$. With this value for β in the solar nebula a becomes 1.13, and the spirality of the solar nebula ranges itself naturally with those for the three nebula taken as typical.

It is interesting to recall the fact that the logarithmic spiral is the orbit of a particle moving under an attraction varying inversely as the cube of the distance from the center. It appears then that, after the greater part of a nebula has gathered into a central star, we may regard the larger part of the gravitation as producing cohesion of the whole and along with the general rotation providing the general rigidity, while at the same time part of the attraction of gravity is so modified by collisional viscosity as to operate like an unresisted attraction varying inversely as the cube of the distance from the center.

A rather important point in observational astronomy raised by the present theory is the desirability of searching systematically for uncondensed vestiges of the original solar nebula not only near the sun but out as far as *Neptune*. Does the earth at intervals of nearly a year pass through the remnant of the segment of the spiral or each of the two spirals from which it was condensed? Two crucial observations needed to verify the present theory are that of inward motion along the spirals of all spiral nebulae and the demonstration of a process of regular segmentation of the spirals in some of them.

MELBOURNE, AUSTRALIA
April 1911

POSTSCRIPT added July 27.—The trend of previous thought concerning a connection between the structure of the solar system and that of spiral nebulae may be indicated by two references: first, to William Thomson's summary of three papers of 1854 in Vol. II, art. lxvi, of his *Mathematical and Physical Papers*, where, in discussing the origin of the sun's heat from the falling of matter into it, he states that a metoer would approach the sun by a very gradual spiral; and, second, to F. R. Moulton's paper in this JOURNAL, **22**, 165–181, 1905, on Chamberlin and Moulton's theory of the development of the solar system from a spiral nebula.

SECOND POSTSCRIPT.—August 3: Having seen today an article in *Astronomische Nachrichten*, No. 4503 (**188**, 250, 1911), by E. v. d. Pahlen entitled "Ueber die Gestalten einiger Spiralnebeln," I can add the following information. This author finds that selected nebulae have their shapes definite enough to be taken as logarithmic spirals. He assumes that the selected spirals lie each in a plane, and considers the case where this plane is not at right angles to the line of sight. He takes the spirals of *M 51 Canum Venaticorum* to lie in a plane making an angle of 42° with the plane at right angles to the line of sight, and finds spiralities which are 4.28 and 4.24 when expressed in radians. These are markedly larger than the mean value 2.84 which I have given above on the assumption that the plane is at right angles to the line of sight. He finds that *M 33 Trianguli* and *M 74 Piscium* are at right angles to the line of sight, and have spiralities 1.16, 1.25, and 4.08, 3.53. The values 1.16 and 1.25 for the two spirals of *M 33 Trianguli* are close to 1.13 estimated above for the solar nebula. He finds the spirals of other nebulae to be logarithmic, but does not give the coefficients from which their spiralities can be calculated.

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VARIOUS SCALES FOR COLOR-ESTIMATES

By J. G. HAGEN, S.J.

The variety of symbols and scales now in use for expressing the colors of stars, and the desirability of reaching more uniformity, are too apparent to need any comment. This paper is intended to make a step forward in that direction. It begins with a historical review of the work done in this branch; then it defines the elements of color and their symbols; the scales of one and two dimensions and their co-ordination to each other, are then explained; and finally some proposals are submitted for the future.

The present exposition of the matter was occasioned by the re-observation of the star-colors contained in Sestini's Catalogue, with a view of republishing the latter in more convenient form. The volume will appear in the course of next year, as No. 3 of the new series, "Specola Vaticana," and will contain a more exhaustive treatise on the scales of color-estimates. It may be agreeable to many readers to have its substance presented in English. By referring to the forthcoming Italian publication, we have the advantage of not incumbering the present article with too many references to the sources, and of saving a graphical representation, which will be found there.

I. HISTORICAL SKETCH

Colors of stars are mentioned in four different groups of astronomical literature, of which only one will concern us here. There are the *double stars*, the colors of which were first observed by the two Herschels (1782 and 1847), then by Dunér (1876), O. Struve (1878), and others. Most of O. Struve's and Dembowski's estimates are found in Burnham's *General Catalogue*. All these observations were made occasionally, without the purpose of establishing any systematic method or nomenclature. Then there are the *red stars*, so called, as distinguished from the great mass of yellow stars by their deeper orange or reddish hue. It is well known that four of our most conspicuous reddish fixed stars are mentioned in the Almagest, as standing out from the yellowish tint of the rest. Lalande gave (1807) a list of 33 "red stars." The first who made a critical collection of all that was known about red stars, was Schjellerup (in 1866 and 1874). Then followed a catalogue by Schmidt (1873), the *Prodromo* of Secchi (1876), and observations and catalogues by Birmingham (1877), by Lindemann (1879 and 1882), and by F. Krüger (1893). Comprehensive bibliographies have been published by Knobel, Espin, and Krüger.

Colors of *variable stars* are first mentioned in connection with the *Nova* of 1572, by Tycho Brahe. Argelander did not observe them much himself; but proposed the first scale in 1844, which was used for thirty-six years by his disciple Schmidt. Colors of variable stars were observed a great deal by the English astronomers Pogson, Baxendell, and Hind. Lists of those colors are found in the "Notes" to the catalogues of Pogson (1856) and of Schönfeld (1866, 1875). Special columns for the color were first used by Gore (1887) and Chandler (1888, 1893, 1896). In the Harvard catalogues of variable stars the spectral type is given instead of the color, but Table V of the Second Catalogue contains a column for the color.

The three groups of literature hitherto mentioned treat of colored stars, not of *star-colors* in general. Of the latter we now possess four published catalogues: the first by Sestini (1845 and 1847), which contains all the stars of Baily's Catalogue, down to −30° declination; then Vol. XIV of the *Harvard Annals* (1884), with a column of observations by Franks; further the *Photometric Durchmusterung* of the northern sky, by Müller and Kempf (1894–1907), and finally Osthoff's Catalogue (1900) of all the bright stars within the first five magnitudes, down to −10° declination.

What follows deals only with direct estimates of color by the eye and, consequently, excludes all kinds of *colorimeters*, such as were either described or used by Secchi (1852–1856), by Kincaid (1867), by Zöllner (1868), by Christie (1874), by Lindemann (1882), and by Chandler (1888), and also the *photographic* method, which is being experimentally studied by Schwarzschild, Parkhurst, Bergstrand, and others.

II. THE ELEMENTS OF COLOR AND THEIR SYMBOLS

1. While in the physical theory of colors three *elements* have to be distinguished, brightness, tone, and shade, in practical astronomy the brightness of the stars has been considered separately from color, and is known under the name of magnitude. The other two elements, tone and shade, cannot be separated in a theory of color, be it the chemical theory of pigments or the physiological theory of color-perception, the physical theory of the spectrum, or finally, the astronomical theory of star-colors.

Much ambiguity, however, prevails as to nomenclature. The terms tone, hue, shade, and tint are given as synonyms in most dictionaries. To remove all ambiguity from this article, tone and shade are here used in their technical meaning as the *qualitative* and the *quantitative* elements, respectively. The quality of color depends on its position in the spectrum and the quantity on its mixture with the two auxiliary colors: white and black. The technical designation of the two elements should conform to their definitions. The colors of the spectrum are often compared to the gamut in music, and to this analogy it is due that Newton, in his *Optics*, distinguished seven colors of the spectrum. If then the spectrum is a chromatic gamut, its colors are the tones. In like manner, if the quantity of color depends on its mixture with white or black, it may properly be called shade or *nuance*, which latter word is derived from *nubes*, "cloud." French writers, like Chevreul and Lacouture, use the expressions *ton* and *nuance* in exactly the reversed sense. Maxwell restricted the term "shade" to admixtures of black, and called the paler shadings "tints." We shall avoid the double term, for fear that the reader might consider tone, shade, and tint as three essentially different elements of color, while in reality the latter two mean only more or less shading.

In measuring colors, both elements—the qualitative and the quantitative—must be referred to certain *scales*. The scale for the quality of color consists in the selection of *fundamental* or simple colors to which all the others can be referred. The selection will be a different one for different theories of color. In the art of painting, red, yellow, and blue are usually considered the primary colors. Leonardo da Vinci added green as a fourth one; but this has been declared a mixed color by Waller (1686), Chevreul (1851), and all following writers. It has been mentioned that Newton established seven principal colors in the physical theory. Later on, his distinction between blue and indigo was dropped. In physiological optics, Young chose red, green, and violet as the primary sensations; in all probability, because red and violet form the extremes and green the middle of the solar spectrum. From experiments on color-blindness, however, Helmholtz and Maxwell were led to replace violet by blue. Sir William Abney returned to violet and modified the places of the primaries in the spectrum.

The astronomical theory of star-colors cannot accept its scale of tones from either chemistry or physics or physiology; it has to take the fundamental colors from what the stars actually show, after their light has passed through our atmosphere. This maxim was followed by Argelander in 1844, as we shall see in the next section.

The quantitative element of color must likewise have its scale or gradations. Newton represented it graphically, by arranging the colors of the spectrum along the circumference of a circle, whose center represents white. The quantity, or saturation as he called it, of any color within the circle is then proportional to the length of its radius, i.e., to its distance from white. Newton considered likewise the mixture with black as a quantitative element of color, but did not comprise it in his geometrical design. We shall return to this point presently. It follows from Newton's way of representing the two elements of color, that the solar spectrum was to him the normal, and at the same time the maximum, saturation.

From this normal quantity all the colors diminish in two directions: toward white and toward black. The *paler* colors (Maxwell's tints) are also called undersaturated, while the *darker* shades are oversaturated. They form the scale for the quantity of color.

Returning to Newton's chromatic circle we hope to show that it leads naturally to a chromatic sphere, with the spectral colors along the equator and the auxiliary colors, white and black, as its poles. Newton gave a diagram of the colors in Fig. 11 of Tabula III of his *Optics*. The seven chromatic sectors are not all of the same width (he makes them proportional to the seven intervals of the gamut), and are not homogeneous either in radial or in angular direction. The center represents white and the line separating two sectors represents the mixture of the two. The middle color of any sector is modified toward either of its two limiting radii by adjectives. Thus the yellow sector has the two modifications orange-yellow and greenish-yellow. Newton's Latin expressions *flavus aurescens* and *flavus viridescens* show that in Latin, as in all its derived languages, the modification stands on the right of the principal color, while in all the Germanic languages the order is reversed. This has to be remembered, when O. Struve's Latin color designations are rendered into English. Newton found by experiment that the colors which lie in opposite directions on the circle, or on the same diameter, when mixed in proper proportions, produce gray but never white. The designation "complementary" for such colors was probably introduced by Chevreul in 1839.

The want of the other pole for black, in Newton's diagram, was pointed out by Lambert (1772). Curiously enough he supplied it by the vertex of a cone erected over the circle as a base. In the successive parallel sections the pole white gradually changes into black. It is hard to see why Lambert preferred the cone to the sphere. The latter is, to the writer's knowledge, not mentioned in the literature on the subject. Only Chevreul speaks of a "construction chromatique-hémisphérique;" yet all his diagrams seem to represent a plane. On the chromatic sphere, with the two poles white and black, and the spectral colors along the equator, any two half-meridians include a constant tone, and any two parallel circles a constant shade. Each diameter designates two complementary colors, giving gray together. Hence white and black are likewise complementary.

Where only three primary colors occur, as in the theory of color-sensation, the triangle has offered itself as a simple diagram. The primaries are assigned to the three angles, and white is placed in the center of gravity. The diagram might be extended to represent the dark colors as well. It is only necessary to replace the triangle by a tetrahedron, with white at the fourth angle. A second tetrahedron would have black in the place of white, and both would be brought base to base so as to make the primaries coincident.

2. On the chromatic sphere the scales of the two elements of color are at once apparent. The scale of the tones consists in the meridians, and the scale of the shades in the parallel circles. The next question is to find proper *symbols* for them. Four kinds of symbols have been in use: (*a*) technical words; (*b*) colored diagrams; (*c*) numbers, and (*d*) letters.

a) *Words* have been used to define the tones of colors by designating Fraunhofer's lines in the spectrum, or by naming natural objects of corresponding coloration. Much less definite are the words used for designating tint and darkness of colors.

b) A first but very imperfect *diagram* of chromatic tones and their shadings was published by Waller, in 1686. Much more artistic representations of colors were given by Chevreul (1839) for industrial purposes. Those of his follower, Lacouture, are unsurpassed in beauty and correctness. Colored diagrams, especially designed for star-colors, are found in Smyth's *Sidereal Chromatics* (1864); they give six different tones, each with four degrees of shading, or twenty-four pictures in all. The many representations of the solar spectrum are intended for tones only, while the shading necessarily inherent in them may serve as a measure of their artistic perfection.

c) *Numbers*, as steps in the chromatic scale, were first used by Klein (1868), with the explicit purpose of imitating Argelander's magnitude-scale. Dealing only with strongly colored stars, he divided the interval yellow to deep, fiery red into five steps, from 0 to 5. Schmidt extended the scale to all the stars, and, keeping practically Klein's steps, moved the numbers backward toward white and ended with 10=red. This was in 1872. Safarik (1879) speaks in one place of twenty grades that he used. Köhl (1878) had a scale of eleven steps, from 1=deep red to 12=white. Innes (1898) passes from 1=red, over 8=white, to 14=reddish violet.

Even less uniformity prevails in the numerical designation of shades. Chevreul counted ten grades of saturation on either side of the normal one, or twenty from white to black. The ten grades of Chevreul were afterward (1890) reduced to six, on either side of the spectral colors, by Lacouture. In *Sidereal Chromatics* Smyth chose four grades from deep saturation toward white, and put the numbers in the form of exponents over the diagrams as well as over the words designating the quality of color. The same notation was followed by Espin (1886), Franks (1887), and Backhouse (1905)—the latter two, however, in the opposite direction from Smyth, calling 1=white-tinged and 4=deepest tint. Krüger counts 1, 2, 3, in the same direction with Franks.

d) *Letters* came into use as symbols of the quality of colors in the years 1883–1887. Naturally they were always the initials of the corresponding words. It will be noticed that the English language is the only one that furnishes these initials without ambiguity. The priority of letter symbols is due to H. W. Vogel (1883). He started from the three primary or simple colors:

W, G, R,

and, by means of combinations and permutations of the symbols, interpolated four mixed colors between them; thus:

W, GW, WG, G, RG, GR, R.

Following Argelander, he considered white as one of the principal sidereal colors; he made a mistake, however, by omitting one of Argelander's colors between white and red. Whether his G mean yellow or orange, one of the intervals W–G or G–R is necessarily too large. Vogel gave no symbols for the quantitative element.

The next year the "Harvard Photometry" appeared (*H.A.*, **14**, 1884). On p. 94 the letter symbols are defined as the initials of the English names of all the colors used by Mr. Franks. They are the six principal colors of the spectrum:

R, O, Y, G, B, P,

with the additional colors W = white, and L = lilac. Combinations of letters are avoided, but the shading of the colors is expressed in three grades, by capital Latin, small Latin, and small Italian types, in the order from strong to faint saturation.

A combination of the symbols R and O was used the first time by Espin (1886).

Mr. Franks proposed afterward (1887) to keep the six principal types mentioned above, with a few modifications. White should be designated by O, as in Newton's chromatic circle; this necessitated expressing orange by Or. The letter P was afterward changed into V. The shades were to be indicated by numerical exponents, following Smyth. Krüger, in his Catalogue of 1893, held to Vogel's symbols, but improved them by putting O between G and R.

In Lacouture's *Répertoire chromatique* (1890), we meet the French initials, and for the first time also a special symbol for black. White and black are used to indicate the shadings. Mixed pale or dark colors were thus expressed by three letter symbols.

We said before, that the English initials (as first used by Professor E. C. Pickering) are the only ones that admit of no ambiguity. If the auxiliary color black is introduced, the initial B cannot be used. Here the letter S would recommend itself, as the initial of "somber" (= *sub-umbra*). It would also accord with the French *sombre*, the German *schwarz*, and the Italian *scuro*.

III. COMPARISON OF THE VARIOUS SCALES

There have been two scales in use in sidereal chromatics, which may be called the *linear* and the *areal* scales. The symbols of the former are mainly numerical, those of the latter literal. Once the two sets of symbols are correlated to one another by some rule, the comparison between the two chromatic scales will be obvious.

1. The *areal*, or two-dimensional, scale has its origin in Newton's chromatic circle, and has been developed by English astronomers. Smyth, Franks, and Backhouse agree in the choice of six *fundamental* colors:

red, orange, yellow, green, blue, violet (or purple), and of four degrees of chromatic intensity; but the arrangement of tones and shades is different in the case of Smyth from that of the other two. First, the diagram of Smyth is rectangular, that of Franks and Backhouse circular: the former using parallel co-ordinates, as it were, the latter polar co-ordinates. The normally saturated colors are also marked differently: they have the index 1 in the rectangle, and the index 4 on the circumference of the circle.

The symbols of the six fundamental colors lend themselves at once to the designation of mixed colors by combinations of any two of them, and, if desired, by permutations, putting the symbol of the modifying color to the left of the predominant one.

A peculiarity of the two-dimensional chromatic scale, although not a necessary one, consists in not using white and black as auxiliary colors. We shall presently see that white was proposed by Argelander as one of the fundamental colors, and we remember that it was adopted by Vogel in 1883. In the arts of painting and dyeing black is considered as a color, and in physiological optics a special sensation is attributed to it by Helmholtz. A symbol (N) was assigned to black by Lacouture, and the combination of this symbol with those of the spectral colors gave the shadings of the darker colors. For variable stars, in particular, the darker shadings should be contained in the scale; and hence the chromatic sphere would seem to be a better system of two co-ordinates than the circle. Besides this, we hope to show in the last section that the two symbols W and S will express all the grades of shading that an observer can estimate directly.

The *linear* or one-dimensional scale owes its origin to Argelander. In his "*Aufforderung*" of 1844 he proposed the chromatic scale:

purpur, rot, orange, gelb, weiss, bläulich,

"with two or three gradations between them." Here we find white, where the solar spectrum places green. The scale seems to represent what Argelander actually saw in the stars, independently of any chromatic theory. It is quite likely that by *purpur* Argelander meant to express what he observed in the variable stars when they decline toward their minimum brightness, i.e., a darker shade of red. In like manner his *bläulich* is not a fundamental color;

it means the faintly bluish tint in which white stars sometimes appear.

Argelander's scale was used by Schmidt for nearly forty years. Schmidt divided the interval between white and red into ten grades and fixed the four fundamental colors on the following numbers:

white = 0, yellow = 4, orange = 6, red = 10.

We hope to show that the numbers 0, 4, 6, 10, which follow no law, could advantageously be replaced by the arithmetical progression: 0, 3, 6, 9. These numbers would respond better to the "two or three gradations" that Argelander proposed between the fundamental colors.

The four primary colors of Argelander have almost universally been adopted in Europe and America. They were used by Secchi in his *Prodromo* (1875), without a proper scale; by Safarik, with ten or twenty divisions; by Chandler and Yendell, with nominally ten, but in reality only six grades; by Krüger (1893) and Osthoff (1900), who both followed Schmidt's gradations.

Independent of Argelander's proposition are the observations of Sestini, whose *Primo Tentativo* (1843) antedates the *Aufforderung;* then those of Lindemann, who describes the colors like Sestini; and finally those of Vogel, in so far as he omitted one of the two colors between W and R, and reduced Schmidt's scale from ten to six grades, in order, as he said, not to have "too many subdivisions." In the course of the Potsdam *Durchmusterung* it was found, however, that the subdivisions were insufficient, the chromatic steps being on the average almost twice as large as those of Schmidt. The remedy which was applied is well known: by means of the suffixes ± each of the six intervals was divided into three, which gave a scale of eighteen grades between W and R, with an average value of nearly two-thirds of Schmidt's chromatic grade.

2. If now some mathematical *correlation* could be established between the literal and the numerical symbols, the result would be, not only a comparison between the scales of one and two dimensions, but even an easy transition from one to the other without ambiguity. Fortunately this correlation can be made simply by changing the

four irregularly distributed numerals of Schmidt: 0, 4, 6, 10, into the arithmetical progression:

0=W, 3=Y, 6=O, 9=R.

Between these four cardinal points the combinations and permutations of the four letters will exactly fill the numerical intervals, as shown in the following table:

TABLE I

White			Yellow			Orange			Red		
BW	W	YW	WY	Y	OY	YO	O	RO	OR	R	SR
−1	0	1	2	3	4	5	6	7	8	9	10

The two limiting shades −1 and 10 are supposed to correspond to Argelander's limits, *bläulich*=bluish-white and *purpur*=somber-red. The scale may be called the *chromatic scale of nine grades*, not as if it were limited to so many grades, but because it divides the interval R to W into nine steps; just as the Centigrade thermometer has its name, not from the actual number of degrees, but from its division between boiling and freezing points. The difference between this scale and that of Schmidt consists mainly in that it puts the yellow one grade nearer to white. The difference in the red will hardly ever be noticed on account of the scarcity of stars which deserve the symbol R. In Vogel's scale one of the two middle sections of Table I is cut out. Chandler has put orange on 4; consequently his large grades would reach red with the number 6. While Krüger and Osthoff adhered strictly to Schmidt's scale, that of Safarik agrees better with ours; for he has:

3=straw-yellow, 6=orange, 9=blood-red.

Klein's numbers are exactly five units smaller than in our scale, his 0, 1, 2, 3, 4, 5 being identical in definition with our 5, 6, 7, 8, 9, 10.

The advantage of the nine-grade scale consists in its symmetrical construction. The observer has only to remember the principal grades 3, 6, 9—all multiples of 3. Practically he has to impress on his mind only the two standard colors corresponding to 3 and 6.

Once he has fixed the two cardinal points, he will add to each one grade forward and one backward, to acquire the two middle sections of Table I. Adding the same two modifications to the limiting colors W and R, he will be in possession of the entire scale from −1 to 10.

3. After having slightly modified the linear scale, we suggest a similar change in the scale of two dimensions. It will consist in reducing the four degrees of saturation to three, in order to co-ordinate them to the three shadings:

YW, WY, Y.

For this purpose we construct on the chromatic sphere, between pole and equator, two parallel circles representing these degrees of saturation. It is now an easy matter to trace the linear scale upon the sphere. It will be a line broken in the three points: W, Y, R. We begin by putting the number 0 on the pole W. The numbers 1, 2, 3 will then run along the meridian marked Y, cutting the two parallels in 1 and 2. Number 3 will lie on the equator; and along that fundamental circle the linear scale will run as far as R=9. From this point it will pass along the meridian marked R to the other hemisphere as far as the first parallel, where we place number 10. There remains number −1, which we reach by producing the scale backward from the pole W on the meridian B as far as the first parallel. A large diagram, printed in two colors, and representing the linear scale within the scale of two dimensions, will be found in the forthcoming volume of the Vatican Observatory.

The combination of the two scales in one diagram shows that the linear scale is not exclusively one of tones or one of shades, but a mixture of both. Its *raison d'être* has been illustrated by comparing the colors actually seen in the stars to those which a solid mass apparently passes through, when cooling off from white incandescence to a deep red (see *American Journal of Science*, **19**, 467, 1880). We say "apparently," for although blue and green are not perceived in the cooling process, the presence of these rays can hardly be doubted. It seems as if the rays of shortest wave-length were the first to disappear in the process and those of largest wave-length the last, as in fact the heat rays remain after all the visible rays have vanished. When the blue rays in a white star have disappeared, the green in combination with the red will produce the sensation of yellow. It would seem, therefore, that the linear scale corresponds to what is actually seen in the stars, at least in a general way. It remains true, however, that sometimes the scale exercises coercion on the observer.

Combinations of colors like BY, RY, and even RW, which do not contain the intervening tones of the linear scale, are observed, as is testified by Sestini, Franks, Krüger, and by the writer, although other observers seem to doubt it. As there are no numbers provided for the combinations mentioned, the observer will, in all such cases, have to pass from the numeral symbols to letters.

A great difference seems to prevail between the two scales with regard to taking the mean of a number of observations. Arithmetically the operation is obvious in the linear scale, although it would lose its physical meaning, if the estimated numbers differed more than two or three units. In the circular or spherical chromatic scale the method of taking the mean has never been explained. Vector analysis suggests a method. In that scale, colors are designated by vectors (straight or spherical). The half-sum of two vectors is a third vector drawn to the middle point of the line which joins their extremities. Incidentally this operation shows that the degree of saturation would be better designated in the form of coefficients than of exponents. Thus, the symbolical operation:

$$\frac{1}{2}(y^1+y^2)=y^{\frac{3}{2}}$$

would look better mathematically if written:

$$\frac{1}{2}(1y+2y)=\frac{3}{2}y\ .$$

IV. SUGGESTIONS

Should there ever be question of introducing more uniformity in the notation of sidereal colors, the following suggestions may serve as a preparation:

1. The foundation for further deliberation seems to consist in the following points:

a) The two scales of one and two dimensions may be allowed to exist together. The latter has proved itself useful mainly in the observation of double stars, while the former is more applicable t single stars and especially to variables.

b) Accordingly both numerical and literal symbols may be used, the latter being the initials of the English denominations, with the auxiliary S = somber.

c) The two scales should be correlated by a mathematical rule, so as to make the transition from the one to the other easy and without ambiguity. In this way they will supply the defects of each other, and may be considered as two modifications of one general scale.

d) The correlation can be effected by reducing the ten grades of Schmidt to *nine*, and the four shadings of Franks to *three*.

Table I (page 271) will present the solution of the problem.

2. A secondary question is the notation of the shading of colors. The following points are proposed:

a) In every chromatic scale it must be possible to express the degree of saturation. This, however, seems to be needed only in the fundamental or simple colors, because there alone it is clearly perceptible. In mixed colors a distinction must be made between the relative saturation of the components and the absolute saturation of the compound. The relative shading will appear to the observer as the predominance of one component over the other, an impression which he will denote by the permutation of the literal symbols or by the correlated numerals. Thus, if in a mixture of O and R, the latter has a deeper shade than the former, he will denote it by OR = 8; if on the contrary O is more saturated than R, he will write RO = 7. If the compound of O and R shows a tendency towards W, it will, first of all, appear as RO = 7 and not as OR = 8. The contrary will take place when the tendency is towards a darker shade. The question then turns about a pale RO and a dark OR. Now experience seems to show that it is very difficult to distinguish between a pale RO = 7, and O = 6; or between a dark OR = 8, and R = 9. Experience seems likewise to show that the errors of observation are larger than the differences indicated. In the *Memoirs of the British Astronomical Association* a very pale orange-yellow is denoted thus: OrY^1, which stands for $(OrY)^1$, and is the nearest tint to white. It seems to be questionable whether, in that very pale mixture, the component orange can be perceived beside the yellow. The difficulty of perception is not so great in the simple colors: a pale R can be distinguished from OR, and a pale O from YO. Likewise there is a difference between a dark O and RO, and between a dark R and violet red.

The advantage of restricting the notation of shade to the fundamental colors lies in the fact that any tone or shade of these colors can be expressed by two symbols at the most, while shades of mixed colors require three symbols.

b) Once the notation of chromatic shades is limited to three degrees on either side of normal saturation, and to the fundamental colors only, there is the choice between two sets of symbols:

The numerals 1, 2, 3, 4, 5 express the shades from very pale to very dark. On the chromatic sphere the two poles are $W=0$, $S=6$, and the index 3 belongs to the equator. In the British Astronomical Association the numerals are used in the form of exponents over the letters. Another set of symbols was used in Table I of the preceding section, viz., the two letters W and S. It will be remembered that the numerals −1, 1, 2, and 10 in the linear scale of nine grades express shades. They are here repeated, with the two sets of symbolical notation:

TABLE II

Areal scale	B^1	Y^1, Y^2	R^4
Linear scale	−1	1, 2	10
Correlation	BW	YW, WY	SR

c) The suggestion we wish to make is contained in Table II. The letters W and S, combined with any letter symbol of the fundamental colors, will express a pale or dark shading of that color, and the permutation of the combined letters will express two degrees of each shading, beside the normal saturation (without W and S). On the chromatic sphere the permutations with W or S represent the two parallels between equator and pole.

The application of the suggestion to the areal scale would be to replace the exponents by the letters W and S. The application to the linear scale would mean to employ letter symbols beside the numerals, wherever needed, and to consider the correlation established between them in Table I as a bridge for easy transition from one system to the other.

3. The suggestions made in paragraphs 1 and 2 would yield the following advantages:

a) In the two-dimensional scale all the tones and shades would be expressed by two symbols at the most. The letter symbols would be understood without a key, at least in the Germanic languages.

b) The linear scale would supplement its own deficiencies through the correlation between letters and numerals, for instance in the following combinations, WR, WO, SO, BY, etc., which are not expressible in the numerical scale.

c) The same correlation would make the reading of observations easy for all, no matter what scale they used themselves. And all that has to be remembered is: Y=3, O=6, R=9.

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THE ABSORPTION SPECTRUM OF SELENIUM VAPOR, AND THE EFFECT OF TEMPERATURE UPON IT

BY E. J. EVANS AND G. N. ANTONOFF

The first investigation of the absorption spectrum of selenium vapor was carried out by D. Gernez,[1] who found that below 700° C. the vapor gave complete absorption except in the red. Above 700° C. absorption bands, which correspond with bands in the emission spectrum, appeared. Lockyer and Roberts[2] found that the vapor gave a fluted absorption spectrum. Since selenium is one of those substances giving anomalous values for the vapor-density, it is interesting to determine the changes produced in the absorption spectrum by rise of temperature. At 860° C. the vapor-density is 110.7, and at 1420° C. the vapor-density is 81.5, which nearly corresponds to the normal density of the diatomic molecule Se_2 (78.5). Selenium vapor at comparatively low temperatures consists of a mixture of diatomic molecules and aggregates of greater molecular complexity, but at temperatures above 1500° C. the vapor consists almost wholly of diatomic molecules. Graham,[3] as the result of his investigation of the absorption spectrum of sulphur vapor between 500° C. and 900° C., was able to determine the spectra belonging to the S_2 and S_8 molecules respectively. The diatomic molecule S_2 gives bands between λ 2600 and λ 3415, while the more complex molecule S_8 gives bands between λ 3985 and λ 4775.

In the case of selenium the experimental difficulties are great, because of the high temperature at which the normal vapor-density is obtained and besides, as far as the present authors are aware, there is no definite knowledge of the nature of the molecular complexes present at comparatively low temperatures. In the present work the absorption spectra of selenium vapor at high and low temperatures (vapor-pressure constant) will be described. Experiments were also carried out with a known weight of selenium placed in a quartz tube, and the absorption spectrum photographed at different temperatures. The effect of temperature on the absorption spectrum was carefully noted, and in certain cases the wave-lengths of the absorption bands were measured. The values obtained are, however, only approximate, being correct to a few Ångström units.

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[1] *Comptes Rendus*, **74**, 803, 1190, 1872.

[2] *Proc. Roy. Soc.*, **23**, 344, 1875.

[3] *Ibid.*, A, **84**, 311–324, 1910.

EXPERIMENTAL ARRANGEMENT

It is unnecessary to give a full description of the experimental arrangement as it is similar to the one previously employed in an investigation of the effect of high temperature on the absorption spectrum of iodine.[1] The selenium was introduced into a quartz tube, which was provided with transparent quartz ends, and also a side-tube of small diameter. When the absorption of a constant mass of vapor was studied, the side-tube was short, but when the absorption at constant pressure was studied, this side-tube was several inches long. The quartz tubes (each about 10 cm long and having a capacity of about 30 cc) were dried, and after the selenium was introduced, evacuated to a pressure of a fraction of a millimeter. Dry hydrogen was now allowed to enter the tube, which was again evacuated. The process of filling with hydrogen, and evacuating was again repeated, and when the pressure was only a fraction of a millimeter, the side-tube was sealed off in the oxyhydrogen flame. The quartz tube was placed inside an electric furnace, one wound with an alloy of nickel and chromium for temperatures up to 1200° C., and a carbon-tube furnace for higher temperatures. The temperature was read off in degrees Centigrade on the scale of a galvanometer, to which a *Pt—Pt-Rh* thermocouple was connected. When a constant vapor-pressure was required, the following arrangement was adopted. The side-tube protruding from the end of the furnace was placed inside a larger tube also made of quartz. This tube was wound with nickel wire, and the windings were continued on the side-tube to within a few centimeters of the clear quartz ends of the tube inside the furnace. The pressure of the selenium vapor in the quartz tube is the vapor-pressure corresponding to the coldest part of the apparatus. In the present work the end of the side-tube was made the coldest part of the apparatus, and the temperature was recorded by placing a thermo-couple inside the larger tube, and pushing it just over the end of the side-tube. Unfortunately no data giving the vapor-pressures of selenium at different temperatures could be obtained. The positive crater of the electric arc was employed as a source of light, and two lenses (quartz lenses for ultra-violet) were used to bring the spot of light to a focus on the slit of a concave grating of 1 meter radius and 1500 lines to the inch. Since selenium absorbs strongly in the blue, violet, and ultra-violet, its absorption spectrum was studied photographically by means of Wratten and Wainwright's "panchromatic" plates. The first-order spectrum was employed throughout the present research.

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[1] *Astrophysical Journal*, **32**, 1, 1910.

DIFFICULTIES

Selenium heated in air or oxygen forms on cooling a white oxide (SeO_2), which crystallizes in long needles. This oxide rapidly evaporates at about 300° C., forming a greenish yellow vapor whose absorption spectrum has been investigated by Gernez.[1] It gives a channeled absorption spectrum in the blue and violet. It was therefore necessary in all experiments on the absorption of selenium vapor to heat the selenium in a vacuum, or in an atmosphere of carbon dioxide. The method actually adopted has been described above, and when the quartz tube was ready for use, it only contained very small traces of air and hydrogen. The photographs did not show the characteristic bands of SeO_2 in the blue and violet. Two of the quartz tubes were found to leak at temperatures of 900° C. and 1000° C., and this was evident from the color of the light falling on the slit of the grating spectroscope. When the tube was taken out of the furnace, the surface of the quartz was found to be covered with the white, needle-shaped crystals of the dioxide. In other cases the selenium showed no signs of having been converted into the dioxide, and the vacuum was good after prolonged heating to 1000° C.

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[1] *Comptes Rendus*, 74, 1190, 1872.

EXPERIMENTAL RESULTS

Since selenium dioxide may be formed during the course of experiments on the absorption spectrum of selenium vapor, it was considered advisable to make a brief examination of the absorption bands due to the former. Some of the oxide was prepared by heating selenium in a current of oxygen, and about 0.006 grams of the white, needle-shaped crystals were placed in a tube of Jena glass, which was afterward evacuated and sealed. The tube was then introduced into the electric furnace, and the absorption spectrum of the vapor examined at a temperature of about 350° C. The approximate wave-lengths of the SeO_2 bands from the violet to the blue are as follows: 3930, 3960, 3990, 4020, 4055, 4083, 4117, 4157, 4185, 4220, 4256, 4290, 4330, 4365, 4395, 4437, 4470.

The breadth of the bands varies from about 15 to 40 Å., and the 4437 and 4470 bands are double (4437 and 4470 are the approximate mean wave-lengths of the double lines). The photographs do not show the presence of any bands between 3800 and 3600.

About 0.006 grams of SeO_2 were again weighed out, and introduced into a quartz tube, which was evacuated to about 1 cm pressure, and the side-tube sealed off in the oxyhydrogen flame. The absorption spectrum was photographed at 360° C., 600° C., and also at 900° C. The low-temperature photograph shows the presence of bands in the blue and violet, and in addition bands in the ultra-violet between λ 3800 and λ 3200. The photograph taken at 600° C. only extends from λ 7200 to about λ 4200, and shows a few bands due to SeO_2, and traces of bands due to selenium vapor. Beyond λ 4200 in the violet and ultra-violet there is complete absorption. The third photograph also indicates complete absorption beyond λ 4200, and an examination of the absorption bands in the blue shows distinctly the presence of selenium. The ultra-violet bands between λ 3200 and λ 3800 are considered to be due to the vapor of metallic selenium and not to the dioxide. The SeO_2 crystals employed in the above experiments were not absolutely white, and at the time it was considered probable that the crystals were mixed with small traces of selenium. This explanation of the origin of the ultra-violet bands was also borne out by subsequent experiments on the absorption spectrum of selenium vapor, when care was taken to exclude the presence of oxygen.

THE ABSORPTION SPECTRUM OF SELENIUM VAPOR

When the pressure of the selenium vapor in the quartz tube is comparatively large, the color of the image on the slit is deep red, and the spectrum of the light passing through the vapor extends from λ 7200 to λ 5800. There is continuous absorption below λ 5800, and also no absorption bands are observed in the region 7200 to about 5800. As the vapor-pressure is diminished (temperature of the vapor being kept at 600° C.) absorption bands are observed in the green, blue, and violet, and when the vapor-pressure reaches a low value, bands are observed in the ultra-violet. The ultra-violet bands make their appearance when the temperature of the solid selenium in the coldest part of the side-tube is about 40° above the melting point (217° C.) of the metal. The experiments indicate that the absorption by the vapor increases continuously from the extreme red to about λ 2400 in the ultra-violet. The absorption by thin films of the solid was investigated by R. W. Wood,[6] who found that it increased continuously with diminishing wave-length until λ 2200 was reached. In the ultra-violet absorption spectrum of the vapor no bands were detected of shorter wave-length than 3200 (i.e., between λ 3200 and λ 2300).

EXPERIMENTS WITH CONSTANT MASS OF VAPOR IN TUBE

Series I

In the first series of experiments about 0.005 grams of selenium were introduced into a quartz tube (10 cm long, volume about 20 cc), which was evacuated to a pressure of 0.5 cm of mercury before sealing off in the oxyhydrogen flame. The tube was then heated in a carbon-tube furnace, and the absorption spectrum was observed visually at various temperatures from 350° to 900° C. At 450° the spot of light falling on the slit of the grating was yellow in color, becoming deep red at 600°. In the latter case only the red end of the spectrum could be observed, but no dark bands were visible. As the temperature was raised, the color of the light falling on the slit of the grating became yellow, and the spectrum observed extended almost up to the blue. The visual method of observation was soon discarded, as selenium vapor absorbs chiefly in the blue, violet, and ultra-violet. Two photographs of the absorption spectrum of the vapor were then taken under the following conditions:

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[6] *Phil. Mag.* (6) 4, 607, 1902.

Experiment 1.—Photographs of the absorption spectrum at 540° and at 1070° C. were taken and examined. The low-temperature absorption spectrum does not show any bands, but indicates the presence of a general absorption increasing in intensity toward the violet, and becoming complete beyond λ 5500. The spectrum extends from λ 7200 to about λ 5500. The photograph taken at 1070° shows the presence of absorption bands between λ 4700 and λ 5300. Also the absorption in the orange and yellow is very much diminished, and the spectrum extends from λ 7200 to about λ 4650.

Experiment 2.—This experiment was very similar to the previous one, the only difference being that the first photograph was taken when the temperature of the furnace was rising from 600°–700°. The spectrum (600°–700°) extends to λ 4600, and shows the presence of bands between λ 4600 and λ 5300. There is however much greater absorption in the region λ 5000 to λ 5700 at the low temperature (650° C.) than at the high temperature (1000° C.).

Series II

A combustion-tube containing 0.0015 grams of selenium was exhausted to a pressure of a fraction of a mm and sealed. Then the absorption spectrum was photographed at a temperature of 600° C. On examining the photograph it is found that there is complete absorption between λ 4000 and λ 3600, and absorption bands appear between λ 4000 and λ 4500. These bands are quite distinct from the SeO_2 bands previously described.

Series III

A very small quantity (0.0004 grams) of selenium was placed in a quartz tube, which was filled with hydrogen, and evacuated according to the method previously described. In the first place, experiments were conducted at various temperatures far removed from the boiling point (680°) of the metal, and thus the absorption spectrum at different pressures could be determined. When the temperature of the furnace was 360° C., the absorption bands extended from about λ 3250 to λ 3750. Below λ 3250 there are no absorption bands, and for the small vapor pressure (corresponding to 360° C.) there are also no absorption bands between λ 3885 and the extreme red. When the temperature of the furnace was 380° C. the same absorption bands were photographed, but the intensity of the bands between λ 3580 and λ 3885 was stronger. The general absorption below 3250 was also much stronger. A photograph was also taken when the temperature of the furnace was 410° C. and it shows the presence of a few bands beyond 3580 in the ultra-violet. All the absorption bands (between λ 3200–λ 3600), which are visible in the 360° photograph are not present in the 410° photograph, because the absorption is too intense. However, the absorption bands in the region λ 3580–λ 3885 are very much stronger than in the lower temperature photographs. The approximate wave-lengths of the bands in the ultra-violet are: 3240, 3255, 3280, 3295, 3317, 3338, 3363, 3387, 3412, 3435, 3460, 3483, 3510, 3537, 3592, 3614, 3640, 3663, 3684, 3700, 3715, 3730, 3742, 3755, 3763, 3774, 3802.

The absorption lines 3640 $\begin{Bmatrix} 3635 \\ 3645 \end{Bmatrix}$, 3663, 3684 are double lines, the wave-lengths given being the mean between the two components. The average width of the absorption bands between λ 3250 and λ 3580 is about 10 A°. At 600° C. all the selenium placed in the tube was in the state of vapor, and consequently experiments were made at this temperature, and also at much higher temperatures to determine the effect of temperature on the absorption spectrum of a constant mass of the vapor. The absorption spectrum at 600° C. shows the presence of absorption lines between λ 3700 and λ 4100, and complete absorption between λ 3700 and λ 2900. If photographs of the absorption spectra at 600° C. and 1070° C. are compared, it is found that they are almost exactly the same in the region λ 4000–λ 2400, but between λ 4000 and λ 4200 the absorption lines are stronger in the photograph taken at the high temperature. The quartz tube containing 0.004 grams of selenium was finally heated to 1300° C., and the absorption spectrum photographed. The photograph shows the characteristic absorption bands of selenium vapor, and not the SeO_2 bands. There is also a distinct tendency for the absorption bands not to photograph quite as far in ultra-violet as before (600° C.). The main difference between the absorption spectrum at 1300° C. and at 600° C. is that the absorption bands extend much farther toward the red (to λ 4650) in the former.

CONSTANT-PRESSURE EXPERIMENTS

The pressure of the vapor inside the quartz tube was controlled by the current passing around the nickel wire, and the temperature of the coldest part of the apparatus was also measured. The description of the constant pressure experiments will be divided into two parts: (1) comparatively high pressures; (2) low pressures. Two photographs were taken at each pressure, one at a temperature of about 600° C., and the other at about 1000° C.

Experiments at Comparatively High Pressures

A
(1) 3.5 amperes passing through the nickel wire (temperature of coldest part of apparatus 370° C.). Temperature of furnace = 580° C.
(2) Same conditions as (1) except that the temperature of the electric furnace was 1020° C.

In the first photograph (temperature of furnace 580°) the absorption bands extend from λ 3885 to λ 4400. The light is completely absorbed between λ 3885 and about 2800. The absorption bands in the second photograph do not extend quite as far toward the ultra-violet as they do in the first. Also the absorption bands between λ 4200 and 4400 (i.e., toward the red) are more distinct in the second photograph (temperature of furnace 1020° C.).

B
(1) 3.8 amperes passing through the nickel wire (temperature of coldest part of apparatus 420°). Temperature of furnace = 600° C.
(2) Same conditions as in (1) except that the temperature of the furnace was 1015° C.

The absorption spectrum at 600° C. shows the presence of bands between λ 4200 and λ 4600. Below 4200 toward the ultra-violet there is complete absorption, and above 4600 (to λ 7200) there are no absorption bands. The absorption spectrum at the high temperature shows slightly greater absorption in the violet, and the absorption bands above 4500 (toward the red) are more distinct.

C
- (1) 4.1 amperes passing through nickel wire (temperature of coldest part of apparatus 460°). Temperature of furnace=600°.
- (2) Same conditions as (1) except that the temperature of furnace was 1010° C.

In the high- and low-temperature spectra, complete absorption occurs below λ 4500 (i.e., in blue-violet and ultra-violet). Absorption bands are visible between λ 4500 and λ 5160, and in each case a continuous spectrum is photographed between λ 5200 and λ 7200. The absorption bands between λ 4700 and λ 5200 are much more distinct in the photograph taken at the high temperature.

Another photograph was taken at a comparatively high temperature (940° C.), when 4.7 amperes were passing through the nickel wire (temperature of coldest part of apparatus 520°). The absorption spectrum shows the presence of bands between λ 4640 and λ 5200. The photograph between λ 4600 and λ 5200 consists of dark patches separated by fairly broad and lighter spaces. The absorption bands really consist of a large number of fine absorption lines. When 5.3 amperes were passing through the heating wire (temperature of coldest part of apparatus 583° C.) no absorption bands were visible. Below λ 5600 (i.e., in green, blue, etc.) there is complete absorption.

LOW-PRESSURE EXPERIMENTS

In these experiments a current of 2.5 to 2.8 amperes was passed through the heating wire, and the temperature of the coldest part of the apparatus was also measured. When the current was 2.5 amperes (temperature of coldest part of apparatus about 230° C.) weak absorption bands were photographed between λ 3200 and λ 3600, and the intensity of these bands rapidly increased as the current was raised to 2.8 amperes (temperature of coldest part of apparatus 280° C.). In the latter case the bands also extend from λ 3200 to λ 3800. The pressure was then kept constant (2.65 amperes through heating wire), and photographs of the absorption spectra at 500° and 1100° C. were taken. In the latter case the absorption bands (λ 3200–3700), which were present at the low temperature, had almost competely disappeared. Similar experiments were made at a higher pressure (2.8 amperes through heating wire), when the temperatures of the furnace were 600°, 1100°, and 1200° C., respectively. At the highest temperatures the absorption bands between 3200 and 3700 were much weakened in intensity.

DISCUSSION OF RESULTS

The discussion of the relation between the absorption spectrum and the nature of the various molecular complexes is rendered very difficult by the lack of information concerning the densities of the vapor under the conditions employed. It is known that the vapor-density diminishes with increase of temperature, finally attaining a value corresponding to Se_2 at temperatures above 1500° C. On the other hand, the vapor-density, and the nature of the molecules present at low temperatures (300° C.) and pressures are unknown. It is perhaps allowable to postulate that at these comparatively low temperatures, the molecules are more complex (Se_4, Se_6, or Se_8), and if such is the case, there are reasons for considering the absorption bands between λ 3200 and λ 3700 to be due to molecules more complex than Se_2. These bands, which in all probability are due to Se vapor, make their appearance at temperatures in the neighborhood of 300° C., and also rapidly diminish in intensity with increase of temperature. When the pressure of the vapor corresponds to 2.65 amperes through the heating wire, the absorption bands actually disappear at about 1100° C., but when the pressure is higher (2.8 amperes) the bands do not disappear even at 1200° C. The absorption spectra of 0.0004 grams of selenium vapor at 600° C. and 1300° C. indicate that the breaking up of the more complex molecules into Se_2 causes the absorption bands to become more distinct toward the red, and brings out new bands not present at the lower temperature. Also complete absorption in the ultra-violet commences at a slightly higher wave-length. It can be stated without much doubt that the absorption in the ultra-violet is not appreciably diminished with increase of temperature. The evidence of the photographs seems to point to a slight increase. At 1300° C., and 1.5 cm pressure (this would be the pressure of the *Se* vapor in the tube at 1300° on the assumption that all the vapor was diatomic) most of the molecules are diatomic, and probably the absorption bands between λ 3800 and λ 4700 are due to Se_2.

SUMMARY OF RESULTS

1. If the temperature of the selenium vapor is kept constant at 600°, absorption bands appear between λ 3200 and λ 3880, when the pressure is low. As the pressure increases there is complete absorption in the above region, and bands appear higher up the spectrum (i.e., toward the red). The same process is repeated as the pressure of the vapor is increased, until finally only the red end of the spectrum is let through.

2. The ultra-violet bands (λ 3250–λ 3800) diminish in intensity with increase of temperature, and for the low pressure corresponding to 2.65 amperes passing through the heating wire disappear at 1100° C. If the pressure is greater, the bands do not disappear altogether even at 1200° C.

3. For comparatively high vapor-pressures (i.e., complete absorption between λ 4200 and λ 2800), the effect of rise of temperature is to increase the intensity of the absorption bands toward the red and to cause the commencement of complete absorption in the blue, violet, or ultra-violet (depending on the pressure) at a point in the spectrum having a larger wave-length.

4. It is difficult to discuss the changes produced in the absorption spectrum by increase of temperature in relation to the corresponding changes of vapor-density, because of insufficient data. There are some reasons for regarding the bands between λ 3200 and λ 3800 as being due to molecules of greater complexity than Se_2 and the bands between λ 3800 and λ 4700 as being due to Se_2.

We are greatly indebted to Professor Rutherford for the interest he has taken in the work, and for placing the necessary facilities at our disposal.

Physical Laboratory
Manchester University
August 10, 1911

NOTE ON THE GROUPING OF TRIPLET SEPARATIONS PRODUCED BY A MAGNETIC FIELD[1]

By HAROLD D. BABCOCK

In recent papers on the Zeeman effect for chromium[2] and vanadium[3] attention was called by the writer to the grouping of the separations of the outer components of triplet lines. It was shown that for those metals the values of $\Delta\lambda/\lambda^3$ for triplets occur chiefly in two easily distinguishable groups. The completion of tables of magnetic separation for iron and titanium by King[4] has led to further inquiry along this line, which was subsequently extended so as to include all the data available. The additional material here discussed is found in papers by B. E. Moore on zirconium, yttrium, osmium, barium,[5] and thorium;[6] by W. Miller on manganese;[7] by H. M. Reese on nickel;[8] and by R. Jack on tungsten and molybdenum.[9]

The values of $\Delta\lambda/\lambda^2$ for the triplet lines of each metal were arranged in order of magnitude and a curve was drawn showing the frequencies of their occurrence. Lines whose classification or measurement was doubtful were omitted. As a considerable portion of the separations referred to the distance between the outer components instead of between the central and either one of the outer components, all the numbers were reduced to this system and the factor one-half applied at the end. The abscissas for the

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 57.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 52; *Astrophysical Journal*, **33**, 217, 1911.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 55; *Astrophysical Journal*, **34**, 209, 1911.

[4] *Contributions from the Mount Wilson Solar Observatory*, No. 56; *Astrophysical Journal*, **34**, 225, 1911.

[5] *Astrophysical Journal*, **28**, 1, 1908.

[6] *Ibid.*, **29**, 144, 178, 1909.

[7] *Physikalische Zeitschrift*, **7**, 896, 1906.

[8] *Astrophysical Journal*, **12**, 120, 1900.

[9] Göttingen Dissertation, 1908.

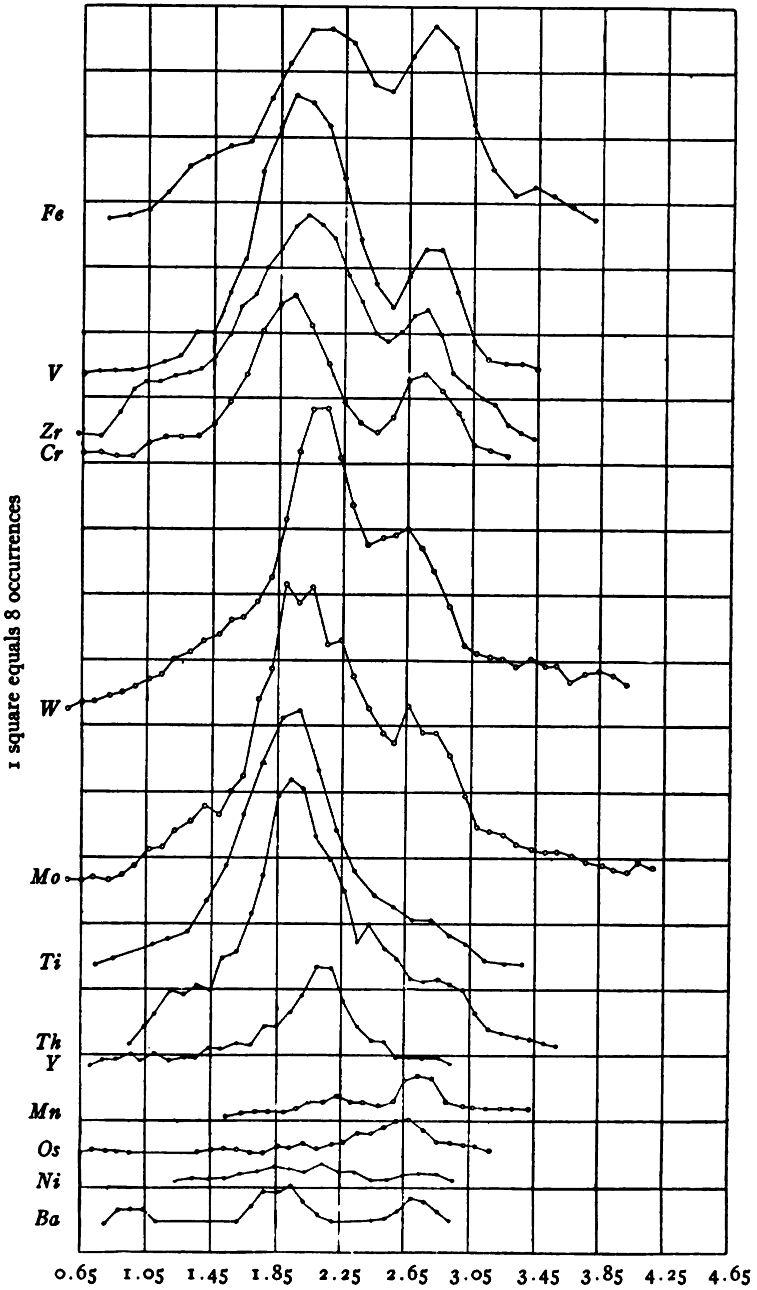


FIG. 1.—Grouping of Triplet Separations
Interval in $\Delta\lambda/\lambda^2$ equals 0.10. Field 20,000 gausses

curves are equal intervals in $\Delta\lambda/\lambda^2$. A given ordinate was obtained by averaging the number of occurrences for its own abscissa with those for the two adjoining ones. As first drawn the curves related to the various field-strengths employed by different observers. The abscissas were accordingly reduced to a standard field of 20,000 gausses, making them directly comparable, and they are thus displayed in Fig. 1. The scales are, for the abscissas: interval in $\Delta\lambda/\lambda^2$ equals 0.10, one square being equal to four intervals; for the ordinates: one square equals eight occurrences.

The alignment of maxima and minima, first noticed for chromium and vanadium, is shown here extending in general to these other metals. The first maxima lie between the limits $\Delta\lambda/\lambda^2=1.80$ and $\Delta\lambda/\lambda^2=2.20$, while the second are included between the values 2.60 and 2.80. The abscissas corresponding to the two maxima are thus seen to have the ratio 2:3. Although some curves show the second maximum only slightly and others have only a suggestion of the first, none exhibits a distinctive grouping except barium. This metal has in addition to the two usual groups a third, the abscissas being in the ratio 1:2:3. Small weight attaches to the curves for osmium, barium, manganese, and nickel on account of the few triplet lines which have been measured in their spectra.

It is to be observed that the data studied here were collected by six observers working in five laboratories, and that the field-strengths were different and were measured by different methods. In addition it is likely that some lines are classified as triplets which are really more complex. So close a similarity among the curves is therefore remarkable, and a combination of the results should be of considerable interest. Accordingly the position of the minimum was carefully determined on each curve and the values of $\Delta\lambda/\lambda^2$ thus divided into two groups. The means for each group, weighted according to the number of lines upon which they depend, when combined yield as values of $\Delta\lambda/\lambda^2$, $1.892\pm0.014_7$ for group I, and $2.794\pm0.018_4$ for group II, the appended numbers being probable errors. Using the equation

$$\frac{\Delta\lambda}{\lambda^2}=\frac{e}{m}\cdot\frac{H}{4\pi v},$$

where H is the field-strength in C.G.S. units and v is the velocity of light, the resulting values of e/m are $1.783 \pm 0.013_8 \times 10^7$ and $2.633 \pm 0.017_4 \times 10^7$, the former being based upon measurements for 2043 triplets and the latter upon those for 827. Two-thirds of the larger value is 1.755 ± 0.116, which, when combined with the first, with weights proportionate to 2043 and 827, yields the final weighted mean 1.775 ± 0.0132. Table I is a summary of the work here indicated.

TABLE I

GROUPING OF TRIPLET SEPARATIONS

METAL	FIELD	GROUP I			GROUP II		
		Limits	Number of Lines	Mean $\frac{\Delta\lambda}{\lambda^2}$ H = 20,000	Limits	Number of Lines	Mean $\frac{\Delta\lambda}{\lambda^2}$ H = 20,000
Titanium......	17,500	1.00–2.25	214	1.876	2.25–3.20	25	2.872
Iron...........	16,000	1.20–2.00	139	2.079	2.00–2.90	108	2.908
Yttrium........	24,400	1.70–3.10	56	1.955	3.10–4.00	5	2.702
Zirconium......	24,400	1.70–3.00	307	1.821	3.00–4.00	91	2.760
Osmium.......	24,400	1.80–2.60	4	1.843	2.60–4.00	26	2.620
Barium........	24,400	2.00–2.50	18	1.860	2.50–3.50	8	2.700
Thorium.......	24,450	1.20–2.90	277	1.803	2.90–4.20	87	2.702
Manganese.....	23,850	2.00–2.85	11	2.160	2.85–3.70	20	2.750
Nickel.........	28,300	2.00–3.40	12	1.970	3.40–4.25	2	2.760
Chromium.....	20,000	1.30–2.40	104	1.900	2.40–3.00	39	2.700
Vanadium.....	20,000	1.30–2.50	249	1.971	2.50–3.10	72	2.780
Molybdenum...	23,850	0.50–3.00	360	1.907	3.00–4.08	156	2.928
Tungsten......	23,850	0.80–2.85	292	1.863	2.85–4.80	188	2.726
			2043	$1.892 \pm 0.014_7$		827	$2.794 \pm 0.018_4$

$\frac{e}{m}$ $1.783 \pm 0.013_8 \times 10^7$ $2.633 \pm 0.017_4 \times 10^7$

It should be said that, before deciding upon the positions of the minima for these curves, they were drawn with various intervals in $\Delta\lambda/\lambda^2$, with and without the normal point ordinates described above; and the method of drawing herewith presented was selected as best representing the data. The minima were in general selected upon the original curves, before reduction to the standard field. The outer limits of the two groups are of little importance relatively, on account of the few occurrences there. A very few values of $\Delta\lambda/\lambda^2$ which were nearly one-half the value corresponding to the first maximum, or twice that value, were omitted from both groups.

The most important instance of this procedure is in the case of barium. The small distinctive group clustering around the value $\Delta\lambda/\lambda^2 = 1.90$ is not included in group I.

The appearance of a typical curve like that for chromium suggests a combination of two symmetrical overlapping curves. If this is the case, the positions of the maxima being the same for all the metals, but their relative heights different, the division of groups I and II at the minimum is not an equivalent process for all cases. For the ratio of the number of values assigned to group I but really belonging to group II, to the number assigned to group II but belonging to group I, would depend upon the relative heights of the two maxima, as would the position of the minimum itself. When the ratio of the height of the second maximum to that of the first decreases, the minimum moves out toward greater values of $\Delta\lambda/\lambda^2$, increasing group I at the expense of group II. The mean value of $\Delta\lambda/\lambda^2$ for group I is thus raised and at the same time given greater weight, while the opposite happens to group II. For nearly all of the curves shown here the second maximum is lower than the first, so that if this source of error could be removed, the two values

TABLE II

VALUES OF $\frac{e}{m}$ BY VARIOUS OBSERVERS

Färber*	1.710×10^7	Gmelin**	1.771
Lohmann†	1.750 1.720	Classen‡‡	1.776
Gehrcke and v. Baeyer‡	1.740	Bucherer¶¶	1.763
Stettenheimer††	1.791	Simon***	1.865
Weiss and Cotton¶	1.767	Proctor†††	1.859
		Mean	1.774

* *Annalen der Physik*, 9, 886, 1902.

† Stettenheimer, Dissertation, Tübingen, 1907.

‡ *Verhandl. der Deutschen Physikalischen Gesellschaft*, 10. Corrected by v. Baeyer, *Physikalische Zeitschrift*, 9, 831, 1908.

†† Dissertation, Tübingen, 1907.

¶ *Journal de Physique* (4), 6, 429, 1907.

** *Annalen der Physik*, 28, 1079, 1909.

‡‡ *Physikalische Zeitschrift*, 9, 764, 1908.

¶¶ *Ibid.*, 9, 760, 1908.

*** *Annalen der Physik*, 69, 589, 1899.

††† *Physical Review*, 30, 53, 1910.

of *e/m* upon which the final result depends would be more accordant. It does not seem likely, however, that the final mean value would be much changed.

Table II is a collection of other determinations of *e/m*, the average of which is 1.774×10^7. If the values of Proctor and of Simon be omitted, however, the average falls to 1.754×10^7, but the divergence of my result from this is only a little over 1 per cent.

Mount Wilson Solar Observatory
June 1911

THE SOLAR PROMINENCE OF OCTOBER 10, 1910

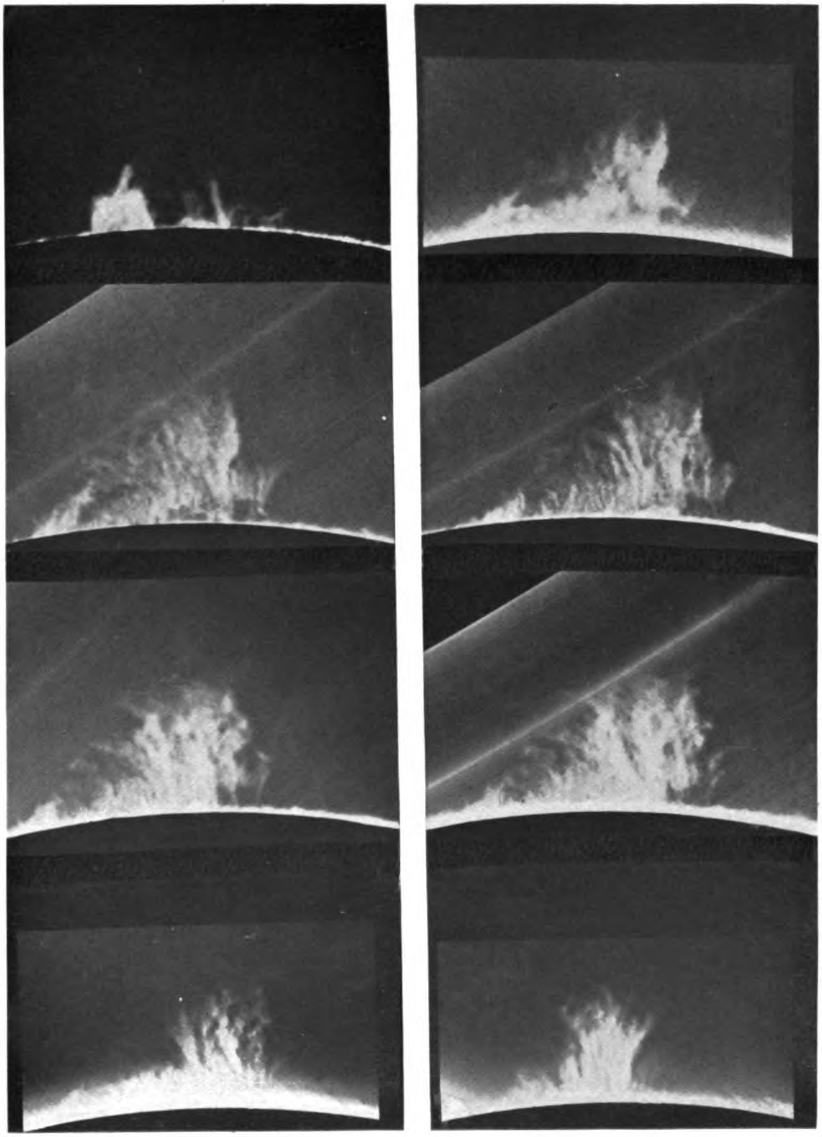
By FREDERICK SLOCUM

A series of photographs of an interesting solar prominence was obtained with the Rumford spectroheliograph on October 10, 1910. Plates XIV and XV show some of the best views. All were taken in the light of the H-line of calcium except Figs. 2, 7, and 8, which were photographed in the $H\alpha$-line by Mr. Ellerman at the Mount Wilson Solar Observatory, and very kindly sent to me by him for comparison with my calcium images.

The prominence was first noticed on the plates of October 8, the first exposure of that date showing a bright mass 52,000 km high, extending from position angle 121°5 to 125°5, and a group of jets and filaments between 114° and 121°, rising to the same height (Fig. 1, Plate XIV). Upon another plate, exposed at $5^h\,59^m$ G.M.T., the mass had assumed the form of a cone, and the filament at 119° had increased to 70,000 km in height, curving over toward the apex of the cone.

No observations were made on October 9, as the electric power is not available during the daytime on Sundays, but the first plate of October 10 showed a great development (Fig. 3, Plate XIV). The prominence extended from 114° to 130°, that is, from latitude —24° to —40°, and reached a height of 105,000 km. The appearance is that of streams of gas rising from a great number of points on the surface, scattered over an area about 240,000 km in extent along a meridian. After ascending to some height most of the longer streams are inclined toward the south, while a few of the longer and most of the shorter streams are bent in the opposite direction. This may be due to horizontal currents in opposite directions at different levels, or the appearance may be produced by a local vortex. If the lower streams are in the background, as they seem to be, the direction of rotation would be clockwise, as seen from above, corresponding to the direction of terrestrial cyclones in the southern hemisphere.

PLATE XIV



1
Oct. 8
$5^{h}59^{m}$G.M.T.
Ca

2
Oct. 10
$2^{h}\ 50^{m}$
Hα

3
Oct. 10
$2^{h}\ 58^{m}3$
Ca

4
Oct. 10
$4^{h}\ 28^{m}2$
Ca

5
Oct. 10
$5^{h}\ 18^{m}7$
Ca

6
Oct. 10
$5^{h}\ 35^{m}1$
Ca

7
Oct. 10
$6^{h}\ 10^{m}4$
Hα

8
Oct. 10
$12^{h}\ 0^{m}$
Hα

Calcium and Hydrogen Spectroheliograms of Solar Prominences on October 8 and 10, 1910
Scale: Sun's Diameter = 240mm

Mr. Ellerman's hydrogen image (Fig. 2, Plate XIV) was taken only eight minutes earlier than my calcium image (Fig. 3, Plate XIV). The general outline and dimensions of the two are approximately the same, but the details are quite different. The hydrogen prominence shows none of the filamentous structure which is the most conspicuous feature of the calcium image. This is not a general characteristic of hydrogen prominences, however, for Figs. 7 and 8, Plate XIV, show the filamentous structure to a very marked degree.

Between 2^h 58^m and 5^h 35^m G.M.T., ten calcium photographs of the prominence were obtained, three of which are reproduced in Figs. 4, 5, and 6, Plate XIV. No great change occurred during this interval, but many variations in the smaller details may be noted on all of the plates, even though some of them are separated by an interval of only 90 seconds. This shows the necessity of using the shortest possible exposure-time in prominence photography. The length of exposure for the calcium images in Plates XIV and XV, that is, the time required for the image of this prominence to pass across the first slit, was approximately ten seconds.

Some time after noon, 6^h G.M.T., the prominence became very active, but unfortunately not enough plates were secured, either here or at Mount Wilson, to trace all of the changes. From the Mount Wilson hydrogen photographs at 6^h and 12^h, G.M.T. (Figs. 7 and 8, Plate XIV), one might infer a continuous gradual decline of the prominence, but calcium photographs taken in the interval between these two show interesting activity. Six photographs were secured between 7^h 53^m and 8^h 6^m G.M.T., two of which are shown in Plate XV on a slightly larger scale than the figures of Plate XIV. The most interesting features of the prominence at this time were the central arch, the funnel beside it, and the long plume curving off to the right.

The arch was apparently decreasing in height at the rate of 23 km per second. Whether this formation is really an arch in the bright structure or whether it is a cloud of relatively dark vapor projected against the brighter prominence, it is impossible to decide. The sharp edges favor the latter alternative, while the faint structure seen under the arch may indicate the former.

A similar doubt exists in my mind in regard to the explanation of the round black dots which appear in both photographs of Plate XV. They may be either holes or absorption effects.

The change from the V-shape to the U-shape in the formation to the right of the arch in Plate XV is shown by intermediate plates to have taken place progressively. It presents the appearance of a vortex somewhat beyond the main part of the prominence, but I have no proof of vortical motion.

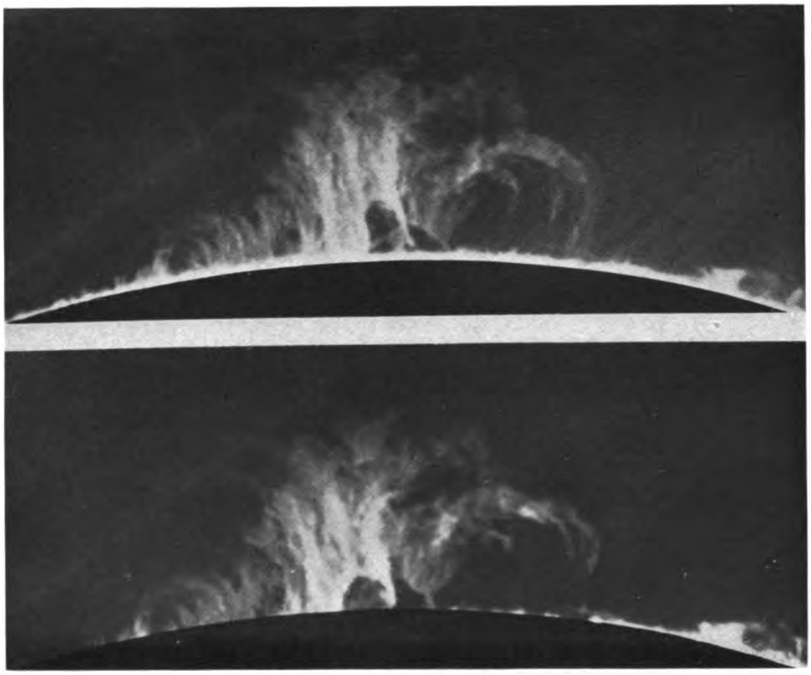
The plume evidently has its origin in the upper part of this vortex. If its form is due to the motion of particles, these particles must be attracted by the sun. This attraction may be electrical or may be purely gravitational, implying that the matter of which the plume is composed is heavier than the atmosphere in which it floats. In either case the plume differs from the main part of the prominence, in which the particles appear to be repelled, or to be lighter than the surrounding atmosphere, so that they rise like the flames from a terrestrial fire.

SPECTRUM OF THE PROMINENCE

At 5^h G.M.T. I obtained a series of photographs of the H and K region of the spectrum of the prominence. The second slit of the spectroheliograph was opened to its full width, 8 mm = 72 A, and exposures were made with the first slit across the prominence in an east-and-west direction. Between exposures the image of the prominence was moved 1 mm = $5.''3$ in declination. In this way twenty exposures were made.

The bright $H\epsilon$ line can be traced to a height of $10''$ in all of the spectra. The H_2 and K_2 lines of calcium at the limb show the normal chromospheric appearance with maximum widths of 1.03 and 1.31 A respectively. Beyond the limb the lines taper down for a distance of $15''$ to $18''$ and then maintain an approximately uniform width of 0.92 A for the H_2 line and 0.95 A for the K_2 line out to the limit of the prominence. In the chromosphere the K_2 line rises to a somewhat greater height than the H_2 line, but in the prominence they are equal in length. In intensity the K_2 line is stronger than the H_2 line, but both vary greatly in different

PLATE XV



G.M.T

$7^{h}\,56^{m}.8$

$8^{h}\,6^{m}.4$

Calcium Spectroheliograms of Solar Prominence on October 10, 1910
Scale: Sun's Diameter = 340mm

parts of the prominence, the variation apparently being somewhat greater in the H than in the K line.

Measurements of the calcium lines with respect to the neighboring metallic lines of the sky spectrum were made. The lines of the latter are weak in comparison with the emission lines of the prominence and the results are not conclusive, but they seem to indicate that there is no appreciable displacement of the H_2 and K_2 lines as a whole, but that local displacements up to ±0.15 A occur in both lines.

The absorption lines H_3 and K_3 appear fine and sharp at the sun's limb and are displaced 0.13 A toward the blue with respect to the H_2 and K_2 lines. Both the H_3 and K_3 lines can be traced throughout the length of the H_2 and K_2 lines, their average widths being 0.21 A and 0.19 A, respectively; the K_3 line thus appears narrower than the H_3, while the K_2 line is wider than H_2. The absorption lines are not straight, but follow an irregular sinuous course within the emission lines, the local relative displacements amounting to ±0.20 A.

If these displacements are manifestations of the Doppler effect, they correspond to velocities of about 15 km per second in the line of sight. Such velocities would be of the same order as the transverse movements shown by the spectroheliograms; but if the vapor which produces the absorption lines exists only at a high level, as has generally been assumed, the movements indicated, at least those which cause the displacements near the limb of the sun, must be entirely independent of the prominence.

If, for example, we assume that the absorbing vapor is at a height of 2′ or about 43,000 km, and that the prominence is exactly at the limb, the material producing the H_3 and K_3 lines will be, for different parts of the prominence, from 90,000 km to 170,000 km nearer to the observer than the bright vapor which produces the H_2 and K_2 lines. For an absorbing stratum at a height of 3′ the figures would be 215,000 km for lines formed at the northern limit of the prominence and 241,000 km for those at the southern limit.

It is not at all certain, however, that motion in the line of sight is the only cause that will explain the displacements in this case.

W. Michelson[1] and A. Cotton[2] have shown that changes either in density or refractive power, or transverse movements of any transparent medium through which light passes, will produce displacements in the lines due to the source of light, and it is possible that some such process may be concerned in producing the irregular relative displacements of the emission and absorption lines of calcium.

On October 11 the site of the prominence was occupied by a group of prominences about 40″ in height, extending continuously from position-angle 92° to 134°.

YERKES OBSERVATORY
May 22, 1911

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[1] "On Doppler's Principle," *Astrophysical Journal*, 13, 192, 1901.

[2] "On Doppler's Principle in Connection with the Study of Radial Velocities on the Sun," *ibid.*, 33, 375, 1911.

THE WIDENING OF THE HYDROGEN LINES BY HIGH PRESSURES

By R. ROSSI

It has been known for some years that when an electric arc burns between electrodes of certain metals in an atmosphere of hydrogen, and when the light is examined with a spectroscope, the hydrogen lines are brilliantly seen in addition to the metallic spectrum.[1] The lines are rather broad, with diffused edges, which shade off gradually; only the first four or five members of the hydrogen series can be noticed; the more refrangible ones being probably too wide and too faint to impress the photographic plate.

In the course of some experiments on the effect of pressure on metallic arc spectra in an atmosphere of hydrogen[2] it was noticed that the lines of that gas were much broadened by high pressure, sometimes to the extent of some hundreds Ångström units.

The following work is a rough attempt to find the rate of widening with pressure. The source of light was a copper arc burning in hydrogen at pressures ranging from $\frac{1}{4}$ to 26 atmospheres (absolute). Observations were confined to the more intense lines, viz., $H\alpha$, $H\beta$, $H\gamma$, and $H\delta$. Owing to the haziness of the lines a small-dispersion spectrograph was chosen, the distance on the photographic plate from $H\alpha$ to $H\delta$ being only 1.6 cm.

The widths were measured with a Kayser measuring machine, the middle of the shadings on each side of a line being taken as edges.

A difference in the time of exposure or development will give different values for the widths of the lines, if measured in this way, owing to changes of intensity on the plate. To overcome this difficulty in part, at least, several photographs were taken at each pressure, each with a different exposure; and the measurements were then taken on a set of plates on which the average intensity of the copper lines had remained the same. At low pressures, when the lines were still comparatively narrow, their width was estimated from the widths of the copper lines at atmospheric pressure.

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[1] Crew and Basquin, *Proc. Amer. Acad.*, **33**, 18, 1898; and Basquin, "The Spectrum of Hydrogen Given by the Metallic Arc of Tin, Copper, Silver, etc.," *Astrophysical Journal*, **14**, 1, 1901.

[2] *Phil. Mag.* (6), **21**, 499, 1911.

As the pressure increases, the lines are broadened out, especially the more refrangible ones, which finally melt into one another, thus rendering impossible a measurement of their width at high pressures. They all seem to broaden out symmetrically, with the exception of *Hα*, which broadens out more toward the violet end of the spectrum. This is especially noticeable at the higher pressures. No reversals seem to take place, but the small resolving power of the instrument used may not enable one to notice them. There are no striking changes in the relative intensity of the lines, only a slight enhancement of *Hα*. The accompanying table gives the widths found at the various pressures. These values are plotted in Diagram I, widths as ordinates, pressures as abscissas.

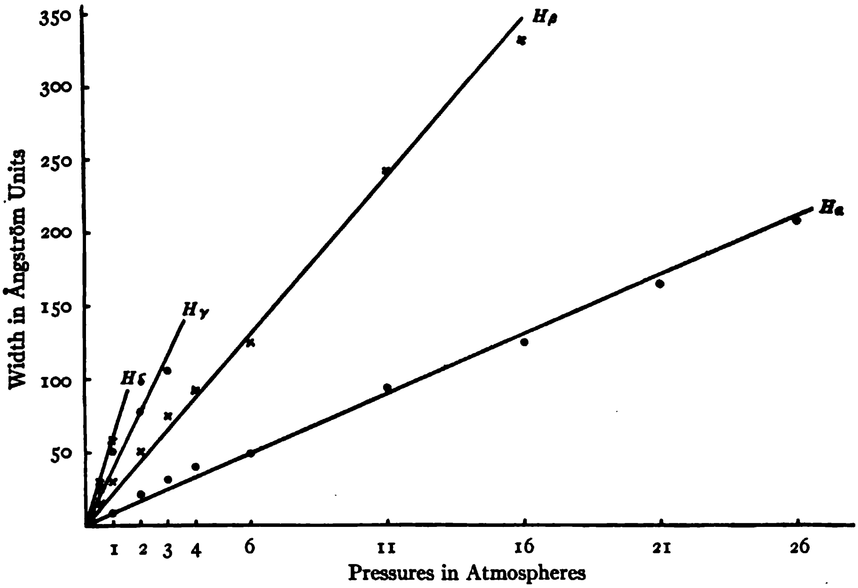

DIAGRAM I

WIDTHS IN ÅNGSTRÖM UNITS†

Pressures in Atmospheres	$\frac{1}{4}$	$\frac{1}{2}$	1	2	3	4	6	11	16	21	26
H_α	..	..	8*	(21)	31	39	48	92	121	163	206
H_β	..	16	30	50	74	91	124	241	(330)	..	..
H_γ	8*	24	50	77	105	..	..	..	..	..	..
H_δ	8*	29	58	..	..	..	..	..	..	..	..

†An asterisk (*) indicates that the width was estimated from the widths of the copper lines at atmospheric pressure. Figures in parentheses denote doubtful readings.

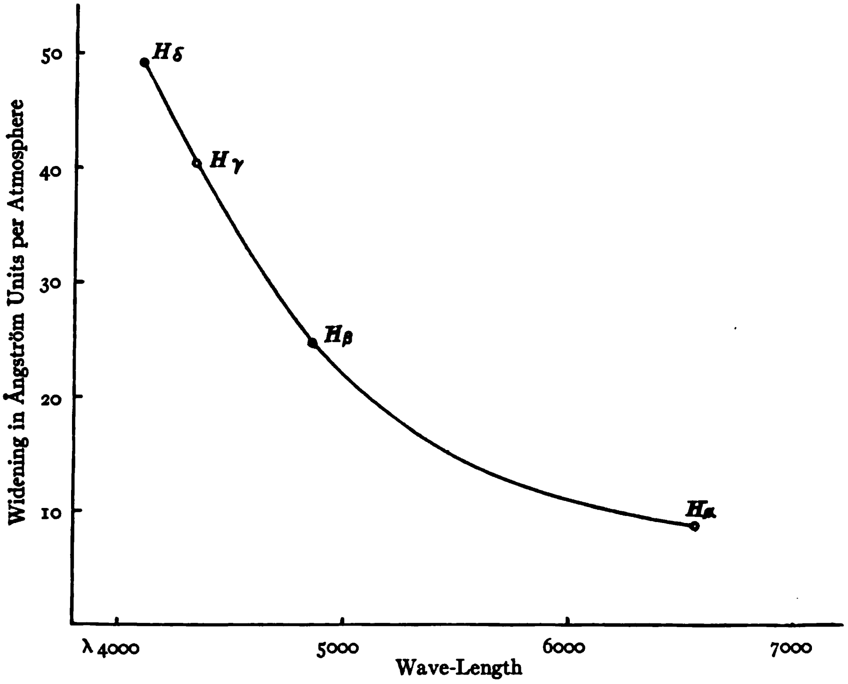

DIAGRAM II

Within the limit of experimental error, the width increases linearly with the pressure; it is interesting to note how this law, found by Michelson for low pressures, in the spectrum of the hydrogen vacuum tube, holds also at these high pressures. It is also seen from Diagram I how the rate of widening with pressure is larger for the more refrangible lines. The mean values of the broadening in Ångström units per atmosphere are respectively:

$H\alpha$	$H\beta$	$H\gamma$	$H\delta$
8.69	24.7	40.4	49.3

Too much weight should not be attached to the last two figures, the measurements for $H\gamma$ and $H\delta$ being taken over too small a range of pressures. These rates of broadening are plotted against wave-lengths in Diagram II. These four values are not strictly comparable, owing to the different sensibility of the photographic plate for different radiations (tending to make the first values smaller, compared with the others) and the slight changes in relative intensity of the lines (tending to make the first value larger). Assuming them to be comparable, we find that for hydrogen the broadening varies as the inverse third or fourth power of the wave-length.

I express my thanks to Professor Rutherford and Professor Schuster for their interest in this research, and for placing the necessary apparatus at my disposal.

PHYSICAL LABORATORIES
MANCHESTER UNIVERSITY
May 1911

THE RADIAL VELOCITY OF *α CYGNI*

By OLIVER J. LEE

When the binary character of *α Cygni* was announced,[1] the suggestion was made that the H and K lines of calcium, which in this star are strong and narrow, deserved special investigation. A series of one-prism plates was made with the Bruce spectrograph during the summer of 1910. Most of these spectrograms were taken on the "Seed 23" emulsion and all were fully exposed in the region from λ 3900 to λ 5000. These plates, with 3 others taken several years ago, in all 19 plates, were measured. The calcium lines did not show any anomalous behavior and the publication of the results has been deferred.

In a paper[2] published last winter Neuimin and Belopolsky find apparently real residuals when the velocities derived from the hydrogen and helium lines are compared with those obtained from the metallic lines. From 11 plates Neuimin finds a mean difference for metallic lines minus $H(\beta,\gamma,\delta)$ of $+9.1\pm1.2$ km, and Belopolsky from 5 of the same plates computes the residual for metallic lines minus $H(\beta, \gamma, \delta, \epsilon)$ and *He* λ 4472 of $+7.1\pm0.9$ km. Recently Mr. Frost called my attention to these papers and suggested that I examine my measures concerning this point.

Although *α Cygni* shows about 70 measurable lines on our one-prism plates, only 14 have been considered in this paper. Of these 10 are metallic lines, mostly enhanced, and so situated in the spectrum that there can be no doubt of their purity. The other 4 are the β, γ, δ, and ε lines of hydrogen. The helium line λ 4472 is too weak and diffuse on our plates to be relied upon and hence has not been included. The accompanying table (p. 304) gives all the usual data for these plates.

In the next table (p. 305) I have followed to some extent the arrangement used in the *Mitteilungen* for the sake of simplifying the comparison of results.

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[1] *Astrophysical Journal*, **31**, 176, 1910.

[2] *Mitteilungen der Nicolai-Hauptsternwarte zu Pulkowo*, No. 38, 1911.

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	Quality of Plate
					km	
IB 1074.........	1907 June 1	21^h39^m	Fox	9	− 2.5	v.g.
1075.........	June 1	21 53	Fox	13	− 3.7	v.g.
2177.........	1909 Nov. 8	15 18	L.	12	−10.9	v.g.
2342.........	1910 May 20	21 38	L.	12	−10.4	v.g.
2347.........	May 23	20 32	B.	12	− 8.7	Overexp.
2360.........	May 30	21 30	B.	14	− 6.4	v.g.
2369.........	June 6	21 45	M.	11	− 3.8	g.
2376.........	June 10	21 15	L.	11	− 8.3	g.
2409.........	June 27	21 47	M.	12	− 5.4	Overexp.
2418.........	July 4	19 42	L.	12	− 4.5	v.g.
2444.........	July 25	16 12	L.	10	− 4.7	v.g.
2450.........	July 29	16 19	B.	14	− 9.1	v.g.
2459.........	Aug. 1	17 17	L.	11	−11.6	v.g.
2487.........	Aug. 19	15 08	B.	12	− 5.2	v.g.
2490.........	Aug. 25	15 54	B.	14	− 1.9	v.g.
2497.........	Aug. 27	14 39	L.	14	− 7.3	v.g.
2500.........	Sept. 5	14 21	B.	11	− 7.6	v.g.
2504.........	Sept. 19	13 00	L.	14	− 9.3	v.g.
2750.........	1911 June 29	19 53	L.	13	+ 2.8	v.g.

B = Barrett, L = Lee, M = S. A. Mitchell.

The average magnitude of the differences given in the line Metal—Hydrogen is +2.32 km. A glance at the range of the velocities given by the hydrogen lines on each plate shows this difference to be well within the limit of observational error.

The average of the probable errors for the mean of the metallic lines is ±1.01 km. The corresponding average for the hydrogen lines is about twice as large when computed and probably 3 to 4 times as large in reality.

The differences of the mean of all the lines measured on a plate from the mean of the lines representing each element were averaged for the 19 plates. They are:

Mean —*Fe*	−1.04 km
Mean —*Ca*	−0.09 km
Mean —*Mg*	−2.09 km
Mean —*Ti*	−0.52 km
Mean —*Cr*	−0.92 km
Mean —*H*	+1.42 km

These residuals cannot be regarded as anything but accidental errors.

A discrepancy is found by Neuimin in the wave-length of the *Mg-Cr* blend at λ 4352 and is corrected by him by assuming

RADIAL VELOCITY OF *a CYGNI* AS DERIVED FROM DIFFERENT ELEMENTS

Elem.	Wave-Length	1074	1075	2177	2342	2347	2360	2369	2376	2409	2418	2444	2450	2459	2487	2490	2497	2500	2504	2750
		km	km	km	km	km	km	km	km	km	km	km	km	km	km	km	km	km	km	km
	4045.935	−16	−13	+ 5	−28	....	−24	....	....	−10	−14	....	− 8	....	− 7	− 7	− 8	− 6	− 9	− 4
Fe	4508.455	−26	−31	+ 1	−31	−21	−19	−20	−30	−22	−16	− 4	−20	−14	− 4	+ 2	− 5	− 3	− 2	− 5
	4515.508	....	−24	+ 3	−14	−19	−16	−13	−17	....	−18	− 5	−18	−13	− 9	+ 5	− 9	− 4	+ 6	−13
Ca	K 3933.825	−14	−12	+11	−22	−20	−19	−26	−20	−16	−14	−13	−13	−15	− 7	− 7	− 6	− 7	− 3	−14
	H 3968.625	−16	−10	− 6	−23	−24	−23	−20	−24	−20	−19	−15	−16	−15	− 8	− 6	− 9	− 6	0	−12
Mg	4481.400	−14	−10	+ 9	−25	−21	−21	−20	−24	−15	−14	−12	−18	−11	− 5	+ 4	−10	0	− 6	− 9
Ti	4501.445	−22	−24	....	−19	−24	−18	....	−22	−19	−16	....	−17	....	....	+ 1	− 2	....	−10	−12
	4563.939	....	−28	+10	−25	−25	−22	−15	−27	....	−21	....	−11	....	....	− 7	+ 3	....	+ 6	−12
Cr	4558.827	....	−21	+10	−24	−26	−27	−13	−22	−12	−18	−10	−14	−12	− 5	+ 2	−14	− 3	+ 2	− 6
	4588.381	....	....	....	....	....	−22	....	....	−15	....	....	−21	−24	0	− 4	− 5	− 6	− 3	....
	3970.213	−12	−14	+ 1	−20	−17	−14	−19	−12	−14	− 7	− 7	− 6	−24	− 5	+ 4	− 2	− 3	0	0
H	4101.900	−17	−14	+ 1	−39	−31	−21	−20	−28	−17	−17	−21	−18	−20	− 9	− 3	−11	−11	− 5	−10
	4340.634	−17	−18	− 6	−31	−31	−28	−16	−22	−27	−18	−22	−16	−19	−11	−13	−12	−.9	−11	− 7
	4861.527	....	−19	+ 5	....	....	....	....	....	−27	....	−16	−28	−29	−17	− 3	−11	....	−15	−19
Means of metallic lines...........		−18.1	−19.2	+ 5.4	−23.4	−22.5	−21.1	−18.1	−23.2	−16.1	−16.7	− 9.8	−15.6	−14.9	− 5.6	− 1.7	− 6.5	− 4.4	− 1.9	− 9.7
Means of hydrogen lines...........		−15.3	−16.2	+ 0.2	−30.0	−26.3	−21.0	−18.3	−20.7	−21.2	−14.0	−16.5	−17.0	−23.0	−10.5	− 3.8	− 9.0	− 7.7	− 7.8	− 9.0
Metal−hydrogen...		− 2.8	− 3.0	+ 5.2	+ 6.6	+ 3.8	− 0.1	+ 0.2	− 2.5	+ 5.1	− 2.7	+ 6.7	+ 1.4	+ 8.1	+ 4.9	+ 2.1	+ 2.5	+ 3.3	+ 5.9	− 0.7
Means of all lines..		−17.1	−18.3	+ 3.7	−25.1	−23.5	−21.1	−18.2	−22.5	−17.8	−16.0	−12.5	−16.0	−17.8	− 7.2	− 2.3	− 7.2	− 5.3	− 3.6	− 9.5
Reduction to sun...		+14.64	+14.62	−14.60	+14.74	+14.83	+14.70	+14.40	+14.18	+12.39	+11.51	+ 7.80	+ 6.91	+ 6.15	+ 1.95	+ 0.37	− 0.06	− 2.32	− 5.71	+12.28
Radial velocity....		− 2.5	− 3.7	−10.9	−10.4	− 8.7	− 6.4	− 3.8	− 8.3	− 5.4	− 4.5	− 4.7	− 9.1	−11.6	− 5.2	− 1.9	− 7.3	− 7.6	− 9.3	+ 2.8

intensities of 2 and 5 respectively for the two components instead of 5 and 5 as given by Rowland. There is, however, an iron line at λ 4351.94 which is missing in the arc and rises in the spark to intensity 2 on the scale of 10 for the strongest lines present. Concerning this line Sir Norman Lockyer and Mr. Baxandall say:[1]

> In stars of the solar type the line is probably of a complex origin, *Cr* 4351.93, *Mg* 4352.08, and p *Fe* 4351.93 all being involved. In higher temperatures stars like *a Cygni*, *Sirius*, and *Rigel* there is abundant evidence in favor of a proto-*Fe* origin and little or none for either chromium or magnesium. Thus, other lines of *Cr* and *Mg*, which are similar in intensity and behavior in their respective spectra to those mentioned above, are all unrepresented in these stellar spectra, whereas all the enhanced *Fe* lines of similar intensity and behavior to the line 4351.93 are strongly represented in the same stellar spectra.

The wave-length 4294.273 has been used for the blend of *Ti* 4294.204, intensity 2, and *Fe* 4294.301, intensity 5. A comparison of these lines in the arc and spark[2] shows that while the intensity of the *Ti* line changes from 4 to 6, the *Fe* line is unaffected. Hence the adopted wave-length is probably a little too large. A few of the lines used by Neuimin, for example λλ 4290.377, 4303.337, 4308.081, and 4338.084, are known to have closely neighboring lines that are enhanced at the higher temperatures. It is not probable that the elimination of a few doubtful lines would have materially affected the mean. The precaution has been taken, however, of using chiefly enhanced metallic lines and in every case lines that are sufficiently isolated from other lines to be unaffected by them.

Isolated faint companion lines were seen and measured on several plates, but only on one plate were they consistent enough to be regarded as real. This exception is No. 2360, which shows a satellite at the lines γ, δ, ϵ, and *Mg* 4481 about 0.6 Å. U. toward the red. A re-examination of our plates was made to see if faint companions to the stronger lines used in measurement could be detected on more of the plates. Could it be shown that such satellite lines occur repeatedly, this might furnish an explanation of the anomaly in velocity suggested by the observations of Neuimin and Belopolsky, for obviously a faint line lying close on the edge of a heavier line will vitiate measures of the center of this line. Several pairs of plates, comparable in point of density and grain-structure, have hydrogen lines which differ slightly in width and in sharpness of the edges.

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[1] *Proc. R.S.*, **74**, 260, 1904.

[2] *Astrophysical Journal*, **19**, 325, 330, 1904.

The following conclusions may be drawn:

1. The present observations do not confirm the conclusion reached by Neuimin and Belopolsky and the mean residual from 19 plates of +2.32 km cannot safely be interpreted as real.

2. The discrepancy found by them may be due to the complexity in the hydrogen lines as indicated.

YERKES OBSERVATORY
August 1911

RESIDUAL RAYS OF SELENITE

BY H. M. RANDALL

In a recent paper J. Koch[1] has reported the results of a new determination of the wave-length of the residual rays ("Reststrahlen") from selenite. The method makes use of a Jamin interferometer by means of which a direct comparison is made between corresponding numbers of interference bands due to the "Reststrahlen" and to the sodium line $\lambda = 0.58948\ \mu$ (*vacuo*). The following results were obtained:

(1) $\lambda = 8.6795\,\mu$ (*vacuo*) (mean of 10)
(2) $\lambda = 8.6770\,\mu$ (*vacuo*) (mean of 5)
(3) $\lambda = 8.6789\,\mu$ (*vacuo*) (mean of 5)

Mean $\lambda = 8.6787\,\mu$ (*vacuo*)

The relatively close agreement of the various values is interesting and raises the question as to whether other methods will yield the same result; that is, whether the "Reststrahlen" from selenite may be employed as a standard wave-length in the infrared region. Energy-curves of this radiation have been obtained accordingly by using both the grating and the prism method. As these methods are familiar, the experimental details will be sufficiently indicated by the accompanying diagram, Fig. 1. S, the source of the radiation, is a 220-volt Nernst lamp, whose image is focused by the silvered mirror M upon the slit S', of the mirror spectrometer after three reflections at the polished selenite surfaces g, the angle of incidence being about 45°. M_1 and M_2 are silvered mirrors 10 cm in diameter and of 50 cm focal length, while L is a plane Rowland grating with 40 lines per mm. The spectrometer was made by Fuess and has been previously described.[2] The linear thermopile T was of the Paschen type[3] and is used in connection with a Paschen galvanometer, with which a deflection of 1 mm corresponded to a current of 0.7×10^{-10} amperes, the scale being approximately 2.5 m from the galvanometer. The deflections of the galvanometer were proportional to the current throughout the length of the scale. The slits S and S' were kept equal and gradually increased in width till sufficient radiation passed through to give satisfactory deflections of the galvanometer. A relatively wide slit, 1.7 mm, was found necessary. This corresponds to a spectral region of about 800 Å.U. The wave-length, λ, is given by the equation ($\lambda = C \sin \theta$), C being determined by using the *He* line λ 5875.870 in the 30th and 40th orders. C was found to be 504817 Å.U.

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[1] *Annalen der Physik*, **26**, 974, 1908.

[2] E. Giesing, *Annalen der Physik*, **22**, 333, 1907.

[3] F. Paschen, *ibid.*, **33**, 736, 1910.

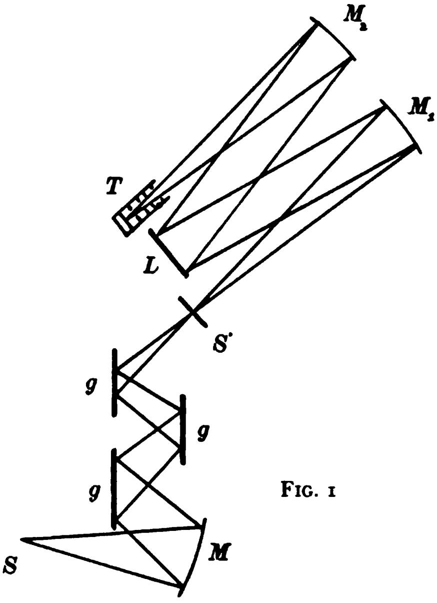

FIG. 1

Fig. 2 represents the energy-curves to the right and left of the undeviated image, the abscissas are the spectrometer readings and the ordinates the galvanometer-throws when the shutter before the slit S is removed. The wave-length corresponding to the center of mass of the triangles is $\lambda = 86940$ Å.U. (*vacuo*). A second independent determination gave $\lambda = 86950$. These values are in accord with that of Aschkinass[1] which was 8.69 μ.

The grating was then replaced by a fluorite prism, and the slits were narrowed to 0.28 mm. In this case the slit-width corresponded approximately to 180 Å.U. The curves of Fig. 3 were obtained with this arrangement. The observed throws of the galvanometer are indicated by the points of the lower curve. These have been increased by multiplying by the proper factor to compensate for the absorption of the fluorite,[1] which is large in this region, and the upper curve represents the values which would have been obtained had there been no absorption. This curve gives $\lambda = 86785$ Å.U. (*vacuo*) for the wave-length corresponding to the center of mass of the triangle. This is also quite accurately the value of the wave-length corresponding to the maximum of the curve. It will be noticed that this result is in close agreement with those of Koch, indicating the correctness of the determinations of Paschen on the dispersion and absorption[2] of fluorite in this region. While the value obtained by the grating is perceptibly larger than that given by the prism, due possibly to the alterations in the energy-distribution of the spectrum which a grating may introduce, it is interesting to observe that the wave-lengths obtained from those parts of the bands carrying the maximum energy are nearly the same in the two cases, being in fact also in close agreement with the values of Koch. Thus, if the values of λ are obtained from the intersections of the lines AB with the curves, the following results are obtained:

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[1] *Annalen der Physik*, 1, 42, 1900.

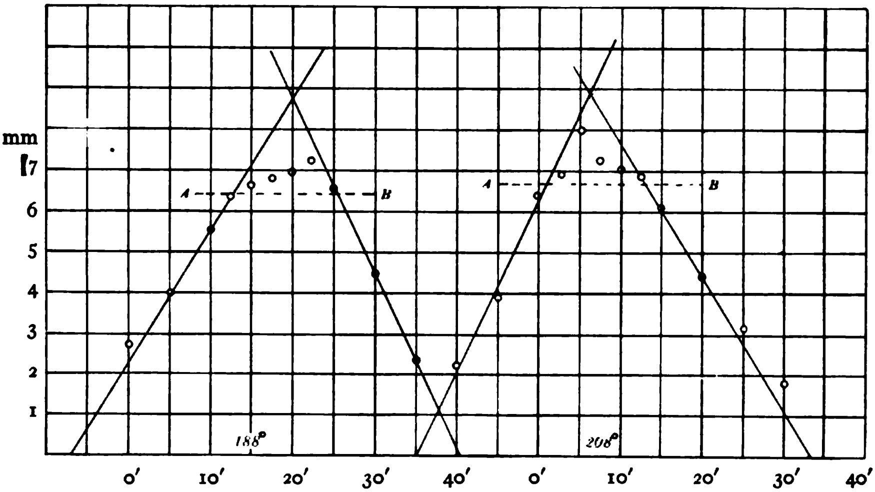

FIG. 2

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[1] F. Paschen, *Sitzungsberichte der K. preuss. Akademie der Wissenschaften*, 1, 417, 1899.

[2] F. Paschen, *Annalen der Physik*, 53, 301, 812, 1894.

Grating $\lambda=86793$ Å.U. (*vacuo*)
Prism $\lambda=86770$ Å.U. (*vacuo*)
while from Koch $\lambda=86787$ Å.U. (*vacuo*).

It is to be noted also that the approximate breadth of the band in the case of the prism is 0.5 μ and in the case of the grating 0.6 μ, values which are in substantial agreement with the value 0.5 μ obtained by Koch from visibility-curves.

While the values from the energy-curves by either of the methods here used may be repeated under definite conditions with variations not exceeding 20 Å.U., still the fact that the different methods give appreciably different results and also the fact that the band seems to have a rather broad maximum make it unsuitable for a standard of wave-length. In fact, the form of this somewhat broad maximum suggests the possibility of a double maximum for the residual rays of selenite in this region. The data do not warrant, however, any positive conclusions in this respect.

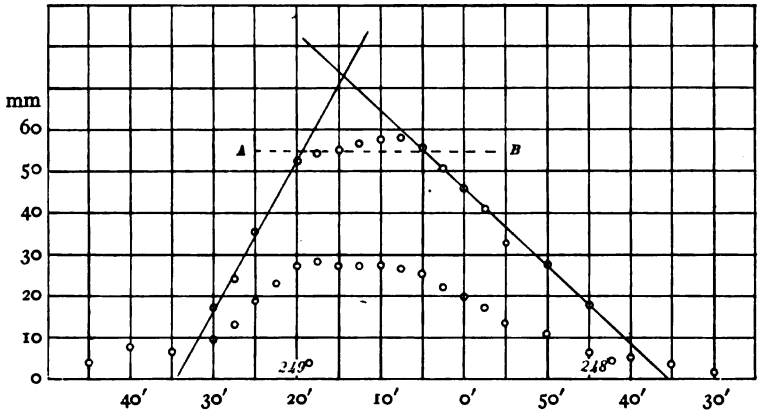

FIG. 3

These data were obtained in the Physical Laboratory of the University of Tübingen and the writer wishes to express here his deep obligations to its director, Professor F. Paschen.

PHYSICAL LABORATORY
UNIVERSITY OF MICHIGAN
August 1911

NOTE ON THE ZEEMAN EFFECT FOR CHROMIUM

BY J. E. PURVIS

In the *Astrophysical Journal* for April 1911 (33, 217) a description is given by H. D. Babcock of the measurements of the constituents of a series of chromium lines in the visible regions of the spectrum. He also gives a table comparing his values of $\frac{d\lambda}{\lambda^2} \cdot 10^8$ for four chromium lines with those measured by myself,[1] by W. Hartmann,[2] and by W. Miller.[3]

The differences are considerable, and I have remeasured the separate constituents of the four cited lines which were photographed on another plate about the same time as those which were measured and quoted in my earlier paper. I have reduced the determinations to a field-strength of 23,850 C.G.S. units so as to compare the numbers with the others, and the results of a comparison of the triplets are given in the following table:

COMPARISON OF SEPARATIONS OF TRIPLETS

$\frac{d\lambda}{\lambda^2} \cdot 10^8$ for 23,850 C.G.S. Units

λ	Purvis. Earlier Measurements	Purvis. Later Measurements	Miller	Hartmann	Babcock
4646.39	+0.93 0 −0.93	+0.92 0 −0.92	+1.31 0 −1.31	+1.21 0 −1.21	+1.42 0 −1.42
4558.90.....	+0.92 0 −0.92	+0.94 0 −0.94		+1.19 0 −1.19	+1.29 0 −1.29
4351.97.....	+0.94 0 −0.94	+0.95 0 −0.95	+1.29 0 −1.29	+1.18 0 −1.18	+1.31 0 −1.32
4344.70.....	+0.89 0 −0.98	+0.92 0 −0.92	+1.20 0 −1.20	+1.11 0 −1.11	+1.25 0 −1.25

4344.7 is a fairly weak line; and the constituents are not easy to measure.

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[1] *Proc. Camb. Phil. Soc.*, **14**, I, 41–84, 1906.

[2] Das Zeeman-Phaenomenon im sichtbaren Spektrum von Kupfer, Eisen, Gold, und Chromium. (Dissertation, Halle, 1907.)

[3] *Annalen der Physik* (4), **24**, 105–136, 1907.

It is difficult to explain the differences of the various investigators, although a probable cause may come from impurities, like iron, which would interfere with the energy of the field, both by absorption and by the attraction of small particles to the poles of the magnet. The experimental error from the strength of the field would be less in my observations than in the others; the field-strength I used was 39,980 C.G.S. units, and there can be no doubt of its accuracy; the maximum field-strength used by Miller was 32,300 units, that by Hartmann was 23,850 units and that used by Babcock was 23,400 units. The higher value of the field-strength is important in these close measurements; for, of course, the stronger the field, the more widely separated the constituents will be, with the consequent more exact measurements. Both Miller and myself used a Rowland large concave grating; Babcock, a plane grating; and Hartmann, an echelon instrument.

But whatever is the cause of the differences, it is desirable to place on record a comparison of the measurements of the separate constituents of the four lines on two different photographic plates taken at about the same time and under similar conditions, in order to indicate that the discrepancy does not come from very faulty measurements.

Chemical Laboratory
Cambridge, England

MINOR CONTRIBUTIONS AND NOTES

WILLIAMINA PATON FLEMING

Williamina Paton Fleming was born in Dundee, Scotland, on May 15, 1857. Her parents were Robert and Mary (Walker) Stevens. Her education was obtained in the public schools of Dundee, where, at the age of fourteen, she was one of the pupil-teachers which were a part of the training used there at that time. After her marriage in early womanhood to James Orr Fleming, they came to this country and settled in Boston. In 1881, Mrs. Fleming, finding it necessary to support herself, entered Harvard College Observatory as a copyist and computer. It was on the eve of the development of a new and remarkable method of attacking astronomical problems; for celestial photography, which had been experimented with for several years, was about to be seriously undertaken at Harvard by the director of the observatory, Professor Edward C. Pickering. By placing a prism in front of the object-glass of the telescope, the spectra of all the brighter stars were obtained. Thus an almost untrodden field lay open for the investigator, a field that was destined to produce a vast amount of material for the astronomical discussions of the future. As the work progressed, Mrs. Fleming was put in charge of the original examination of the photographic plates, their care and storage, and the classification of objects found upon them. Gifted with great keenness of vision and a clear and logical mind, she at once gave evidence of ability for the work. Each photographic plate may be likened to the only existing copy of a valuable book, and, being very fragile, must be safely stored, and at the same time must be accessible, so as to be consulted readily at any moment. When a sufficient amount of material was at hand, the first general photographic classification of stellar spectra was undertaken, and was assigned to Mrs. Fleming. The results are published in the "Draper Catalogue," which forms Vols. XXVI and XXVII

Yours very truly,
W. P. Fleming

of the Harvard *Annals*, and gives the spectra of 10,351 stars, with measures of their photographic light. She observed, in classifying these spectra, that while a large proportion of them fall in classes resembling a few typical stars, there are also many that are peculiar. By the presence of bright lines in their spectra, she thus discovered ten Novae, and more than three hundred stars that vary in light. In the early days, when celestial photographs were rare, and some of these discoveries were attributed by skeptics to defects on the film, she never doubted the validity of the photographic evidence. Her industry was combined with great courage and independence.

She also found fifty-nine gaseous nebulae, ninety-one stars of the fifth type, Class O, and sixty-nine stars of the Orion type having bright hydrogen lines. She became interested in the red stars, whose spectra are of the fourth type, Class N, and appear very short on photographs taken with the objective-prism. Other spectra which resemble those of Class N, in the region between $H\beta$ and $H\gamma$, but which contain as much blue light as stars of the second type, she assigned to a new type called sixth type, or Class R. Just before her life-work was suddenly ended, she was engaged upon a portion of the *Annals*, called "Peculiar Spectra," which gives tables of all these objects with descriptions of their peculiarities. This is nearly all in print, and it is hoped it will soon be finished.

In a "Photographic Study of Variable Stars," Vol. XLVII of the *Annals*, Mrs. Fleming gave the results of her laborious work in measuring the position and magnitude of sequences for observing two hundred and twenty-two of the variable stars she discovered. Most of these variables were found by the presence of bright hydrogen lines, traversing a banded spectrum of Class M. This combination of lines and bands, which she called Md, was always an object of interest to her, and was considered as a certain evidence of the star's variability. She gave much attention to this class of spectrum, and divided it into eleven subclasses, depending upon the relative brightness of the hydrogen lines and the position of the maximum intensity of the continuous spectrum. A large part of her time was spent upon the preparation of the numerous volumes of the Harvard *Annals* for the printer, and her power of grasping facts quickly and clearly was useful in final readings and corrections of copy and proof.

In 1899, Mrs. Fleming's work was recognized by the Harvard Corporation, and she was given an official position on the observatory staff, with the title of Curator of Astronomical Photographs. Successive honors followed. She was made an honorary member of the Royal Astronomical Society of London in 1906, and soon after, she was appointed honorary fellow in astronomy of Wellesley College. Only a few months before her death, the Astronomical Society of Mexico presented her with the Guadalupe Almendaro medal for her discovery of new stars. She was a member of the Astronomical and Astrophysical Society of America and of the Société astronomique de France.

Mrs. Fleming's health had been impaired for several years, but her great natural vitality and courageous spirit had sustained her through trying illnesses. Early in May she was evidently not so well as usual, and some days later, she went to a hospital for rest. Her condition was soon found to be critical, and pneumonia developed, which proved fatal on May 21, 1911. She left one son, Edward P. Fleming, who graduated as a mining engineer from the Massachusetts Institute of Technology in 1901, and is now chief metallurgist for a large copper company in Chile.

Mrs. Fleming was possessed of an extremely magnetic personality and an attractive countenance, enlivened by remarkably bright eyes. Although most of her life was spent in the routine of science, yet her human interests were numerous. Fond of people and excitement, there was no more enthusiastic spectator in the stadium for the football games, no more ardent champion of the Harvard eleven. Industrious by nature, she was seldom idle, and long years of observatory work never unfitted her for the domestic side of life. As much at home with the needle as with the magnifying eyepiece, she could make a dainty bag, exquisitely sewed, or dress a doll in complete Scotch Highland costume. She was never too tired to welcome her friends at her home or at the observatory, with that quality of human sympathy which is sometimes lacking among women engaged in scientific pursuits. Her bright face, her attractive manner, and her cheery greeting with its charming Scotch accent, will long be remembered by even the most casual visitors to the Harvard College Observatory.

ANNIE J. CANNON

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CONCERNING THE ECHELETTE GRATING

In a paper in the May number of this *Journal* (**33**, 338, 1911), which has just come to my notice, Messrs. Anderson and Sparrow make two criticisms of a paper by R. W. Wood and myself which has appeared in the *Philosophical Magazine* (6), **20**, 886, 1910, and the *Physikalische Zeitschrift*, **11**, 1161, 1910. As it seems to me that Messrs. Anderson and Sparrow have misunderstood the intended scope of our investigation, I feel bound to reply to their criticisms.

The papers of Anderson and Sparrow and of Trowbridge and Wood both deal with the distribution of energy diffracted from a reflecting grating whose groove-form is supposed to be known, but the scope of the two investigations is very different. Whereas Messrs. Anderson and Sparrow calculate on diffraction principles (assuming certain simple conditions of reflection) the distribution of energy among the various orders, the work of Mr. Wood and myself was primarily experimental and consisted in the measurement of the distribution of the energy diffracted by a number of reflecting gratings of known groove-form.

In our analysis of our experimental results we at first attempted to secure a quantitative agreement with the theory of diffraction and developed equations for our experimental conditions which were similar to the equations published by Anderson and Sparrow.

We, of course, soon realized that in our unavoidable ignorance of the exact conditions under which reflection was taking place it was utterly out of the question to expect a quantitative agreement as regards energy distribution, but that the simple theory did predict whether or not a given order was strong or weak.

In preparing the paper for publication we preferred to present the matter of strong, weak, or absent spectra in a manner similar to that in which the subject has already been treated by Schuster. It is to this treatment of the subject that Messrs. Anderson and Sparrow object, stating that "such reasoning ignores the process of diffraction altogether and is in our opinion not valid."

We ourselves drew attention to this circumstance in our paper, a fact which appears to have been overlooked by our critics. Referring to our paper we find the objection raised by Messrs. Anderson and Sparrow expressed as follows:

> We shall in the present treatment discuss the results by the Fresnel method, considering the interference between secondary wavelets originating on the surface of the wave-fronts reflected from the oblique edges of the grooves. As Lord Rayleigh has pointed out, this method holds only when the width of the groove considerably exceeds the wave-length of light. In the present case, with our closest ruling, the groove-width was 1.5 times the wave-length of our longest waves, and it appears probable that in this case we are very near, if not beyond, the point at which we may safely employ the Fresnel treatment.

Further on we say:

> It is very questionable whether the upper wave-front 4, 3, 2, 1 will behave as a plane wave, i.e., travel out without diffraction; for each one of the elements of which it is composed has had to travel one or more wave-lengths before uniting with its neighbor.

The object of the investigation by Mr. Wood and myself was largely utilitarian in that we wished to determine the necessary conditions for obtaining intense grating spectra, and we have found that Schuster's treatment of the problem is qualitatively in accord with experiment; further that a treatment similar to that of Messrs. Anderson and Sparrow is also qualitatively but not quantitatively in accord.

The second criticism of Messrs. Anderson and Sparrow is that the energy distribution in grating spectra obtained by keeping the collimators fixed and by rotating the grating is not a "true" energy distribution.

We also drew attention to this in the following words:

> In order to make a thoroughly satisfactory study of the distribution of the energy it would be advisable to keep the angle of incidence fixed (for example, normal), and to swing the bolometer through the spectra. With the instrument at our disposal this was impossible, and it was necessary to make the spectra pass across the bolometer by rotating the grating. This complicates the

discussion of the results in no small degree, for the energy distribution varies with the angle of incidence, as can be seen with an ordinary grating.

However, there is this practical advantage to be gained in our method of observation: The angle between the collimators enters as a parameter in the expression for the retardation over a single element of the grating and so by varying this parameter the region in which concentration takes place may be varied, thus securing a wider range of wave-lengths in which the energy may be localized.

There is nothing in the paper by Messrs. Anderson and Sparrow to indicate that either of the difficulties which they raise had been foreseen by us.

A. TROWBRIDGE

PRINCETON UNIVERSITY
September 6, 1911

REVIEWS

Mathematical and Physical Papers of Lord Kelvin. Vol. V. Edited by SIR JOSEPH LARMOR. London: The Cambridge University Press, 1911. 8vo, pp. 602. 18*s*.

This collection contains five groups of the less important papers of Lord Kelvin and forms, not the last, but next to the last, volume in the series of his Mathematical and Physical Papers. For the student who is seeking a first-hand account of those great discoveries and principles which must always be associated with the name of Kelvin this volume will be of minor interest, for the reason that these epoch-making papers have already been reprinted in the preceding volumes; but for him who is interested mainly in the historical development of these principles the work under review possesses great value.

The first 134 pages, which have to do with thermodynamic problems, will be read mainly by the physical chemist and the physicist. As might be expected, this section covers a wide range in both time and topic, including a paper on "Stirling's Air-Engine," dated 1847, and another on the "Animal Thermostat," published fifty-six years later. This latter paper deals with an extremely difficult and attractive physiological problem, namely: How does an animal automatically keep its temperature down to blood-heat even when surrounded by a saturated atmosphere at a temperature higher than blood-heat?

Another paper, dated 1853, deals with the thermodynamic heating of buidings, and clearly anticipates methods which are only just now being employed to cool some of our best constructed houses.

Paper No. 119, although more than a quarter of a century old, apparently contains the fundamental idea involved in the theory of logging pipes recently worked out and experimentally verified by Professor A. W. Porter, of London. The at first astonishing theorem here proved is that a given amount of clothing will sometimes make a body warmer, sometimes cooler, depending upon the size of the body and the ratio of the thermal conductivity of the clothing to its emissivity.

The next 150 pages are devoted to papers on cosmical and geological physics, dealing largely with the secular cooling of the earth and with the sun's heat. Here is to be found Kelvin's presidential address before the Geological Society of Glasgow in which he shows a clear conception of the variation of latitude certainly as early as—and probably much earlier than—1874, long before its actual discovery. As illustrating the author's scientific instinct and sense of caution we may cite the closing sentence of his B.A. address in 1861 on the "Possible Age of the Sun's Heat," a statement which clearly leaves room for the discovery of radium some forty years later: "As for the future, we may say with equal certainty that inhabitants of the earth cannot continue to enjoy the light and heat essential to their life for many million years longer, *unless new sources, now unknown to us*, are prepared in the great storehouse of Creation" (p. 144). That the subsequent discovery of very appreciable quantities of radium in the earth's crust has greatly diminished—not the scientific value—but the relative importance of these researches is now well recognized.

Homer Lane's problem—"the problem of convective equilibrium of temperature, pressure, and density in a wholly gaseous spherical fluid mass kept together by mutual gravitation of its parts" (p. 225)—comes in for elaborate discussion of great interest to all students of the solar surface.

The following 70 pages are given to "Molecular and Crystalline Theory," dealing mostly with problems of crystallography. Here may be found one of his best known papers "On the Size of Atoms."

A generation ago the following paragraph taken from this lecture was quoted in nearly every textbook of physics: "To form some conception of the degree of coarse-grainedness indicated by this conclusion, imagine a rain-drop or a globe of glass as large as a pea, to be magnified up to the size of the earth, each constituent molecule being magnified in the same proportion. The magnified structure would be coarser grained than a heap of small shot, but probably less coarse grained than a heap of cricket balls" (p. 296).

The remainder of the volume, some 250 pages, is occupied with electrical problems, largely concerning electrical measurements and questions of engineering. Many of these contributions antedate the divorce of electrical science and electrical engineering and are, for that very reason, full of interest.

It will be remembered that the verdict of the then Sir William Thomson and Professor Joseph Henry concerning the telephone, as exhibited at the Centennial Exposition in 1876, virtually established the success of that instrument. From the same point of view, the author's report on the Gramme machine, also exhibited in America for the first time at the Centennial Exposition, is worthy of notice, especially for its prophetic insight. The last paragraph runs as follows: "En outre de ces relations théoriques tres-intéressantes, les machines Gramme exposées ont des applications pratiques à l'électro-métallurgie, à l'éclairage des manufactures, des phares et des navires, et serviront peut-être aussi, dans l'avenir, à l'éclairage des villes et des maisons d'habitation" (p. 413). In this section will be found the story of his series of ampere-balances, the first satisfactory commercial and absolute instruments for the measurement of electric currents; here also the history of the Kelvin Bridge and the Electrostatic voltmeter.

That this eminent Scotch-Irishman possessed not only great flexibility of mind but also that tenacity of opinion, characteristic of his race, is evinced by his persistent defense of the direct current, as against the alternating, for the transmission of power. The following remark addressed to the Institute of Electrical Engineers in 1907, only a few months before his death, will serve to illustrate both his generosity and his persistence of view. "I am reminded," he says, "of a little prophetic conversational statement made by Lord Rayleigh a good many years ago. He rejoiced to see the use of alternating current coming in: 'Because,' he said, 'the whole world will now learn the subtleties of electrical science, which they had no chance of learning before.' That prophecy has been fulfilled. 'And,' he added, 'after that they will come back to the continuous [direct] current.' I do not know that you will all agree with that anticipation, but you will, I am sure, all enthusiastically agree with Lord Rayleigh in rejoicing that we have had this twenty years of alternating current, and of electrical science in its most interesting characters, including in fact, wireless telegraphy, put before beginners in electrical science as they now are in practical schools of electricity" (p. 595).

Concerning the volume as a whole it will suffice to say that it is made up in the faultless style of The Cambridge University Press, and that it shows at many points the kindly and scholarly editorial supervision of Sir Joseph Larmor.

H. C.

A History of the Theories of Aether and Electricity from the Age of Descartes to the Close of the Nineteenth Century. BY E. T. WHITTAKER, London and New York: Longmans, Green & Co. (Dublin University Press), 1910. Pp. xiv+475. $4.50.

This book, by the accomplished Royal Astronomer of Ireland, from whose pen we have had already a work on analysis and one on dynamics, in no way disappoints the expectations which these previous works would naturally raise. The task of the historian of science is not easy. He must add to a thorough knowledge of the scientific side of the subject the qualities of historian and philosopher. For it is not enough merely to chronicle the steps of progress; it is necessary to see each advance or misstep in its true historical perspective, and to estimate its effect upon the science of the time as well as its bearing upon later developments. That the author has succeeded to a noteworthy degree in these respects will be conceded by all his readers.

The first three chapters deal successively with the beginnings of our knowledge of light, electrostatics and magnetism, and electric currents (galvanism). The progress is sketched in bold outlines—that of light from Descartes to Huygens, that of electrostatics and magnetism from Gilbert of Colchester down to the introduction of the potentials by Green, and that of currents from Galvani to Ohm, including the chemical theories of Davy and Berzelius and the classic experiments of Ampère. In each of these three lines, as yet independent, we follow the development of the theory up to a point where mathematical analysis begins to lay powerful hands upon it. The next two chapters deal with this mathematical development of the theory of the ether in the hands of Fresnel and his followers. These chapters make interesting reading. The subject is one which makes considerable demands upon the mathematical knowledge of the reader, and the author does not hesitate to make free use of the brevity and directness which equations offer in expounding these theories; but only those who have attempted to delve into some of the original memoirs of Cauchy, MacCullagh, and Kelvin will realize how signally the author has succeeded in presenting briefly the main points of these researches.

In the next chapter, devoted to the work of Faraday, we find electrostatics and galvanism united to form a single subject. It is to be regretted that the author does not give more than a small footnote to the work of Henry, whose discoveries, though not adequately recognized at the time, were by no means unimportant. After a chapter on the mathematical development of electromagnetism under the influence of the ideas of action at a distance, a return is made to the history of Faraday's ideas and of Maxwell's development of them. In Maxwell's theory we find at last the theories of light and electricity united on a common basis, and the known physical properties of the ether expressed in a single system of equations. The successes of this theory, as well as the shortcomings when applied to dielectrics, are well summarized. But the English physicists have not been content to rest on Maxwell's equations—indeed we find that Kelvin, to the end of his long life, "never thoroughly believed it"—and in the succeeding chapter we have an account of the various attempts to construct mechanical models of the ether. It is a pity that the author did not see fit to include here some account of Osborne Reynolds' theory of a "granular ether," for it is surely worth at least passing mention. The followers of Maxwell, particularly Hertz, are discussed at some length. The last two chapters deal with the development of the modern theory of electrons, which finds its inception in the theories of electrolytic conduction of Faraday, as well as in the ideas of Faraday and Mossotti on the polarization of a dielectric. The treatment here is necessarily brief, but by no means inadequate, and the clearness of the presentation nowhere shows to better advantage.

It seems a pity that more attention was not paid to the physical theories of gravitation. It is true that none of these has been found adequate, and that the problem remains as yet detached from electromagnetic theory; but the subject would fall naturally into a history of the theories of the ether, and the author of the present work is so peculiarly fitted to discuss the question that the omission will be a source of regret to many.

That some slight errors or inaccuracies should creep into a book of this character is only to be expected, but the one or two we have observed are of too trivial a character to deserve mention, and affect in no way the general excellence of the work. The book is attractively printed, and remarkably free from misprints.

C. M. Sparrow

Johns Hopkins University

THE
ASTROPHYSICAL JOURNAL
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AND ASTRONOMICAL PHYSICS

VOLUME XXXIV DECEMBER 1911 NUMBER 5

ON THE INTERNAL AND CLOUD-VELOCITIES FOR GROUPS OF STARS OF DIFFERENT TYPES OF SPECTRA

By H. A. WEERSMA

In his paper, "On Certain Statistical Data Which May Be Valuable in the Classification of the Stars in Order of Their Evolution,"[1] Professor Kapteyn finds, as far as the available data warrant a conclusion, that the linear peculiar velocities of the stars increase with age.[2] This conclusion has been confirmed later from much more extensive data as to radial velocity by Professor Campbell.[3] Professor Kapteyn further points out that this phenomenon very probably entails a gradual dissipation of the star streams with advancing age, and is confirmed in his view by Dyson's[4] observation that the stars of Type I "diverge less from the general drift of the streams than the other stars."

The large extension given to the available number of well-determined astronomical proper motions by the publication of the *Preliminary General Catalogue of 6188 Stars*, by Professor Lewis Boss, and the extreme importance of the two closely connected phenomena renders it exceedingly desirable to test their reality by its aid. To do this is the aim of the present paper.

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[1] *Contributions of the Mount Wilson Solar Observatory*, No. 45; *Astrophysical Journal*, 31, 258, 1910.

[2] This *assumes* that the successive stages of evolution, beginning with the earliest, are in the order of the spectral classes B, A, F, G, K, M.

[3] *Lick Observatory Bulletin*, No. 196, Table II.

[4] *Proceedings of the Roy. Soc. of Edinburgh*, 29, 390, 1910.

The spectra have been taken, as far as possible, from the catalogues: *Harvard Annals*, **28**, Part 1, Table 12 (Miss Maury); *ibid.*, Part 2, Table 17, and **56**, No. 4, Table 1 (Miss Cannon), and if not contained in these, from *H.R.*, **50**.

I have confined myself provisionally to the stars of spectral classes A and K whose magnitude, according to Boss, is 6.0 or brighter, and all of the stars of the spectral class M. The helium stars are being investigated by Professor Kapteyn himself, and have not been included for that reason.

For the stars of these groups the angles of position of the proper motions have been computed from the data of the *Preliminary General Catalogue*. These angles of position have been used in the way explained below.

Explanation of the method.—Theoretically we have proceeded from the following set of hypotheses, which are identical with those explicitly stated or implied by Mr. Eddington in his paper, "The Systematic Motions of the Stars":[1]

1. The stars are distributed over two drifts of which the centers of gravity are in relative motion.

2. The internal velocities of the stars of each drift are distributed according to Maxwell's law.

3. The modulus, h, is the same for the two drifts.

For the positions of the apex and of the true vertex we have taken for all types of spectra:

$$\text{Apex} \quad A = 269^\circ\!.7, \quad D = +30^\circ\!.8$$
$$\text{True vertex} \quad A' = 94^\circ, \quad D' = +10^\circ$$

the equinox being that for 1875.

We will now suppose the sky to be divided into a number of areas and we will call, for the center of any arbitrary area:

α = the right ascension,
δ = the declination,
λ = the distance from the apex,
λ_1 = the distance from the true vertex,
γ = the angle between the direction to the antapex and that to the true vertex taken positively in the sense of a motion from north through west to south.

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[1] *Monthly Notices R.A.S.*, **67**, 34, 1906.

The quantities λ, λ_1, and γ may easily be calculated for every area by means of the adopted co-ordinates of the apex and the vertex.

Supposing that only stars of a *determinate class of spectrum* are considered, we will further introduce the following notations:

p = the velocity of the center of gravity of *all* the stars relative to the sun;

g_1 and g_2 = the velocities of the centers of gravity of the first and second drifts, respectively, relative to the center of gravity of all the stars;

g = the velocity of the center of gravity of the first drift relative to that of the second.

Consider a definite small area A of the sky having its center in O. In the tangential plane in O we will fix a rectangular system of co-ordinates OXY, taken in such a way that the sense of a motion directed from the positive end of the axis X to the positive end of the axis of Y is the same as that of a motion from north through west to south. The direction of OX will be left provisionally indeterminate.

We will further put

ω = the angle of position of the direction to the antapex, counted from OX;

ξ_1, η_1, and ξ_2, η_2 = the projections of the velocities of the center of gravity of the first and second stream, respectively, relative to the sun;

h = the modulus entering into the distribution according to Maxwell's law;

u, v = the projections of the internal velocities of an arbitrary star on OX and OY;

N_1, N_2, N_3, N_4 = the number of the stars in A, belonging to the first drift, for which the angle of position of the proper motion counted from OX lies in the first, second, third, and fourth quadrant, respectively;

N = the total number of the stars in A, belonging to the first drift;

n_1, n_2, n_3, n_4, and n = the corresponding quantities for the second drift.

Let A'' represent an area lying diametrically opposite to A and having the same dimensions.

The center O'' is also taken as the origin of a system of co-ordinates $O''X''Y''$ of which the axis $O''X''$ has the same direction as OX, $O''Y''$, a direction opposite to OY.

Further, let ξ_1'', η_1'', ξ_2'', η_2'', N_1'', N_2'' . . . . N'', n_1'', n_2'' . . . . n'' represent for A'' quantities corresponding to those given above for A.

In A as well as in A'' the angles of position of the proper motions are taken positive in the sense north-west-south.

We put further

$$\left.\begin{array}{lll} N_1'=N_1+N_4'' & n_1'=n_1+n_4'' & \mathbf{N}_1=N_1'+n_1' \\ N_2'=N_2+N_3'' & n_2'=n_2+n_3'' & \mathbf{N}_2=N_2'+n_2' \\ N_3'=N_3+N_2'' & n_3'=n_3+n_2'' & \mathbf{N}_3=N_3'+n_3' \\ N_4'=N_4+N_1'' & n_4'=n_4+n_1'' & \mathbf{N}_4=N_4'+n_4' \\ N' \;=N\;+N'' & n' \;=n\;+n'' & \mathbf{N}\;=N'\;+n' \end{array}\right\} \qquad (1)$$

Then we have

$$\left.\begin{array}{l} \xi_1=p\sin\lambda\cos\omega+g_1\sin\lambda_1\cos(\omega+\gamma) \\ \eta_1=p\sin\lambda\sin\omega+g_1\sin\lambda_1\sin(\omega+\gamma) \end{array}\right\} \qquad (2)$$

Considering further that the angle of position of the observed proper motion lies in the first quadrant if u lies between $-\xi_1$ and ∞ and at the same time v between $-\eta_1$ and ∞, etc., we have

$$\left.\begin{array}{l} N_1=N\dfrac{h^2}{\pi}\displaystyle\int_{-\xi_1}^{+\infty}e^{-h^2u^2}\,du\int_{-\eta_1}^{+\infty}e^{-h^2v^2}\,dv \\ N_2=N\dfrac{h^2}{\pi}\displaystyle\int_{-\infty}^{-\xi_1}e^{-h^2u^2}\,du\int_{-\eta_1}^{+\infty}e^{-h^2v^2}\,dv \\ N_3=N\dfrac{h^2}{\pi}\displaystyle\int_{-\infty}^{-\xi_1}e^{-h^2u^2}\,du\int_{-\infty}^{-\eta_1}e^{-h^2v^2}\,dv \\ N_4=N\dfrac{h^2}{\pi}\displaystyle\int_{-\xi_1}^{+\infty}e^{-h^2u^2}\,du\int_{-\infty}^{-\eta_1}e^{-h^2v^2}\,dv \end{array}\right\} \qquad (3)$$

From these we derive easily the following equation:

$$N_1+N_3-(N_2+N_4)=\frac{4N}{\pi}\int_0^{h\xi_1}e^{-t^2}\,dt\int_0^{h\eta_1}e^{-t^2}\,dt. \qquad (4)$$

As

$$\xi_1''=\xi_1,\quad \eta_1''=-\eta_1$$

we get in the same way

$$N_1''+N_3''-(N_2''+N_4'')=-\frac{4N''}{\pi}\int_0^{h\xi_1}e^{-t^2}\,dt\int_0^{h\eta_1}e^{-t^2}\,dt. \qquad (5)$$

Subtracting (5) from (4), we get for the first drift

$$N_1'+N_3'-(N_2'+N_4')=\frac{4N'}{\pi}\int_0^{h\xi_1} e^{-t^2}\,dt\int_0^{h\eta_1} e^{-t^2}\,dt. \qquad (6)$$

For the second drift, we get in the same way

$$\left.\begin{aligned}\xi_2&=p\sin\lambda\cos\omega-g_2\sin\lambda_1\cos(\omega+\gamma),\\ \eta_2&=p\sin\lambda\cos\omega-g_2\sin\lambda_1\sin(\omega+\gamma),\end{aligned}\right\} \qquad (7)$$

$$n_1'+n_3'-(n_2'+n_4')=\frac{4n'}{\pi}\int_0^{h\xi_2} e^{-t^2}\,dt\int_0^{h\eta_2} e^{-t^2}\,dt. \qquad (8)$$

Therefore, taking the two drifts together,

$$\left.\begin{aligned}\mathbf{N}_1+\mathbf{N}_3-(\mathbf{N}_2+\mathbf{N}_4)=\frac{4N'}{\pi}\int_0^{h\xi_1} e^{-t^2}\,dt\int_0^{h\eta_1} e^{-t^2}\,dt+\\ \frac{4n'}{\pi}\int_0^{h\xi_2} e^{-t^2}\,dt\int_0^{h\eta_2} e^{-t^2}\,dt.\end{aligned}\right\} \qquad (9)$$

Hough and Halm have shown that the mixture of the two drifts is not constant for the whole sky.[1] According to these authors the law of proportion for the different parts of the sky may be tolerably accounted for by an equation of the form

$$\frac{n}{N+n}=s_0+\epsilon\cos\beta \qquad (10)$$

s_0 and ϵ being constants, β the distance from the center of the area considered to a point where the maximum density of the second-drift stars occurs. We may write this

$$N-n=(N+n)a+(N+n)C\cos\beta \qquad (11)$$

for the area A, and

$$N''-n''=(N''+n'')a-(N''+n'')C\cos\beta \qquad (12)$$

for the area A'', a and C being constant for the whole sky.

Therefore by addition

$$N'-n'=(N'+n')a+\{N+n-(N''+n'')\}C\cos\beta. \qquad (13)$$

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[1] "A Spectroscopic Determination of the Systematic Motion of the Stars," *Monthly Notices R.A.S.*, **70**, 85, 1909; "On the Systematic Motions of the Bradley Stars," *ibid.*, **70**, 568, 1910.

If now we assume that the distribution of our stars over the sky is such that diametrical areas A and A'' contain equal total numbers of stars, a condition that is approximately fulfilled in our case, we have

$$N+n=N''+n''. \tag{14}$$

Therefore, according to (13), n'/N' will be constant. We will adopt

$$\frac{n'}{N'}=0.68, \tag{15}$$

the average value found by Eddington[1] from the stars of the *Preliminary General Catalogue.*

We thus get

$$N'=0.596\mathbf{N}, \qquad n'=0.404\mathbf{N}. \tag{16}$$

We have further as a consequence of the definitions of g_1 and g_2,

$$N'g_1=n'g_2. \tag{17}$$

Therefore

$$\left.\begin{aligned} hg_1&=0.404h(g_1+g_2)=0.404hg,\\ hg_2&=0.596h(g_1+g_2)=0.596hg.\end{aligned}\right\} \tag{18}$$

We may still dispose arbitrarily of the directions of the axes OX of the systems of co-ordinates in the areas A. We will choose these in two different ways, first in such a way that for every region A the angle of position of the direction to the true vertex counted from OX is 45°.

In this case

$$\omega=45^\circ-\gamma. \tag{19}$$

The quantities $\mathbf{N}_1$, ... etc., $\mathbf{N}$ are completely determined by the empirical data and may easily be derived for every pair of diametrical areas.

From (2), (7), (18), and (19) we derive

$$\left.\begin{aligned} h\xi_1&=hp\sin\lambda\cos(45^\circ-\gamma)+0.286hg\sin\lambda_1\\ h\eta_1&=hp\sin\lambda\sin(45^\circ-\gamma)+0.286hg\sin\lambda_1\\ h\xi_2&=hp\sin\lambda\cos(45^\circ-\gamma)-0.421hg\sin\lambda_1\\ h\eta_2&=hp\sin\lambda\sin(45^\circ-\gamma)-0.421hg\sin\lambda_1\end{aligned}\right\} \tag{20}$$

Furthermore, from (9) and (16)

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[1] "The Systematic Motions of the Stars of Professor Boss's Preliminary General Catalogue," *Monthly Notices R.A.S.*, **71**, 38, 1910.

$$\left.\begin{aligned}\mathbf{N}_1+\mathbf{N}_3-(\mathbf{N}_2+\mathbf{N}_4)=\frac{4\mathbf{N}}{\pi}\Big\{0.596\int_0^{h\xi_1}e^{-t^2}\,dt\int_0^{h\eta_1}e^{-t^2}\,dt+ \\ 0.404\int_0^{h\xi_2}e^{-t^2}\,dt\int_0^{h\eta_2}e^{-t^2}\,dt\Big\}.\end{aligned}\right\}\quad(21)$$

The quantities λ, λ_1, and γ having been computed for every area A by means of the adopted positions of apex and vertex, the quantities $h\xi_1$, $h\eta_1$, $h\xi_2$, and $h\eta_2$ will be determined by the equations (20) in function of the two unknown quantities hp and hg, and the second member of equation (21) will also depend on no other unknown quantities than these two. Taking now all areas together we get a set of equations between the quantities hp and hg that will be especially favorable for the determination of hg, but which will yield a value of hp, the weight of which is very small indeed. In order to obtain a more reliable determination of this quantity we will derive another set of equations.

For that purpose we will now choose our system of co-ordinates in A in another way and let OX coincide with the direction to the antapex.

We have then

$$\omega=0 \qquad (22)$$

and from (2) and (7), with the aid of (18)

$$\left.\begin{aligned}h\xi_1&=hp\sin\lambda+0.404hg\sin\lambda_1\cos\gamma,\\ h\xi_2&=hp\sin\lambda-0.596hg\sin\lambda_1\cos\gamma.\end{aligned}\right\}\quad(23)$$

From equations (3) we derive for an arbitrary area A

$$N_1+N_4-(N_2+N_3)=2\frac{N}{\sqrt{\pi}}\int_0^{h\xi_1}e^{-t^2}\,dt. \qquad (24)$$

By combining this equation with the equivalent one for A'', by the aid of (16) we get

$$N_1'+N_4'-(N_2'+N_3')=\frac{2\mathbf{N}}{\sqrt{\pi}}0.596\int_0^{h\xi_1}e^{-t^2}\,dt, \qquad (25)$$

or combining again with the equivalent one for the second drift

$$\mathbf{N}_1+\mathbf{N}_4-(\mathbf{N}_2+\mathbf{N}_3)=\frac{2\mathbf{N}}{\sqrt{\pi}}\Big\{0.596\int_0^{h\xi_1}e^{-t^2}\,dt+0.404\int_0^{h\xi_2}e^{-t^2}\,dt\Big\}. \qquad (26)$$

In this equation we have of course to take for $\mathbf{N}_1 \ldots \mathbf{N}_4$ the quantities corresponding to the *new* direction of OX. Taking now all areas together we get a second set of equations which combined with (21) enables us to derive the quantities hp and hg by trial. It will be convenient to combine the equations (21) and (26) into a single equation for the whole sky, taking care of course that every pair of diametrical areas shall get its proper weight. In following up this idea we have chosen the weights in such a way that the first and second members of equations (26) were multiplied by $\sin\lambda$, those of equations (21) by $\sin^2\lambda_1 \cdot e^{-h^2p^2\sin^2\lambda}$, adopting in the latter expression for hp a preliminary approximate value.

For the sake of clearness we sum up what has been said about the method which will be applied below, as follows:

We divide the sky into a number of areas which are by pairs equal and diametrically opposite. For every pair A and A'' we derive the quantities $\mathbf{N}$ and further the quantities $\mathbf{N}_1$, $\mathbf{N}_2$, $\mathbf{N}_3$, $\mathbf{N}_4$ in two different ways, viz.:

1. Taking in every area A for OX the direction to the antapex;
2. Taking in every area A for OX such a direction that the angle of position of the direction to the true vertex of the first drift becomes 45°.

Corresponding to these two cases we have then the following sets of equations:

CASE 1

$$\left.\begin{aligned} h\xi_1 &= hp\sin\lambda + 0.404hg\sin\lambda_1\cos\gamma \\ h\xi_2 &= hp\sin\lambda - 0.596hg\sin\lambda_1\cos\gamma \\ \frac{2}{\sqrt{\pi}}\sum\mathbf{N}\sin\lambda\Big\{0.596\int_0^{h\xi_1}e^{-t^2}dt &+ 0.404\int_0^{h\xi_2}e^{-t^2}dt\Big\} = \\ &\Sigma\sin\lambda\{(\mathbf{N}_1+\mathbf{N}_4)-(\mathbf{N}_2+\mathbf{N}_3)\}. \end{aligned}\right\} \quad (27)$$

CASE 2

$$\left.\begin{aligned} h\xi_1 &= hp\sin\lambda\cos(45^\circ-\gamma) + 0.286hg\sin\lambda_1 \\ h\eta_1 &= hp\sin\lambda\sin(45^\circ-\gamma) + 0.286hg\sin\lambda_1 \\ h\xi_2 &= hp\sin\lambda\cos(45^\circ-\gamma) - 0.421hg\sin\lambda_1 \\ h\eta_2 &= hp\sin\lambda\sin(45^\circ-\gamma) - 0.421hg\sin\lambda_1 \\ \frac{4}{\pi}\sum\mathbf{N}\sin^2\lambda_1 e^{-h^2p^2\sin^2\lambda}\Big\{0.596\int_0^{h\xi_1}e^{-t^2}dt\int_0^{h\eta_1}e^{-t^2}dt &+ \\ 0.404\int_0^{h\xi}e^{-t^2}dt\int_0^{h\eta_2}e^{-t^2}dt &= \\ &\Sigma\sin^2\lambda_1 e^{-h^2p^2\sin^2\lambda}\{(\mathbf{N}_1+\mathbf{N}_3)-(\mathbf{N}_2+\mathbf{N}_4)\} \end{aligned}\right\} \quad (28)$$

The quantities hp and hg have then to be solved by trial and error.

Applications of the method.—In applying the method explained above first to the A stars and subsequently to the K and M stars of the *Preliminary General Catalogue*, we have excluded those stars of which it is known that they belong to local drifts. We thus excluded 23 stars belonging to the group of the *Hyades* and furthermore the stars β, γ, δ, ϵ, and ζ *Ursae Majoris*.

The sky has been divided into the following areas, according to the values of Ω and λ, Ω being the angle Vertex-Apex-Star:

TABLE I

Limits of Ω	Limits of λ	Limits of Ω	Limits of λ
A 0° and 360°	0° and 30°	*H* 270° and 315°	30° and 70°
B 0° " 45°	30° " 70°	*I* 315° " 360°	30° " 70°
C 45° " 90°	30° " 70°	*K* 0° " 45°	70° " 110°
D 90° " 135°	30° " 70°	*L* 45° " 90°	70° " 110°
E 135° " 180°	30° " 70°	*M* 90° " 135°	70° " 110°
F 180° " 225°	30° " 70°	*N* 135° " 180°	70° " 110°
G 225° " 270°	30° " 70°		

and further the corresponding diametrical areas A'', B'', C'', etc., together 26 areas. For the centers of the areas *A*, *B*, *C*, etc., we have adopted points not coinciding precisely with the geometrical centers of gravity, but points obtained in the following way. Leaving the co-ordinates of the stars of *A*, *B*, *C*, etc., unaltered, the positions of the stars A'', B'', C'', etc., have been transferred diametrically to *A*, *B*, *C*, etc. By combining now the positions of the stars of *A*, *B*, *C*, etc., with the transferred positions of the stars of A'', B'', C'', etc., we get systems of fictitious positions of stars. The centers of gravity of these systems have been adopted as the centers of the areas, *A*, *B*, *C*, etc. With the co-ordinates of these centers as arguments, the quantities λ, λ_1, and γ corresponding to the different pairs of areas have been calculated. Of course this has been done for the different types of spectra separately.

In combining the equations (27) the areas *A* and A'', which should have hardly any weight and would, moreover, cause some difficulty, have been simply neglected. In order to use them in combining the equations (28), they have been subdivided again, each into four areas according to the following scheme:

TABLE II

sin λ cos (45°−γ)	sin λ sin (45°−γ)	sin λ cos (45°−γ)	sin λ sin (45°−γ)
A_1 +	+	A_1'' +	+
A_2 +	−	A_2'' −	+
A_3 −	+	A_3'' +	−
A_4 −	−	A_4'' −	−

the areas A_1'', A_2'', A_3'', and A_4'' lying evidently again diametrically to A_1, A_2, A_3, and A_4, respectively.

The results.—In the Tables III to VII the main results from the different groups of stars have been condensed. At the head of each table have been given: the designation of the group considered, the total number of stars included in the group, and the resulting values of *hp* and *hg* with their *mean* errors. In the tables themselves we have used for the sake of brevity the following notation. First, every pair of combined diametrical areas has been denoted by one letter, the combination of A_1 and A_1'' being indicated by A_1', of *B* and *B''* by *B'*, etc. This notation has been given in the first column.

Further, we have put

$$\left.\begin{aligned}
P'_{\text{obs.}} &= (\mathbf{N}_1+\mathbf{N}_4)-(\mathbf{N}_2+\mathbf{N}_3)\\
P'_{\text{comp.}} &= \frac{2}{\sqrt{\pi}}\mathbf{N}\left\{0.596\int_0^{h\xi_1} e^{-t^2}dt+0.404\int_0^{h\xi_2} e^{-t^2}dt\right\}\\
O-C &= P'_{\text{obs.}}-P'_{\text{comp.}}\\
Q'_{\text{obs.}} &= (\mathbf{N}_1+\mathbf{N}_3)-(\mathbf{N}_2+\mathbf{N}_4)\\
Q'_{\text{comp.}} &= \frac{4}{\pi}\mathbf{N}\left\{0.596\int_0^{h\xi_1} e^{-t^2}dt\int_0^{h\eta_1} e^{-t^2}dt+\right.\\
&\qquad\qquad\left.0.404\int_0^{h\xi_2} e^{-t^2}dt\int_0^{h\eta_2} e^{-t^2}dt\right\}\\
O_1-C_1 &= Q'_{\text{obs.}}-Q'_{\text{comp.}}
\end{aligned}\right\} \quad (29)$$

The amounts of these quantities have been given in the columns 2, 3 . . . . 7.

The values of $P'_{\text{comp.}}$ and $Q'_{\text{comp.}}$ have been computed for every group of stars by means of the values of *hp* and *hg* at the head of the corresponding table. Further, in deriving the quantities $P'_{\text{obs.}}$, $P'_{\text{comp.}}$, $Q'_{\text{obs.}}$, and $Q'_{\text{comp.}}$ the directions of *OX* have been taken in accordance with the systems denoted above under Case 1 and Case 2, respectively.

The eighth column contains the numbers **N** defined above. In order to reduce the residuals $O-C$ and O_1-C_1 for the different pairs of areas to quantities of the same weight, they have been divided by $\sqrt{\mathbf{N}}$. The quantities obtained in this way are given in the ninth and tenth columns.

TABLE III

A Stars (No Areas Excluded), 1089 Stars

$hp=1.16$, $m.e.=\pm 0.09$; $hg=2.28$, $m.e.=\pm 0.32$

	P' obs.	P' (comp.)	$O-C$	Q' obs.	Q' (comp.)	O_1-C_1	**N**	$\frac{O-C}{\sqrt{\mathbf{N}}}$	$\frac{O_1-C_1}{\sqrt{\mathbf{N}}}$
A_1'	....	......	......	+20	+25.7	− 5.7	55	......	−0.77
A_2'	....	......	......	+15	+13.8	+ 1.2	68	......	+0.15
A_3'	....	......	......	+ 9	+ 7.0	+ 2.0	42	......	+0.31
A_4'	....	......	......	+12	+ 4.1	+ 7.9	26	......	+1.55
B'	+59	+42.8	+16.2	+46	+51.4	− 5.4	98	+1.64	−0.55
C'	+67	+45.5	+21.5	+ 4	+37.7	−33.7	90	+2.27	−3.55
D'	+41	+37.6	+ 3.4	+21	+ 7.2	+13.8	55	+0.46	+1.86
E'	+57	+61.7	− 4.7	− 1	− 2.5	+ 1.5	81	−0.52	+0.17
F'	+58	+61.4	− 3.4	+26	+ 0.2	+25.8	80	−0.38	+2.89
G'	+37	+45.4	− 8.4	+15	+ 7.6	+ 7.4	65	−1.04	+0.92
H'	+22	+20.5	+ 1.5	+18	+14.6	+ 3.4	38	+0.24	+0.55
J'	+24	+22.4	+ 1.6	+34	+28.6	+ 5.4	54	+0.22	+0.73
K'	+74	+71.0	+ 3.0	+68	+48.9	+19.1	108	+0.29	+1.84
L'	+43	+49.0	− 6.0	+29	+25.6	+ 3.4	75	−0.69	+0.39
M'	+34	+45.6	−11.6	+30	+23.9	+ 6.1	70	−1.39	+0.73
N'	+48	+55.5	− 7.5	+36	+38.2	− 2.2	84	−0.82	−0.24

TABLE IV

A Stars (Areas C' and F' Excluded), 919 Stars

$hp=1.22$, $m.e.=\pm 0.09$; $hg=2.58$, $m.e.=\pm 0.21$

	P' obs.	P' (comp.)	$O-C$	Q' obs.	Q' (comp.)	O_1-C_1	**N**	$\frac{O-C}{\sqrt{\mathbf{N}}}$	$\frac{O_1-C_1}{\sqrt{\mathbf{N}}}$
A_1'	....	......	......	+20	+30.0	−10.0	55	......	−1.35
A_2'	....	......	......	+15	+17.7	− 2.7	68	......	−0.33
A_3'	....	......	......	+ 9	+ 9.1	− 0.1	42	......	−0.02
A_4'	....	......	......	+12	+ 4.8	+ 7.2	26	......	+1.41
B'	+59	+38.6	+20.4	+46	+56.9	−10.9	98	+2.06	−1.10
C'	....	......	......	....	......	......	..	......	......
D'	+41	+37.7	+ 3.3	+21	+ 9.8	+11.2	55	+0.44	+1.51
E'	+57	+63.2	− 6.2	− 1	− 2.2	+ 1.2	81	−0.69	+0.13
F'	....	......	......	....	......	......	..	......	......
G'	+37	+45.4	− 8.4	+15	+10.5	+ 4.5	65	−1.04	+0.56
H'	+22	+19.3	+ 2.7	+18	+17.2	+ 0.8	38	+0.44	+0.13
J'	+24	+20.2	+ 3.8	+34	+31.7	+ 2.3	54	+0.52	+0.31
K'	+74	+67.8	+ 6.2	+68	+51.3	+16.7	108	+0.60	+1.61
L'	+43	+46.8	− 3.8	+29	+29.1	− 0.1	75	−0.44	−0.01
M'	+34	+43.4	− 9.4	+30	+27.4	+ 2.6	70	−1.12	+0.31
N'	+48	+53.1	− 5.1	+36	+40.1	− 4.1	84	−0.56	−0.45

TABLE V

K STARS, 1097 STARS

$hp=0.69$, $m.e.=\pm 0.05$; $hg=1.05$, $m.e.=\pm 0.14$

	P' obs.	P' (comp.)	$O-C$	Q' obs.	Q' (comp.)	O_1-C_1	N	$\frac{O-C}{\sqrt{N}}$	$\frac{O_1-C_1}{\sqrt{N}}$
A_1'	....	......	......	+ 1	+ 8.2	− 7.2	62	......	−0.91
A_2'	....	......	......	+ 9	+ 1.9	+ 7.1	49	......	+1.01
A_3'	....	......	......	− 1	+ 1.3	− 2.3	36	......	−0.38
A_4'	....	......	......	+ 7	+ 1.3	+ 5.7	27	......	+1.10
B'	+44	+52.3	− 8.3	+28	+26.9	+ 1.1	112	−0.78	+0.10
C'	+46	+40.5	+ 5.5	+10	+12.1	− 2.1	84	+0.60	−0.23
D'	+41	+32.6	+ 8.4	+ 7	+ 1.2	+ 5.8	63	+1.06	+0.73
E'	+56	+39.8	+16.2	− 3	+ 1.1	− 4.1	73	+1.90	−0.48
F'	+45	+38.2	+ 6.8	+ 5	+ 2.8	+ 2.2	69	+0.82	+0.26
G'	+30	+31.1	− 1.1	+ 1	+ 1.7	− 0.7	59	−0.14	−0.09
H'	+23	+27.7	− 4.7	+17	+ 8.0	+ 9.0	57	−0.62	+1.19
J'	+17	+27.1	−10.1	+ 6	+14.4	− 8.4	58	−1.33	−1.10
K'	+55	+59.0	− 4.0	+23	+25.0	− 2.0	99	−0.40	−0.20
L'	+50	+44.2	+ 5.8	+20	+11.2	+ 8.8	74	+0.67	+1.02
M'	+39	+50.5	−11.5	+ 7	+12.7	− 5.7	85	−1.25	−0.62
N'	+54	+53.6	+ 0.4	+20	+21.5	− 1.5	90	+0.04	−0.16

TABLE VI

M STARS, 240 STARS

$hp=0.76$, $m.e.=\pm 0.06$; $hg=1.38$, $m.e.=\pm 0.39$

	P' obs.	P' (comp.)	$O-C$	Q' obs.	Q' (comp.)	O_1-C_1	N	$\frac{O-C}{\sqrt{N}}$	$\frac{O_1-C_1}{\sqrt{N}}$
A_1'	....	......	......	+ 3	+ 2.2	+ 0.8	11	......	+0.24
A_2'	....	......	......	+ 7	+ 0.9	+ 6.1	9	......	+2.03
A_3'	....	......	......	+ 3	+ 0.7	+ 2.3	9	......	+0.77
A_4'	....	......	......	− 2	+ 0.3	− 2.3	4	......	−1.15
B'	+16	+14.8	+ 1.2	+12	+10.0	+ 2.0	32	+0.21	+0.35
C'	+ 7	+ 5.5	+ 1.5	+ 5	+ 2.5	+ 2.5	11	+0.45	+0.75
D'	+ 7	+ 6.1	+ 0.9	− 1	+ 1.1	− 2.1	11	+0.27	−0.63
E'	+ 7	+ 5.5	+ 1.5	+ 2	+ 0.5	+ 1.5	9	+0.50	+0.50
F'	+10	+10.6	− 0.6	− 1	+ 1.2	− 2.2	18	−0.14	−0.52
G'	+ 5	+ 5.9	− 0.9	− 1	+ 0.7	− 1.7	11	−0.27	−0.51
H'	+ 3	+ 4.4	− 1.4	− 1	+ 1.8	− 2.8	9	−0.47	−0.93
J'	+12	+10.4	+ 1.6	+12	+ 7.8	+ 4.2	24	+0.33	+0.86
K'	+ 7	+ 7.8	− 0.8	+ 9	+ 4.0	+ 5.0	13	−0.22	+1.39
L'	+12	+13.2	− 1.2	− 2	+ 4.5	− 6.5	22	−0.26	−1.39
M'	+22	+18.1	+ 3.9	0	+ 6.1	− 6.1	30	+0.71	−1.11
N'	+ 5	+10.0	− 5.0	+ 3	+ 5.2	− 2.2	17	−1.21	−0.53

In the first solution of the A stars the pairs of areas C' and F' yield somewhat large values for the residual quantities $\frac{O_1-C_1}{\sqrt{N}}$, the former a negative, the latter a positive one (see Table III). Now the factor $\sin^2\lambda_1 e^{-h^2p^2\sin^2\lambda}$ with which the terms given by the

TABLE VII

K and M Stars Together, 1337 Stars

$hp = 0.70$, $m.e. = \pm 0.05$; $hg = 1.11$, $m.e. = \pm 0.12$

	P' obs.	P' (comp.)	$O-C$	Q' obs.	Q' (comp.)	O_1-C_1	N	$\frac{O-C}{\sqrt{N}}$	$\frac{O_1-C_1}{\sqrt{N}}$
A_1'	....	......	......	+4	+10.5	−6.5	73	......	−0.76
A_2'	....	......	......	+16	+2.8	+13.2	58	......	+1.73
A_3'	....	......	......	+2	+1.8	+0.2	45	......	+0.03
A_4'	....	......	......	+5	+1.6	+3.4	31	......	+0.61
B'	+60	+67.0	−7.0	+40	+36.4	+3.6	144	−0.58	+0.30
C'	+53	+45.9	+7.1	+15	+15.0	0.0	95	+0.73	0.00
D'	+48	+38.7	+9.3	+6	+2.4	+3.6	74	+1.08	+0.42
E'	+63	+45.4	+17.6	−1	+1.6	−2.6	82	+1.94	−0.29
F'	+55	+48.5	+6.5	+4	+3.9	+0.1	87	+0.70	+0.01
G'	+35	+37.0	−2.0	0	+2.4	−2.4	70	−0.24	−0.29
H'	+26	+32.1	−6.1	+16	+10.0	+6.0	66	−0.75	+0.74
J'	+29	+37.7	−8.7	+18	+21.2	−3.2	82	−0.96	−0.35
K'	+62	+66.6	−4.6	+32	+29.3	+2.7	112	−0.43	+0.26
L'	+62	+57.3	+4.7	+18	+15.5	+2.5	96	+0.48	+0.26
M'	+51	+68.3	−7.3	+7	+18.3	−11.3	115	−0.68	−1.05
N'	+59	+63.7	−4.7	+23	+26.8	−3.8	107	−0.45	−0.37

different pairs of areas enter into the equations (28) happens to be large for C', and small for F'. The part which these pairs of areas have played in the calculation of hg must therefore be rather large for C', and small for F'. As a consequence we see that the residuals of the other pairs of areas show some preference for positive values. It seems therefore probable that the value for hg obtained in the first solution will be somewhat too small as a consequence of some anomaly in the distribution of the proper motions in C'. The evidence for the reality of an abnormal character of C' is increased by the fact that one of the two diametrical areas of C' lies in *Cygnus* near a region of the sky which also appeared to behave anomalously in the researches of Mr. Eddington.[1] For this reason we have made a second solution for the A stars excluding now C' and F' (see Table IV). The values for hp and hg resulting from this solution will probably be better than those from the first solution. Owing to the small number of M stars, the results for this class are necessarily somewhat uncertain. We have made for that reason another solution based on the K and M stars combined. The results of this solution have been given in Table VII.

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[1] Region $G\gamma$ in *Monthly Notices R.A.S.*, **57**, 52, 1896; region VII in *ibid.*, **71**, 37, 1910.

If now we compare the values of *hg* derived from the different groups of stars, we see that this quantity decreases by a considerable amount if we pass from the A to the K stars. Adopting for the A stars the value of *hg* from the first solution, we get a difference of 1.23 ± 0.35.[1] Adopting the value from the second solution we get 1.53 ± 0.25. Passing now from the K stars to the M stars we get again some increase of the value for *hg*. But we get only a difference of 0.33 ± 0.42, which may therefore be accidental. Finally, taking the K and M stars together, we get

hg for A stars (first solution) − *hg* for K and M stars $= 1.17 \pm 0.34$
hg for A stars (second solution) − *hg* for K and M stars $= 1.47 \pm 0.24$

Adopting for the A stars the value of *hg* from the first solution we get for the *difference* a value 3.4 times larger than the mean error, and adopting the value from the second solution we get a value 6.1 times larger than the mean error. The difference cannot well be accidental, therefore.

Denoting the mean internal velocity of the stars of a drift by Ω, and considering that we have

$$h = \frac{2}{\Omega\sqrt{\pi}},$$

we get:

$$\left.\begin{array}{ll} \text{A stars (first solution)} & : \frac{g}{\Omega} = 2.02 \pm 0.28 \\ \text{A stars (second solution)} & : \frac{g}{\Omega} = 2.29 \pm 0.19 \\ \text{K and M stars together} & : \frac{g}{\Omega} = 0.98 \pm 0.11 \end{array}\right\} \quad (30)$$

We see, therefore, that the second of the hypotheses mentioned at the head of this paper is clearly confirmed by our results. For, taking in every group of stars the mean internal velocity of the stars of the group as unity, the relative velocity of the two drifts expressed in this unit *decreases* with age. Or, stating the result in another way: if for every spectral class we take the relative velocity of the two drifts as unity, the mean internal velocity will *increase* with age. In other words, *the phenomenon of star-streaming*

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[1] We have used throughout the *mean* error, not the probable error.

shows itself in greater purity for the younger spectral types than for the older ones, which is in conformity with what Professor Kapteyn's paper led us to expect.

Of course the results hitherto considered do not enable us to decide whether the effect stated here is caused by an increase of the *absolute* value of the mean internal velocity, or by a decrease in the relative velocity of the two drifts; or, perhaps, by the two causes together. We may, however, express all the velocities in function of what is commonly called the sun's velocity (p). From Tables III, IV, and VII we get:

$$\left.\begin{array}{ll}\text{A stars (first solution)} & : \dfrac{p}{\Omega}=1.03\pm0.08 \\ \text{A stars (second solution)} & : \dfrac{p}{\Omega}=1.08\pm0.08 \\ \text{K and M stars} & : \dfrac{p}{\Omega}=0.62\pm0.04\end{array}\right\} \quad (31)$$

or, combining the values for hp given by these tables with the corresponding values for hg,

$$\left.\begin{array}{ll}\text{A stars (first solution)} & : \dfrac{g}{p}=1.97\pm0.31 \\ \text{A stars (second solution)} & : \dfrac{g}{p}=2.11\pm0.23 \\ \text{K and M stars} & : \dfrac{g}{p}=1.59\pm0.20\end{array}\right\} \quad (32)$$

If it were allowable to assume that the motion of the sun were the same relatively to the center of gravity of the K and M stars as it is relatively to that of the A stars, it would be easy to draw a conclusion. For in this case the quantity p would be equal for the two groups. Therefore the results of (31) would then prove clearly that the quantity Ω increases with age, while those of (32) would yield some evidence for a decrease of g with age, the latter being somewhat doubtful as a consequence of the relatively large amounts of the mean error.

Recent unpublished results obtained by Professor Kapteyn make it doubtful, however, whether the assumption mentioned is permissible. As a consequence we cannot, from the results obtained above, draw safe conclusions about the quantities g

and Ω separately, without taking into consideration supplementary data. In order to meet the difficulty I have tried to derive the values of Ω in kilometers from spectroscopic velocities. The data I have used for that purpose have been taken from a list compiled by Professor Kapteyn mainly from spectroscopic velocities published by the observatories of Bonn and Yerkes, velocities of the centers of gravity of spectroscopic binaries published by Campbell, and some unpublished spectroscopic velocities derived at the Yerkes Observatory. By means of an examination of the corresponding astronomical proper motions, as given in the *Preliminary General Catalogue*, 12 stars from the list, belonging to spectral class A, and 27 belonging to the classes K or M could be shown with very great probability to belong to the first drift. From the spectroscopic velocities of these stars the velocity V_1 of the center of gravity of the first drift relative to the sun has been calculated separately for the A and for the K and M stars. In doing so I have adopted for the co-ordinates α_1 and δ_1 of the apparent vertex of the first drift quantities chosen in such a way that they correspond with the adopted positions for the apex and the true vertex and with the values for g/p found above, viz.:

$$\left.\begin{array}{lll} \text{for the A stars,} & \alpha_1=91\overset{\circ}{.}8, & \delta_1=-12\overset{\circ}{.}5 \\ \text{for the K and M stars,} & \alpha_1=91\overset{\circ}{.}5, & \delta_1=-15\overset{\circ}{.}1. \end{array}\right\} \quad (33)$$

I now made two solutions for every group: (*a*) assuming that the observed radial velocities for stars of a determined spectral class need a constant correction (for pressure-shift ?); and (*b*) neglecting such a term. I found:

$$\left.\begin{array}{ll} \text{A stars, solution } (a), & V_1=19.8 \text{ km} \\ \text{A stars, solution } (b), & V_1=21.1 \text{ km} \\ \text{K and M stars, solution } (a), & V_1=23.1 \text{ km} \\ \text{K and M stars, solution } (b), & V_1=20.8 \text{ km} \end{array}\right\} \quad (34)$$

From the residuals obtained by substituting these values for V_1 in the equations of condition, I derived the values for Ω corresponding to the four solutions of V_1 by multiplying the mean of the residuals (every residual taken with the positive sign) by *two*. I thus found:

$$\left.\begin{array}{ll}\text{A stars, solution }(a), & \Omega=13.4\text{ km}\\ \text{A stars, solution }(b), & \Omega=15.3\text{ km}\\ \text{K and M stars, solution }(a), & \Omega=24.6\text{ km}\\ \text{K and M stars, solution }(b), & \Omega=24.9\text{ km}\end{array}\right\}\quad(35)$$

We see that the values for Ω resulting from the K and M stars are considerably larger than the corresponding values for the A stars. We will adopt:

$$\left.\begin{array}{ll}\text{A stars,} & \Omega=14.3\text{ km (12 stars)}\\ \text{K and M stars,} & \Omega=24.7\text{ km (27 stars).}\end{array}\right\}\quad(36)$$

By combining these results with those of (30) and (31), we find:

$$\left.\begin{array}{lll}\text{A stars, first solution,} & p=14.7\text{ km;} & g=28.9\text{ km}\\ \text{A stars, second solution,} & p=15.4\text{ km;} & g=32.7\text{ km}\\ \text{K and M stars,} & p=15.3\text{ km;} & g=24.2\text{ km}\end{array}\right\}\quad(37)$$

The results of (36) and (37) seem to show that the decrease of $\frac{g}{\Omega}$ with age, stated above, is due in larger part to an increase of Ω, and in less degree to a decrease of g.

Meanwhile, owing to the small number of radial velocities, our results in the form (36) and (37) are quite unreliable. We may, however, confidently expect better results in one or two years. For even now, if only existing observations were published, the data would probably be sufficient for the derivation of fairly trustworthy results. Moreover, observations are now in progress on Mount Wilson for the express purpose of furnishing the data required for investigations closely related to the present one.

ASTRONOMICAL LABORATORY
GRONINGEN
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ON THE DISTRIBUTION OF BRIGHTNESS IN THE TAIL OF HALLEY'S COMET[1]

By K. SCHWARZSCHILD and E. KRON

The expedition of the Potsdam Astrophysical Observatory, which went to Teneriffe in the spring of 1910, in charge of Professor Müller, took with it for photographing Halley's comet only a small stereoscopic camera in which either two Zeiss aplanatic lenses of 23.3 mm aperture and 110 mm focal length, or an anastigmat by Goerz of 20 mm aperture and 180 mm focus could be used. The camera was supplied with a parallactic mounting and driven by a clock-work, but it had no slow-motions for accurately following the movement of the comet, and the arrangements for focusing the camera were not adequate. The views of Halley's comet obtained in Germany in the first months of the year were so unpromising that there seemed to be no reason for equipping the expedition more extensively in this respect, and the principal weight was attached to other problems.

The photographs of the comet obtained by the expedition were a surprise to anyone who had experienced the disappointment as to the appearance of the comet in Germany; but they are indeed not comparable with the plates obtained particularly at the American observatories. However, we were able to derive from them results which appeared to us of interest, in that we could conclude as to the conditions of brightness in the tail of the comet from the blackening of the images on the plate, the necessary data for the photometric comparison having been provided for. Two exposures of the comet were always made simultaneously on the same plate with the two precisely similar Zeiss aplanats, the objective of one having been stopped down to diminish the brightness by 0.4 magnitude.[2] For the plates taken with the Goerz anastigmat, comparison scales were made with a tube photometer, constructed by Professor Eberhard, on plates of the same emulsion which were developed at the same time with the plates of the comet.

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[1] Translated from authors' proof of a paper read before the Kgl. Gesellschaft der Wissenschaften zu Göttingen, 1911.

[2] In consequence of an oversight this reduction was used, instead of that of 1 magnitude, which was intended.

In view of the lack of sharpness of the plates, it would have been of no purpose to derive from them the brightness at every point of the tail. It is also questionable, in view of the complicated structure of the tail, whether much could have been gathered from such a measurement even in case of the best plates, but the following question of a more summary nature seemed more promising.

The brightness of the tail diminishes continuously from the head of the comet outward, aside from irregularities. This is necessarily in part due to the fact that the density decreases in the tail. The decrease of density occurs (1) because the material streaming out from the head is spread out over a constantly increasing cross-section, and (2) because of the simultaneous increase of the velocity with which the particles of the tail passed through this cross-section in consequence of the repulsive force exerted by the sun. Now the question is whether these two circumstances are sufficient to explain completely the diminution of brightness of the tail in a quantitative manner, whence would then follow that the luminosity per unit mass of the material of the tail would remain undiminished during its travels in space; or, on the other hand, whether the decrease in the brightness of the tail is much more rapid than that of the density, which we should a priori regard as more probable for a tail showing essentially a gaseous spectrum, hence a so-called self-luminous tail.

As is evident from the enormously low density of the tail (see below), we observe at each point of the tail the sum of the intensities of the particles lying in the cone of vision, and there is no thought of one particle covering or absorbing the radiation of others in an appreciable degree. We also point out at once that the change of the distance from the sun and hence of the illumination by the sun makes little difference for the relatively short portions of the tail available for measurement.

For settling the above question it suffices to determine "sectional intensities" of the tail, to make a set of photometric measurements of the tail on a line perpendicular to the axis of the tail, and to find the sum or the integral of the intensities found along each such line. The lateral expansion of the tail no longer affects such sectional intensities, and they are dependent only on the velocity of the streaming in the tail, as to which we have sufficient data from other sources.

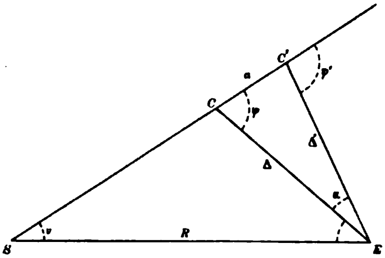
Table I contains a number of these sectional intensities derived from the photographs of the expedition to Teneriffe. The unit is arbitrary, varying from day to day. The distance, *a*, of each cross-section from the head as measured on the plate, is given in degrees. For the determination of these sectional intensities plates that are not very sharp are rather of advantage, inasmuch as they themselves integrate the small irregularities of the structure of the tail.

The following corrections have already been applied to these figures: The intensity of the sky background was determined from the blackening of the plate away from the comet for each section, and was substracted from the total measured intensity. The extinction in the earth's atmosphere for the wave-length 430 $\mu\mu$ was derived from the observations of the sun made on the same day at the same station by Messrs. Müller and Kron. On May 14 only was the weather uncertain and the extinction abnormally large, so that it is easily possible that the comet's head was still more weakened at its low altitude on this day by the mist than would follow from the extinction determined for that day. This would make the intensity for the particles of the tail at a higher altitude still too large in spite of the correction for extinction, and the results of May 14 have accordingly been subsequently given only one-half weight.

It was further to be noticed that the distance Δ' of the different portions of the tail from the earth is not the same, and that the cross-section perpendicular to the apparent axis of the tail as seen from the earth in fact forms a varying angle with the axis of the tail. A change in the distance Δ' causes no change in the surface brightness, but the cross-section of the tail varies inversely proportional to Δ'. Hence all the sectional intensities are to be multiplied with Δ' to reduce them to the same distance. Further, if the angle of the axis of the tail with the line of sight is ψ', the cross-section will be increased as compared with the normal view in proportion to $1/\sin \psi'$, if we regard the tail as cylindrical for a short distance. Hence the sectional intensities are to be multiplied by $\sin \psi'$, in order to reduce them to sections perpendicular to the axis of the tail. The curvature of the tail amounts to only a few degrees in the case of the distances in question here, so that the axis of the tail CC′ may be regarded as coinciding with the prolongation of the radius vector SC from the sun to the comet. But we derive from the triangle SEC′ (sun-earth-particle)

$$\Delta' \sin \psi' = R \sin v.$$

R and v, and hence also the quantity $\Delta' \sin \psi'$, are constant for each plate as we proceed along the tail. Hence it follows that the two effects just considered do not together alter the relations of the sectional intensities which mutually counterbalance in so far as they here come into consideration.



Finally, we investigated (by direct plates of a surface of constant brightness) how the surface brightness of the image depends on the distance from the axis for the objectives employed, and we applied for this a small correction.

The numbers in Table I accordingly give the actual relations of the sectional intensities at the time of each exposure.

We have now to investigate how far the sectional intensities contained in Table I are affected by the increasing velocity of the stream of the tail. If I is the sectional intensity and V the average velocity of the stream through the cross-section, then we must have

$$I \cdot V = \text{a constant}, \tag{1}$$

provided that the stream is stationary and that each particle of the tail retains its luminosity unaltered.[1]

TABLE I

Date........	May 1	May 3	May 7	May 8	May 9	May 9	May 10	May 12	May 14
Objective....	Zeiss	Zeiss	Zeiss	Zeiss	Zeiss	Goerz	Goerz	Zeiss	Zeiss
DISTANCE FROM HEAD a	INTENSITY I OF CROSS-SECTION								
0°26........	....	1.57	....	....	....	....	....	....	....
0.42........	1.06	....	....	....	....	....	....	....	....
0.52........	....	1.11	2.33	2.78	2.52	2.89	2.65	2.23	(2.82)
0.68........	1.27	....	....	....	....	....	....	....	....
0.78........	....	0.76	1.87	1.71	2.00	2.30	2.05	1.69	(2.56)
1.04........	....	0.72	1.38	1.29	1.42	1.93	1.64	1.47	....
1.20........	0.75	....	....	....	....	....	....	....	....
1.30........	....	0.70	1.30	1.30	1.32	1.64	1.41	1.31	(2.23)
1.56........	....	....	1.24	1.06	1.04	1.38	1.25	1.21	....
1.72........	0.61	....	....	....	....	....	....	....	....
1.82........	....	0.58	....	....	....	....	....	....	....
2.08........	....	....	1.01	1.07	0.81	0.97	1.03	1.02	(2.35)
2.34........	....	0.44	....	....	....	....	....	....	....
3.13........	....	....	1.00	0.88	0.69	0.48	0.77	0.77	(2.05)
4.17........	....	....	0.80	0.70	0.53	0.30	0.61	0.74	(2.09)
5.21........	....	....	0.74	0.55	0.41	0.26	0.50	0.64	(1.60)
6.25........	....	....	0.74	0.45	0.32	0.28	0.45	0.56	(1.70)
7.29........	....	....	0.68	....	....	....	....	....	(2.01)
8.34........	....	....	....	....	....	....	....	0.49	....

As to the velocity V, we may assume for the small portion of the tail here in consideration that the repulsive force of the sun is constant and operates in an invariable direction, so that the motion of the particles of the tail relatively to the head is simply a motion of free falling, but solely in the direction away from the sun. The initial velocity being assumed zero, we therefore have for it the formula,

$$V=\text{constant}\cdot\sqrt{a}.$$

In *Publications of the Astronomical Society of the Pacific* (**22**, 125, 1910) H. D. Curtis has communicated the following summary of the observed velocities V of the separate knots in the tail. The values designated by him as uncertain have been omitted.

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[1] This equation is also valid if several streams of different velocities are superposed, provided that the velocity in all cases increases in the same relation with increasing distance from the head.

TABLE II

Date	a	V km/sec	$\frac{V}{\sqrt{a}}$
May 2 to 3	0	0.9	...
May 13 to 14	0.0032	4.9	87
May 27 to 28	0.0042	12.8	198
May 25 to 26	0.0100	19.3	193
June 2 to 3	0.0147	31.8	263
May 28 to 29	0.0186	36.8	270
May 31 to June 1	0.0191	33.3	241
June 6	0.0237	43.8	285
May 26 to 27	0.0270	38.4	234
June 6 to 7	0.0438	70.1	331
May 30 to 31	0.0712	72.4	271
June 7 to 8	0.0905	91.2	303

If we form the quotient $\frac{V}{\sqrt{a}}$, we see that for distances from the head greater than 0.01 the values are nearly constant; also that we may infer from the observations at most a tendency to a more rapid increase of the velocity with the distance from the head as compared with the parabolic formula.

If we assume the parabolic formula, and accordingly form the products $I \cdot \sqrt{a}$, then we obtain the numbers in Table III, in which for clearness the value of the product for a distance from the head of 0°.52 (1 mm on the plates taken with the Zeiss aplanat) has been placed equal to unity. The last column contains the geometrical mean from the results of the separate plates, in which the few values of May 1 have been omitted and those of May 14 have been given only one-half weight.

Contrary to our expectation, it follows from these figures that *the decrease in brightness of the comet's tail is for the most part explained by the falling off in density.* We cannot insist on a precise constancy of the numbers, for the actual relations in the comet's tail are too complicated and too little explained: first of all the streaming is not stationary, as was assumed in deriving equation (1). With increasing activity of the comet, it is presumable that the portion of the tail nearest to the head would become brighter; and with decreasing activity, that the end of the tail would retain a relatively greater brightness. It is also difficult to estimate the possible magnitude of the systematic errors of our measurement.

TABLE III

Date......	May 1	May 3	May 7	May 8	May 9	May 9	May 10	May 12	May 14	May 13-14
Objective..	Zeiss	Zeiss	Zeiss	Zeiss	Zeiss	Goerz	Goerz	Zeiss	Zeiss	
Distance from Head *a*	$I\sqrt{a}$									**Geometric Mean**
0°26......	....	0.63	....	....	....	....	....	....	....	....
0.42......	0.84	....	....	....	....	....	....	....	....	....
0.52......	....	1.00	1.00	1.00	1.00	1.00	1.00	1.00	(1.00)	1.00
0.68......	1.29	....	....	....	....	....	....	....	....	....
0.78......	....	0.83	0.98	0.75	0.97	0.97	0.95	0.93	(1.11)	0.92
1.04......	....	0.91	0.84	0.65	0.79	0.94	0.88	0.93	....	0.84
1.20......	1.20	....	....	....	....	....	....	....	....	....
1.30......	....	0.99	0.88	0.74	0.83	0.90	0.84	0.93	(1.25)	0.89
1.56......	....	....	0.92	0.66	0.71	0.83	0.82	0.93	....	0.81
1.72......	1.16	....	....	....	....	....	....	....	....	....
1.82......	....	0.98	....	....	....	....	....	....	....	....
2.08......	....	....	0.95	0.77	0.64	0.67	0.78	0.91	(1.66)	0.84
2.34......	....	0.85	....	....	....	....	....	....	....	....
3.13......	....	....	1.05	0.78	0.67	0.41	0.71	0.84	(1.77)	0.77
4.17......	....	....	0.97	0.71	0.60	0.30	0.65	0.93	(2.09)	0.71
5.21......	....	....	1.10	0.56	0.44	0.34	0.59	0.87	(2.08)	0.66
6.25......	....	....	1.00	0.63	0.51	0.29	0.60	0.91	(1.79)	0.66
7.29......	....	....	1.09	....	....	....	....	....	(2.66)	....
8.34......	....	....	....	....	....	....	....	0.88	....	....

The constancy of the numbers would moreover be better in the mean if we should give a higher value to the increase of velocity than in the case of a freely falling body, according to the data of Curtis. But even if we select the series decreasing most rapidly, that of May 9, we should still derive from it that the luminosity of the particles of the comet had only decreased to one-third of its value in a period of three days: for this length of time is required according to Curtis' velocities for the particle to proceed to a distance of about 6° from the head of the comet. It appears hardly probable that such a duration of the luminosity could be disputed from the energy of the matter in the tail. But, contrariwise, *our result is immediately explained if we assume that in the tails of comets we are concerned with fluorescent or resonance radiation excited by solar radiation.* Wright (*Lick Observatory Bulletin, No.* 174) very probably observed the resonance radiation of sodium in the spectrum of the tail of Comet 1910 *a*. We should have to assume an analogous origin for the band spectum of *CO* and *CH* in the tail of Halley's comet. The intensity of fluorescence per unit of mass would further not be constant along the tail, but would decrease with the square of the distance from the sun. This gives on the average for the above days of observation a decrease of about 15 per cent for an apparent distance of 6° from the head. This therefore makes little difference qualitatively, but conforms in sense with the observed values.

In what precedes we have investigated only the relative brightness of the different parts of the tail, but the photographs of the expedition to Teneriffe also enable us to derive the absolute brightness in consequence of the fact that on most of the pictures the stars have, in consequence of inaccurate focus, disks of sufficient size so that the diameters (0.1 to 0.3 mm) can be measured, as well as their amount of blackening. Table IV, which follows, gives for each date the comparison star used, and its magnitude according to the *Göttingen Actinometry* reduced to the Harvard system. The mean surface-brightness of the sun was chosen as the unit of surface-brightness, and the value −25.8 mag. was adopted as the photographic stellar magnitude of the sun on the Harvard system, as given by O. Birck in his Göttingen dissertation in 1908. From this was derived the surface-brightness of the comet's tail at the brightest point of the cross-section at a distance of 0°52 from the head; the results are contained in the fourth column of Table IV. The sectional intensities can of course be reduced to the surface-brightness of the sun by applying the same factors. In the column headed α will be found the apparent distance from the comet's head of a point on the radius vector of the comet at an actual distance from it of 0.015 astronomical units. Under the heading I will be found

TABLE IV

Date	Comparison Star	Magnitude	i	α	I	E_0
		m				
May 1.....	10 *Piscium*	4.38	$0.84 \cdot 10^{-11}$	1°0	$1.8 \cdot 10^{-11}$	$2.2 \cdot 10^{10}$
May 3.....	"	4.38	0.70	1.2	1.4	1.7
May 7.....	35 *Piscium*	5.93	1.14	1.5	1.8	1.9
May 8.....	"	5.93	0.89	1.6	1.1	1.1
May 9.....	"	5.93	1.43	1.7	1.7	1.7
May 12....	*x Piscium*	6.28	1.41	2.0	2.1	1.5
May 14....	"	6.28	0.75	2.2	2.3	1.2

the sectional intensities interpolated from Table I for this distance from the head, and referred to the surface-brightness of the sun.

As soon as we attribute the luminosity of the tail to the reflection or resonance of solar radiation, the absolute surface-brightness of the tail gives us a datum for the density of the material of the tail; and the absolute sectional intensities can be used in connection with the determinations of velocity, in order to get some idea as to the amount of matter given off by the comet in forming its tail. Let us at first imagine that the tail consists solely of spherical particles of radius ρ which uniformly diffuse the sunlight in all directions[1] without loss by absorption. Let n be the number of particles to the unit of surface, as seen from the earth; and A be the solar radius, and let the distance of the portion of space from the sun be r, as above. Then the surface-brightness as compared with the surface-brightness of the sun is

$$i=n\pi\rho^2\frac{A^2}{2r^2}\,. \tag{2}$$

If q is the co-ordinate perpendicular to the radius vector and to the line of sight, the sectional intensity will be

$$I=\int i dq=\pi\rho^2\frac{A^2}{2r^2}\int n dq. \tag{3}$$

If we return now to only the actual particles which form the comet's tail and to which we may give a radius ρ_0 and a specific weight s, then we may still ascribe to each particle a certain "effective" radius ρ such that the above formulae will be satisfied. The magnitude $\frac{\rho^2}{\rho_0^2}$ will then represent a sort of albedo. The total mass of the comet's tail which will be projected on the unit of surface as seen from the earth will be

$$\tfrac{4}{3}\pi\rho_0^3 sn=\tfrac{8}{3}\rho_0\left(\frac{\rho_0}{\rho}\right)^2 s\frac{ir^2}{A^2}\,.$$

If we imagine the tail of the comet so compressed along the line of sight that it assumes throughout the maximum density prevailing in its central portion, and if we call the resulting thickness of the tail in the direction of the line of sight, p, then we obtain for this maximum density

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[1] Forward as well as backward, as particles do whose diameters are less than the wave-length of the light.

$$\delta=\tfrac{4}{3}\frac{\pi\rho_0{}^3 sn}{p}=\tfrac{8}{3}\frac{\rho_0}{p}\left(\frac{\rho_0}{\rho}\right)^2 s\frac{ir^2}{A^2}. \tag{4}$$

The total mass which passes through the cross-section of the tail in a unit of time, or the "productiveness," becomes

$$E=\frac{4\pi}{3}\rho_0{}^3 s\cdot v\int ndq=\tfrac{8}{3}\rho_0\left(\frac{\rho_0}{\rho}\right)^2 sv\frac{Ir^2}{A^2}.$$

The unit of length employed above in computing the sectional intensity was 0.2 mm on the plates taken with the Zeiss aplanat, corresponding to 6'.2. The actual magnitude of this unit of length is $\Delta\sin 6'.2$, Δ being the distance of the comet. The above quantities I are therefore still to be multiplied by this. If we finally apply the factor $\sin\psi$, which as stated reduces to normal presentation, then the complete formula for computing the productiveness from the sectional intensity I of the table is

$$E=\rho_0\left(\frac{\rho_0}{\rho}\right)^2 sE_0,\quad E_0=Iv\frac{r^2}{A^2}\Delta\sin\psi\cdot\sin 6'.2. \tag{5}$$

The figures of Curtis for the velocity of streaming at a distance of 0.015 astronomical unit from the head come out about $v=32$ km per second. With these data the values E_0 of Table IV were computed in units of the C.G.S. system. On the average we therefore have, approximately,

$$E=1.5\times10^{10}\rho_0\left(\frac{\rho_0}{\rho}\right)^2 s.$$

If we assume that the section of the tail in space is of approximately cylindrical form, then we should derive from a side view of the tail at 0°.5 distance from the head a thickness of the tail p (in the above explained sense) of about 300,000 km (3×10^{10}cm). Hence with the use of the average value $i=1.0\times10^{-11}$ from Table IV, it follows that

$$\delta=2\times10^{-17}\rho_0\left(\frac{\rho_0}{\rho}\right)^2 s.$$

We shall now follow out two assumptions as to the nature of the particles:

1. In accordance with the theory of Arrhenius as to light-pressure, let the particles be of the order of magnitude $\rho_0=10^{-4}$ cm,

the specific weight being one. The effective radius for such particles will be of the same order as the actual radii, and we will here simply place $\rho=\rho_0$. We here overlook the fact that such particles should chiefly reflect the solar spectrum instead of giving a band spectrum, and we ought to take into account in our computation for them only the fraction of the light of the tail which consists of reflected sunlight.

2. We regard the particles as fluorescing molecules whose radius, ρ_0, is then of the order 10^{-8} cm. It is not improbable that the effective radius of such a molecule is similarly of the same order of size as the actual radius,[1] so that we again place $\rho=\rho_0$ and take 20 as the molecular weight. In the two cases we obtain:

HYPOTHESIS I

$E=1500$ kilograms per second,
$\delta=2\times 10^{-21}$;

HYPOTHESIS II

$E=150$ grams per second,
$\delta=4\times 10^{-24}$.

The free path of the molecules at the last density would be of the order of 10^{19} cm, corresponding to the distance of the nearest fixed stars, so that collisions would no longer occur.

If we expose the earth for a whole day to a current of density 10^{-22} having a velocity of 100 km per second (these figures must roughly hold good according to Curtis' velocities and according to Hypothesis I for the transit of the earth through the comet's tail), then on each square centimeter of the earth's surface there will fall a mass of 10^{-10} grams, carrying an energy of 1/10,000 gram-calories. The total mass caught by the earth amounts then to 250,000 kilograms. Since these figures may be regarded as upper limits, we see how little material action from the transit by the comet's tail was to be expected.

POTSDAM
May 1911

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[1] This holds good at least if the fluorescence is due to the fact that a few electrons in the molecule damped only by radiation swing about their position of rest with frequencies of the order of those of light.

A DETERMINATION OF THE MELTING-POINTS OF TANTALUM AND TUNGSTEN

BY WILLIAM E. FORSYTHE

INTRODUCTION

In measuring temperatures much above 1550° C., the limit so far attained with the constant-volume nitrogen thermometer, the best available method is to determine the temperature in terms of the radiation from a standard source. The standard source that is taken is Kirchhoff's "black body," and the best way of expressing the temperature, at least tentatively, is in terms of a temperature-scale based on either the Stefan-Boltzmann or Wien law of radiation. The usual form of the Stefan-Boltzmann law is

$$E=A\ (T_1T_1^4-T_2T_2^4),$$

where E is the total radiation sent out from the radiator at temperature T_1, to the receiver at temperature T_2, A being a constant. The use of this law requires a measurement of the total radiation falling upon the receiver.

Wien's equation is

$$E_\lambda=c_1\cdot\lambda^{-5}\cdot e^{\frac{-c_2}{\lambda T}},$$

where E_λ is the energy sent out from the radiator corresponding to the wave-length λ and the temperature T, c_1 and c_2 being constants. The use of this equation requires a measurement of the energy corresponding to a particular wave-length.

The temperature-scales as defined by these two relations agree with the nitrogen thermometer up to 1550° C. and have been shown by Lummer and Pringsheim[1] to agree with each other up to about 2000° C. and by Gillett[2] to be in approximate agreement up to about 2500° C.

The temperature of a non-black radiator obtained by measuring the energy corresponding to a particular wave-length is called the

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[1] *Verhandlungen der deutschen physikalischen Gesellschaft*, **13**, 36, 1911.

[2] *Journal of Physical Chemistry*, **15**, 213, 1911.

"black-body" temperature and may be defined as the temperature at which a black body would send out the same amount of radiation of this wave-length. The black-body temperature of any substance will depend upon the condition of the surface, the departure from the true temperature being greater the higher the reflecting power. Since the black-body temperature in general depends upon the wave-length considered, in giving this temperature the wave-length should be specified.

Thus, to determine the true temperature of the melting-point of a metal with an optical pyrometer, either the metal may be melted in a black-body furnace and the temperature taken by observations of the furnace through a small opening, or the black-body temperature of the melting-point may be found by taking observations upon the melting metal with the optical pyrometer, and the difference between the black-body and the true temperature determined by some other means.

The present work gives the results of the direct measurement of the melting-point of tungsten in a black-body furnace and of tantalum and tungsten by a filament method, together with measurements of the black-body melting-point of both tantalum and tungsten by several methods. These black-body melting-points have been reduced to true melting-points by a method to be described later.

ERRORS

An error may be introduced in measuring the temperature of a black-body furnace due to the furnace not being in condition to give black-body radiation, which would make all temperatures as measured from it too low. The furnace will not give out black-body radiation if its walls are not uniformly heated or if there are too many or too large openings in the walls. A simple qualitative test of blackness is given by the degree of invisibility of objects of different radiating powers placed within the furnace. The condition of blackness of the furnace should be further tested by observations at some known temperature.

A second source of error may be introduced by faulty calibration of the pyrometer. This may be due to an error of setting the comparison lamp for a balance with the standard black-body furnace; but if several readings are taken, and care is exercised always to use the same part of the filament of the comparison lamp, this error may be very much reduced. Also the temperature of the standard black body used in calibrating the pyrometer may not be known accurately—an error of one degree at 1500° C. leading to an error of 3° C. at 3000° C.

TABLE I

COMPARISON OF PYROMETERS

Source	Spectral	With Glass
Small furnace	1062° C.	1062° C.
Small furnace	1218	1218
Small furnace	1386	1387
Nernst glower	1861	1858
Nernst glower	1995	2000
Nernst glower	2080	2078
Nernst glower	2080	2085
Nernst glower	2174	2172
Graphite tube furnace	1990	1990
Graphite tube furnace	2370	2380
Graphite tube furnace	2480	2490

In measuring temperature with an optical pyrometer an error may be introduced in determining the wave-length used with the colored glass, as the position of the maximum effective transmission of the glass may depend upon the temperature of the source, that is, on the distribution of energy in the source. To overcome this difficulty Professor Mendenhall[1] designed a direct-vision prism spectroscopic pyrometer by means of which observations can be made with light having a spectral range of not more than 25 Ångström units. Setting this pyrometer for $\lambda = 0.658\ \mu$, which was the wave-length corresponding to the position of the maximum transmission of the colored glass used on the other pyrometer, the spectral pyrometer was calibrated and the two pyrometers compared. In Table I are compared the results for the different temperatures and different sources taken with the spectroscopic and regular pyrometer, which show that within the limits of setting the two agree.

In any determination of a melting-point an error is always liable to be introduced due to an uncertainty as to the exact time of melting; furthermore, some chemical reaction may take place in the furnace between the metal and some gas or vapor that may be present, thus raising or lowering the melting-point. The method taken to guard against the above errors will be pointed out farther on.

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[1] *Physical Review*, 33, 74, 1911.

In comparing high-temperature measurements there is much confusion due to the use of three different values for the melting-point of platinum. In Germany two temperature-scales are in use, the one giving the melting-point of platinum as 1788° C., the other placing this temperature at 1745° C. In this country most of the work is referred to the melting-point of palladium at about 1549° C. and that of platinum at about 1755° C., and in this paper we shall accept these, which are the values of Day and Sosman.[1] The constant c_2 of Wien's equation will be taken as 14,500, while the wave-length used with the optical pyrometer was 0.658 μ.

PREVIOUS DETERMINATIONS

In recent years there have been several determinations of the melting-points of tantalum and tungsten by indirect methods requiring extrapolation of several hundred degrees to reach the true melting-point. The first attempt to measure these high temperatures was in 1906 by Waidner and Burgess,[2] who found a value for the black-body melting-point of tantalum and tungsten by obtaining an equation between the current and the black-body temperature of a filament in a commercial lamp and then noting the current required to melt the filament. As this equation was obtained from measurements made below 1900° C., an extrapolation of about 1000° C. was required to reach their values for the black-body melting-point. To obtain the true temperature they assumed for tantalum and tungsten a relation between the black-body temperature observed with red light and the black-body temperature observed with blue light, a relation which had been found to hold very well for platinum. Their final value was 3080° C. for the melting-point of tungsten, and 2910° C. for the melting-point of tantalum. The method they used to obtain the relation of the black-body temperature to the true temperature may give rise to a considerable error, and moreover von Wartenberg[1] has shown by direct observation that the black-body melting-point of unpolished tungsten is about 100° C. lower than the value they obtained.

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[1] *American Journal of Science* (4), **29**, 93, 1910.

[2] *Journal de Physique* (4), **6**, 830, 1910.

In 1910 von Pirani[2] found for both tantalum and tungsten a relation between the true temperature and the black-body temperature by measuring the true temperature with a thermocouple between twisted filaments of the metal, and the black-body temperature with a Wanner pyrometer sighted at these twisted filaments. Whether the thermocouple would give the true temperature, or a temperature higher or lower than that of the surface, would depend upon how closely the wires were twisted together, while without doubt the black-body temperature as measured would be too high, due to the fact that the openings between the twisted filaments would more or less approach a black body in their radiation. These measurements on the true temperature and the black-body temperature were made below 1700° C., and to extrapolate to the melting-point it was assumed that the energy used in the filament varied as the mth power of the absolute temperature, that is:

$$E=e\cdot i=AT^m, \text{ or}$$
$$\log ei=A^1+m\cdot\log T,$$

where e is the applied voltage and i the current through the filament. This equation, which is linear in log ei and log T was found to hold over a small range of temperature, about 500° C., and was then assumed to hold up to the melting-point, which was 1300° higher. After applying a correction, Pirani's final result was 3250° C. for tungsten and 3000° C. for tantalum, using the scale giving the melting-point of platinum as 1745° C. This correction, of about 200°, added to the results as obtained by extrapolation, was due for the most part to two causes: the first being the conclusion that his temperatures as measured around 1600° C. were too low and the second the fact that in the equation $\log ei=A^1+m\cdot\log T$, m cannot remain constant but must decrease with increasing temperature. A similar extrapolation gave for Waidner and Burgess (*loc. cit.*) a result for the black-body temperature of the melting-point that seems to be about 100° too high, so that a like error may be introduced here. As Pirani gives no values for the black-body melting-point, a direct comparison cannot be made.

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[1] *Berichte der deutschen chemischen Gesellschaft*, **40**, 3287, 1907.

[2] *Verhandlungen der deutschen physikalischen Gesellschaft*, **12**, 301, 1910.

Later,[1] with somewhat similar extrapolations, Pirani found the black-body melting-point of tantalum and also measured this melting-point with a pyrometer. From this black-body melting-point and the reflecting power of tantalum the true melting-point was found by the same method as that used by von Wartenberg, which is described below. This result seems to be free from some of the objections stated above, as no corrections due to errors in the method were made.

Wartenberg (*loc. cit.*) in 1910 obtained the black-body melting-point of tungsten by melting this metal in a vacuum tube, in which a small piece of tungsten was mounted as an anode; the discharge from a Wehnelt cathode sent down upon it raised the temperature to the melting-point. Later he[2] measured the reflecting power of tungsten and was thus able to apply the equation used by Holborn and Henning to compute the true temperature. This equation, derived directly from Wien's radiation equation, is

$$1/S-1/T=c_2/\lambda \cdot \log A/\log e,$$

where T is the true temperature, S the black-body temperature, c_2 the constant of Wien's equation, λ the wave-length used, A the absorbing power, that is $(1-R)$, where R is the reflecting power. Wartenberg tested this relation for several different metals and found that it held within his limits of error up to about 2400° C. By applying this equation to his values for the black-body melting-point of tungsten he obtained about 2930° C. for the true melting-point.

The only attempt to measure the melting-point of tungsten and tantalum by a direct method was by Ruff[3] in 1910, using a vacuum furnace with a heater tube of carbon. In this furnace the heater tube had in it longitudinal slots that served the double purpose of allowing observations to be made upon the temperature inside and also of increasing the resistance. The metals studied were placed in the furnace in the form of Seger cones and the temperature taken when the cones toppled over. Ruff obtained for the melting-point of tungsten by this method about 2650° C. The work of Wartenberg (*loc. cit.*) indicates that this is far too low, as it is 100° C. lower than the black-body melting-point.

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[1] *Verhandlungen der deutschen physikalischen Gesellschaft*, 13, 540, 1911.

[2] *Ibid.*, 12, 105, 1910.

[3] *Berichte der deutschen chemischen Gesellschaft*, 43, 1564, 1910.

DIRECT DETERMINATION

The vacuum furnace used in this work was designed in this laboratory by Mr. Steve and Mr. Barnes along the same general lines as the furnace of Arsem[1] of the General Electric Company and that of King[2] of the Mount Wilson Solar Observatory.

In this furnace, as shown in Fig. 1, the terminals, heater tube, etc., were all inclosed in an air-tight, water-cooled jacket made of iron pipe. The jacket was mounted vertically, with all the working parts of the furnace fastened to the top plate so that they could be easily removed. In order to be able to observe the inside of the furnace there was a window in each end 6 cm in diameter covered with plate glass 0.7 cm thick, held in place between two rubber gaskets. The terminals consisted of copper tubes 1.3 cm in external diameter and 1 cm in internal diameter, each terminal being made of two of these tubes, for inflow and outflow of cooling-water, which were connected at the bottom with a hollow brass casting, to which was fastened the graphite clamps for holding the heater tube. The graphite clamp just fitted the ends of the heater tubes of Acheson graphite which were obtained from the Acheson Graphite Company, turned to the size and shape desired. The heater tubes were 30 cm long, 1.4 cm in internal diameter, and 1.9 cm in external diameter, excepting about 2.5 cm at each end which were 2.5 cm in diameter. This made the thickness of the hot part of the tube wall about 0.3 cm. No allowance was made for the expansion of the heater tube other than slipping in the graphite clamps. To insulate the heater tube from losses of heat to the sides of the jacket it was surrounded by two carbon tubes, as shown in the figure, it being found that when the inner insulating tube was removed the

[1] *Transactions of American Electrochemical Society*, 9, 153. 1906.

[2] *Astrophysical Journal*, **28**, 300, 1908.

maximum temperature obtainable with a given applied voltage was about 400° C. lower than when it was in place.

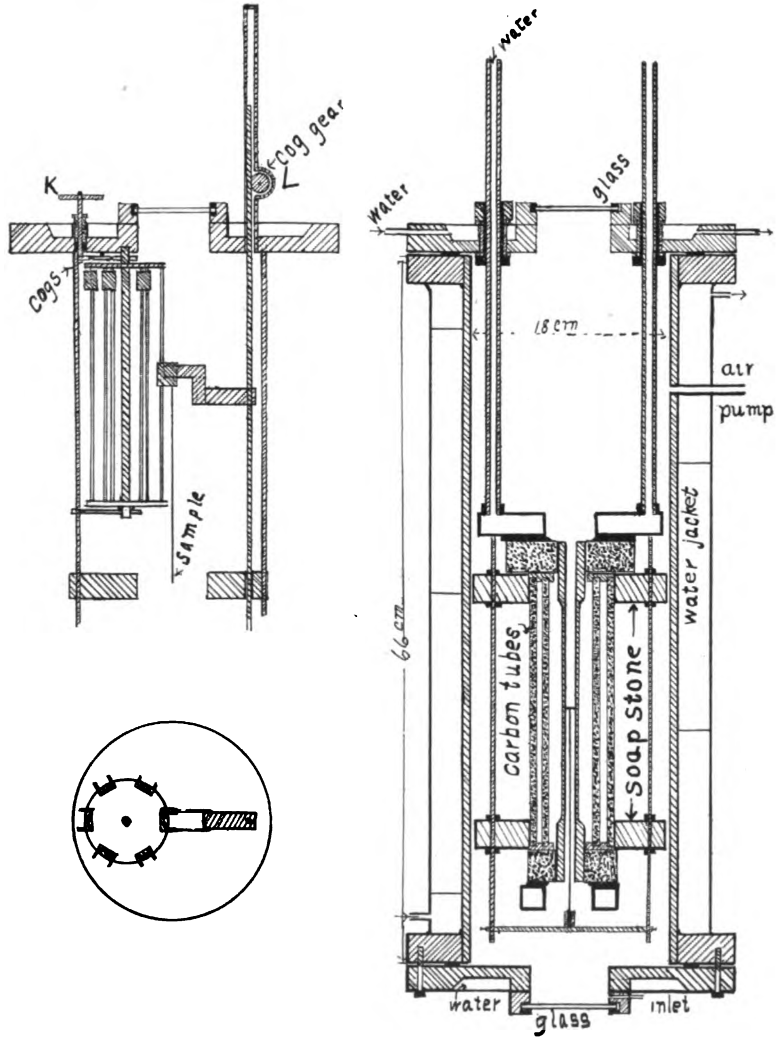

FIG. 1

The heating current was taken from a transformer, the current through the primary being regulated by two electrolytic rheostats of copper sulphate solution in earthenware tanks. These two rheostats were connected in parallel so that a small variation of current could be obtained and the temperature easily controlled. Most of the high-temperature work was done with the transformer set so as to give 30 volts from the secondary, and with this arrangement and all the primary resistance in, the primary would draw about 150 amperes, which gave about 550 amperes through the heater tube. Table II gives the current through the heater tube, the voltage across the copper leads, and the resulting temperature of the furnace. If the temperature of the furnace was not raised above 2950° C. a heater tube would stand two or three runs; i raised much higher would stand but one run, and at a temperature higher than 3000° C. the tube would last but a few minutes.[1]

To obtain the vacuum in the furnace an electrically driven Fluess air-pump was used giving a vacuum of a few tenths of a millimeter pressure, which we were able to hold in the furnace for several days. However, when the furnace was run at the high temperatures, the pressure was hardly ever less than a few millimeters.

TABLE II

Current through Heater Tube	Voltage across Copper Leads	Temperature of Furnace
550 amperes	10 volts	1500° C.
790	13	1780
1000	16	2110
1100	18.3	2425
1175	20	2650
1240	21.5	2900
1400	25	3100

A disk of graphite about 0.1 cm thick was placed inside the heater tube somewhat nearer the lower end of the tube and held in place by a graphite rod 0.6 cm in diameter at the lower end and 0.2 cm in diameter at the top, this rod being insulated from the remainder of the furnace to prevent the formation of an arc at the center of the heater tube.

That the necessary conditions for black-body radiation were satisfied was shown by a careful determination of the melting-point of platinum with a calibrated optical pyrometer. The results indicated a very satisfactory condition of blackness, since the value obtained for the melting-point agreed well with the best accepted values. Again, after the furnace had been run on steady current for a few minutes no difference could be detected between the temperature of the disk and the walls of the heater tube; furthermore, small pieces of magnesium oxide, small pieces of tungsten filaments, and a tungsten cone could not be seen when in the furnace unless they projected over an opening between the disk and the walls of the heater tube.

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[1] Watts and Mendenhall, *Physical Review*, **33**, 65, 1911.

The specimens to be melted were carried by an arrangement shown in Fig. 1, the specimens being in the form of hairpin filaments 0.014 mm in diameter, the free ends being fastened to the two arms of the carriage. When a specimen was wanted in the furnace it was turned in place by the handle *K* and then pushed down in the furnace by the handle *L*. The specimen *s* completed an electric circuit so connected to a relay that a bell would ring when the specimens melted and broke the circuit.

The temperature of the heater tube was measured with an optical pyrometer sighted through the plate-glass window on the disk in the heater tube. This made necessary a correction for the light absorbed and reflected by the plate glass which was easily determined by measuring first the temperature of some steady source with the pyrometer and then the apparent temperature of the same source when sighted through the plate glass. For the first case, from Wien's equation,

$$E_1=c_1 \cdot \lambda^{-5} \cdot e^{\frac{-c_2}{\lambda T_1}},$$

where E_2 is the energy corresponding to the black-body temperature T_1. For the second case, through the plate glass,

$$E_2=c_1 \cdot \lambda^{-5} \cdot e^{\frac{-c_2}{\lambda T_2}},$$

where E_1 is the amount of light passing through the plate glass from the source at the black-body temperature T_1, T_2 being the (apparent) black-body temperature corresponding to E_2. Taking the logarithms of these two equations and substracting the first from the second,

$$\log [E_2/E_1]=\log K=c\ (1/T_2-1/T_1),$$

where K is the fractional part of the light let through by the plate glass and $c=c_2/\lambda(-\log e)$. This equation is linear in $1/T_1$ and $1/T_2$, the graph of $1/T_1$ as a function of $1/T_2$ making an angle of 45° with the axis. Thus a measurement of T_1 and T_2 at but one temperature would give the correction at all temperatures. In practice, however, T_1 and T_2 were measured for several temperatures, using as a source a Nernst glower mounted in a hollow, talc holder. As the glass would get a little fogged at times this correction was determined after each run. For clean glass this correction amounted to about 30° C. at 1600° C. and to about 70° C. at 2800° C.

In measuring the high temperatures use was made of an optical pyrometer of the Holborn-Kurlbaum type.[1] The comparison lamps were 6-volt, 1.3-ampere carbon lamps with pear-shaped bulbs about 2.5 cm in diameter, the horseshoe filament being so mounted that the top of the filament came in the center of the bulb. These lamps were aged for 24 hours on 1.35 amperes before being calibrated. In calibrating these lamps we used the sector method and as the known temperature the melting-point of palladium, this fixed point being determined directly by noting the temperature of melting Kahlbaum palladium. The melting-point of palladium was chosen as the standard temperature because it is about the highest temperature which can be directly observed with a carbon lamp in the pyrometer.

A 1/60 sector was used in the work with platinum, and a 1/180 in the work with tungsten. Starting with 1549° C. (palladium point), the range of the pyrometer with the 1/60 sector was up to 2482° C., with the 1/180 up to 2919° C. The 1/180 sector was 26.8 cm in diameter, which made the opening about 1.7 mm wide at the narrowest point. To determine whether there was an error due to diffraction with this small slit, a sector having ten openings the size of these was made up and points found on the calibration-curve for this sector. No difference was detected in using a sector made up of ten openings and one made with one large opening having the same total area.

Several times the same lamp was calibrated from the gold point

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[1] Mendenhall, *Physical Review*, 33, 74. 1911.

(1063° C.) and from the palladium point, which gave a chance to compare the palladium point with the gold point. As an average of several determinations of the palladium point by extrapolation from the gold point we obtained 1548° C., while an extrapolation down from the palladium point gave 1063° C. for the gold point.

As a test of the furnace and the pyrometer a determination of the melting-point of platinum was made. As it was known that platinum would react with carbon vapor at high temperatures, the platinum was inclosed in small magnesium tubes. The platinum gave, when it broke, all evidence of fusion, the ends being globular and small bright masses of platinum being scattered all along the lower end of the tube. Table III gives the results for the work on platinum in the graphite tube furnace. For the work in the furnace Baker platinum was used and the melting-point of this compared with that of Heraeus by the method of Mendenhall and Ingersoll.[1] The two specimens were melted on a Nernst glower and from their values for the constants of the glower the difference in temperature was calculated. As an average of ten melts the melting-point of Heraeus platinum was found to be six degrees higher than that of the Baker platinum. This gives a final average of 1755° C. for the melting-point of Heraeus platinum, which is in good agreement with the value of 1755° as set by Day and Sosman (*loc. cit.*).

TABLE III

MELTING-POINT OF PLATINUM

1909/12/2	1749° C.
	1744
	1744
	1746
	1744
1910/2/12	1756
	1751
	1749
	1756
Mean	1749° C.

The melting-point of tungsten has been measured in the vacuum furnace described above, indications of a melt being obtained by three different methods. The first was, as outlined above, the use of hairpin filaments, a melt being indicated by the bell ringing when the circuit was broken. The pieces that we were able to get out of the furnace showed every indication of fusion, the ends being tipped with small round bits of the metal. The second method was to bore a small hole in the disk in the center of the heater tube and to lay over this hole a small piece of filament which could be observed to melt and disappear as the temperature was raised. This was undertaken to find out whether direct contact of the tungsten with the carbon made any difference. The results show that the difference, if any, is slight. The third method was to press powdered tungsten into the form of a Seger cone and place this in the heater tube in such a manner that the tip of the cone would project over an opening between the disk and the walls of the tube. The results are given in Table IV.

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[1] *Physical Review*, **25**, 1, 1907.

TABLE IV

MELTING-POINT OF TUNGSTEN IN FURNACE

1911/2/18, Filament	2948° C.
3/1, Filament	2972
3/6, Filament	3010
	3000
3/7, Filament	2948
3/15, Filament	2984
	3000
	3000
3/28, Filament	2917
3/28, Filament on disk	3000
	3000
	2947
3/28, Cone	2937
Mean	2974° C.

MELTING-POINT AS OBSERVED IN A TUNGSTEN WEDGE

8/28	2970° C.

Several attempts were made to melt tantalum in the graphite furnace but with no success. The tantalum was put in the furnace and the temperature run up to between 3000° C. and 3100° C., but when it was taken out it showed no signs of fusion. From the results given below, the melting-point cannot be as high as this. When the tantalum had been in the furnace at this high temperature it did not look at all as it did when put in; indeed, it had more the appearance of a cinder when broken and observed with a microscope. It would seem that some compound with the carbon vapor was formed which had a higher melting-point than the tantalum.

INDIRECT METHOD

A method of obtaining the relation between the true temperature and the black-body temperature has been described by Professor Mendenhall,[1] tested for platinum, and found to give results that agree very closely with the theoretical and experimental results for this relation. This method consists of making a hollow wedge of a thin sheet of the metal and determining the black-body temperature by observing on the outside of the wedge and the true temperature inside of the wedge.

Such a curve giving the relation between black-body and true temperature for tungsten has been determined by Professor Mendenhall and the writer, while Mr. McCauley assisted with the curve for tantalum. These curves were determined with the metals in the lamp described below. The curve for tantalum was run up to the melting-point and observations made on both the true temperature and the black-body temperature, while the tungsten was run up to a black-body temperature of 2500° C., which corresponds to a true temperature of about 2700° C.

The black-body melting-point of tungsten was found by two methods. In the first place it was determined in a manner similar to that used by Waidner and Burgess,[2] excepting that in this case the lamp filaments were balanced photometrically against the heater tube of the furnace and the temperature taken with the optical pyrometer. Several lamps were worked with, but unfortunately only one lamp melted at the point under observation. This method required a correction for the lens used to project the image of the furnace on the lamp filament and also for the walls of the lamp bulb. This latter correction was determined after the melt in order to correct for any blackening of the bulbs. The ends of the filaments gave indications of a true fusion, being tipped with shiny beads of the metal.

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[1] *Astrophysical Journal*, **33**, 91, 1911.

[2] *Bulletin of the Bureau of Standards*, **2**, 319, 1906.

The second method was to mount the metal studied between brass terminals and put these in a water-cooled brass vessel with windows for observing the temperature. The vessel was pumped out with a Gaede pump and calcium heated[1] inside in order to remove the last trace of gas. This vessel was 12 cm in internal diameter and 18 cm high, with windows 2.5 cm in diameter. The lower terminal was connected to the outside with a packed joint which allowed the filament or wedge to be kept straight. After the pump had been run for 24 hours the tantalum would continue for some time at constant temperature, if the current was kept constant; and the drop in voltage across the filament would also remain constant. This was taken to indicate good working condition. The tantalum filaments were rolled from strips obtained from Siemens and Halske, while the tungsten filaments were of two kinds. The first were from a series tungsten lamp from the General Electric Company, and the second were made from ductile tungsten obtained from the same company. All the filaments used were about 3 cm long, the flat ones from 0.7 to 2.0 mm wide and from 0.05 to 0.3 mm thick, and the filaments from the tungsten lamp were 0.3 mm in diameter. The flat filaments were made a little narrower at the middle so that they would heat more at this point.

TABLE V

BLACK-BODY MELTING-POINT OF TUNGSTEN............. $\lambda=0.658$

Filament........................	2827° C.
Strip...........................	2778
	2793
	2814
	2814
Lamp over furnace..............	2791
Strip...........................	2791
	2770
	2770
	2791
	2820
	2800
Mean........................	2797° C.

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[1] Soddy, *Proceedings Royal Society*, A **78**, 429, 1906.

In Table V are given results for tungsten, and for tantalum in Table VI.

Taking the mean of the values for the black-body melting-point of tungsten as given in Table V, and extrapolating the curve showing the relation between the true and black-body temperature to this black-body temperature, we obtained 3030° C. as the true melting-point of tungsten. This is thought to be too high, as the surface of the tungsten used in the work with the wedge changed as a result of being run so long at the high temperatures.

TABLE VI

Black-Body Melting-Point of Tantalum............. $\lambda=0.658$

(Figures in parentheses following values indicate weights given)

Strip..........................	2486 (1)
	2517 (2)
	2530 (3)
	2492 (1)
	2515 (2)
	2495 (1)
	2472 (½)
Mean......................	2511° C.

Melting-Point of Tantalum as Observed from Wedge

	2795° C.
	2786
	2805
	2790 (2)
	2785
	2830
	2805
Mean......................	2798° C.

DISCUSSION OF RESULTS

The results for the melting-point of tungsten as obtained from the graphite tube furnace are seen to have quite a wide variation. This for the most part is thought to be due to the temperature of the furnace increasing in some cases too rapidly because of varying voltage. In this case the disk would not immediately assume the temperature of the walls of the heater tube. The variation of the observed black-body melting-point of both tantalum and tungsten may be due to the pyrometer not being sighted at the exact point on the strip at which melting occurred. The mean of the values given for both the black-body temperature and the true temperature are thought to be accurate to about 1 per cent.

TABLE VII

VALUES FOR THE MELTING-POINT OF TUNGSTEN

Observer	Date	Method	Value Accepted for Melting-Point of *Pt.*	Trials	Value	Average Departure from Mean
Waidner and Burgess	1907	Extrapolation . . .	1753° C.	8	3080° C.	28° C.
Pirani.	1910	Extrapolation . . .	1745	. .	3250	. .
Ruff.	1907	Carbon-tube furnace	1745–55	. .	2650	33
Wartenberg. .	1910	Black-body melting-point and reflecting power	1745	7	2930	30
This work. . .	1911	Furnace	1755	13	2974	27
This work. . .	1911	Observations on wedge	1755	1	2975	. .
This work. . .	1911	Black-body melting-point and wedge	1755	11	3030	16

VALUES FOR THE MELTING-POINT OF TANTALUM

Waidner and Burgess	1907	Extrapolation . . .	1753	5	2910	. .
Pirani.	1910	Extrapolation . . .	1745	. .	3000	. .
Pirani.	1911	Black-body melting-point and reflecting power	(*)	14	2700	30
This work. . .	1911	Wedge.	1755	8	2798	13

*This value has been reduced to a scale with the melting-point of palladium 1549° C. in place of 1575° C. and c_1=14,500 in place of 14,200.

The results for the true and black-body melting-point of tungsten are seen to be about 100° C. lower than the corresponding temperature as found by Waidner and Burgess (*loc. cit.*). As was pointed out above, their work might have an error of that amount. The value for the melting-point of tungsten as found by v. Pirani (*loc. cit.*) is about 300° C. higher than the results given above. As was shown above, his results are in question due to possible errors. The results for the black-body melting-point of tungsten as given by von Wartenberg (*loc. cit.*) check well with the values given in this work, the difference in the final values being about that due to the different scales used. This is a satisfactory agreement, as he actually measured the black-body temperature.

As for the results on tantalum, they do not agree with other determinations of this point. Unless the metal changes in some way at the higher temperatures, it is hard to see why there should be this much difference between the several determinations. In Table VII is given a summary of all the work on these two metals, with values found and the scale used.

SUMMARY

The melting-point of tungsten has been found by two methods which are seen to agree within about 1 per cent, the value obtained by measuring the point directly in a black-body furnace being 2974° C., while that obtained from the relation between the true and black-body temperature as measured from the wedge is 3030° C.

The melting-point of tantalum was determined from direct observations upon a melting wedge of the metal, giving a value of 2798° C.

In conclusion I wish to express my thanks to Professor Mendenhall for his many valuable suggestions and assistance in all parts requiring two observers.

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September 1911

ON THE NEED OF ADJUSTMENT OF THE DATA OF TERRESTRIAL METEOROLOGY AND OF SOLAR RADIATION, AND ON THE BEST VALUE OF THE SOLAR CONSTANT

By FRANK W. VERY

The field of terrestrial meteorology is one of extreme complexity. The several data are so intimately interrelated that changes in one or more of the values will affect the entire series. The *best* value of any one quantity is that which consistently harmonizes the largest number of elements in this complicated interlocking system; and of these related quantities, the one on which many others depend, and the most important astrophysical datum is the solar constant of radiation, now generally recognized as varying slightly, but which nevertheless still maintains a substantial uniformity through the ages which is truly wonderful, and is to be explained only on the supposition that it depends upon some physical constant, such as the boiling-point of the photospheric material or the dissociation temperature of some refractory compound.

The solar constant cannot be directly observed. Its determination, therefore, is open to an unusually wide latitude of interpretation. Such diverse values have been assigned to it that skepticism is natural in regard to even the most plausible results. If the accuracy of the conclusions were at all comparable with the labor which has been expended upon them, we ought to know the value of this fundamental quantity to three or four significant figures, but we are still debating over the first figure.

The reformation of the methods for obtaining the value of the constant introduced by Forbes, given a solid rational basis by Langley, and further improved on the mathematical and observational sides by Crova and Savélief, has apparently been relinquished at the present time in favor of methods which do not differ essentially from those of Pouillet. The elaborate researches of Abbot and Fowle, while purporting to be a continuation and perfection of Langley's methods, are in reality a complete abandonment of the essential principle which was admirably stated by Langley in his paper "On the Amount of the Atmospheric Absorption,"[1] and which has been repeatedly recognized by many able investigators. The principle is—

> that radiant energy is not a single emanation but the sum of an infinity of diverse ones, each with its own separate rate of absorption. It follows that the coefficient of transmission is truly constant only in the case of the absolutely homogeneous ray, which the ordinary photometer or thermometer cannot in the least discriminate; and which the finest linear thermopile or bolometer can but approximately discern, and hence that the original light of the star or heat of the sun, and the amount absorbed, can at best only be found approximately.[2]

The value of approximately 2 calories which is given in the second volume of the *Annals of the Astrophysical Observatory of the Smithsonian Institution* is to all intents and purposes the same as that of Pouillet; for if the positive corrections which are known to be required for the latter, on account of the defects of the water pyrheliometer, are applied, we shall have a value of the solar constant differing very little from that which is now offered to us as an improvement on Langley's result, but which is quite the reverse. The methods used in the *Annals* have all the characteristics of the original Pouillet model, including the admirable agreement of the separate measures and the small apparent probable error of the final result. The fallacy attending such methods may be recognized when it is known that actual reliable measures of solar radiation may be made within the atmosphere which exceed the supposed value outside the atmosphere, as I shall show in a subsequent paper.

While recognizing the elaborate and painstaking character of the researches contained in the second volume of the *Annals*, I cannot admit that they have fixed even the first figure of the solar constant; but, on the contrary, the necessity of harmonizing all of the data proves conclusively that Langley's original figure is nearer to the truth.

One of the weak points in the argument relates to the absorbent power of aqueous vapor. Acting on the hypothesis that the transmission by the aqueous vapor varies as a power of the experimental coefficient of transmission in which the exponent of the power is taken equal to the aqueous mass, Abbot and Fowle draw a curve of t to the twenty-sixth power to represent the aqueous transmission of telluric radiation by the air above Mount Wilson. It is true that they cast some doubt upon the justification of the procedure, saying: "It is barely possible that the water-vapor column through which Rubens and Aschkinass observed was so much more dense than that in the atmosphere that its absorption bands were broadened, owing to the great density of the absorbent, so that there was a stronger absorption than an equal amount of vapor would have produced if contained in a long column of less density."[1]

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[1] *American Journal of Science* (3), **28**, 163, 1884.

[2] *Op. cit.*, p. 167.

This supposition, which is given as "barely possible," is one which I have already demonstrated. Reference may be made to p. 99 of my work on *Atmospheric Radiation*, published by the United States Weather Bureau in 1900, where it is stated that "with a radiating source at the boiling-point of water, 10 microns of liquid water absorb 156 times as powerfully as the same amount of water distributed through about 100 meters of air." Moreover, the transmission of radiation by aqueous vapor at constant tension does not diminish according to a power of t equal to the depth of the vapor traversed, but the initial absorption of the first part of the vaporous layer immensely exceeds the subsequent absorption. Neither does the vapor have to be of considerable tension in order to be effectual. With a dew-point nearly 30° C. below zero, when only a minute amount of aqueous vapor remains in the air, I have found the great water band Ξ in the solar spectrum exerting almost complete absorption between wave-lengths of 5 μ and 8 μ. Owing to these properties of vaporous absorption, the transmission by the aqueous vapor above Mount Wilson cannot be equated to the twenty-sixth power of the aqueous transmission observed with 75 cm of concentrated steam, but is more nearly identical.

A correction of +18.3 per cent is made in the *Annals* to the Allegheny measures of transmission by the aqueous vapor in 110 meters of air, derived from observations on a distant radiator, and the error is attributed to "stray sunlight (probably reflected from the air)." As these measures were made by a method devised by me, which I have since perfected, and which, as now modified, gives reliable results, I may perhaps be permitted to say that the explanation offered is apochryphal, but that the error indicated is about right. The radiator used at Allegheny was freely exposed, and the radiating surface of blackened copper suffered cooling by sudden gusts of wind, and also reflected some solar and sky radiation, because, although lampblack is a very poor reflector, the solar radiation so greatly exceeds that from a surface near the boiling-point of water that even this small reflection cannot be neglected. In my recent measures I have thoroughly protected the radiator from reflected sunlight and wind by a black cloth hood, and a suitable shelter.

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[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, 2, 169.

With columns of moist air longer than 100 meters, the absorption increases very slowly. The case is similar to that of radiation of gases where, as I have shown in the work just cited, one meter of carbon dioxide is a maximum radiant layer. Lest anyone may suppose that this matter is doubtful, I will refer to a recent research on the "Nocturnal Variation of Temperature," by W. Schmidt, who gives in the *Sitzungsbericht* of the Wien Academy of Science for March 1909 a final value of the radiation coefficient of air, derived from meteorological observations, $10^4 \times \sigma = 1.84$, which may be compared with my value from laboratory experiments which gave 2.16.

On p. 172 of the *Annals*, the authors say: "It is highly probable that, considering the greater air-mass attending the oblique passage of many of the rays to space, nine-tenths of the radiation of the solid and liquid surface of the earth is absorbed by the water-vapor of the atmosphere even on clear days."

My value of the transmission of terrestrial radiation is four times as great as that of the *Annals*. The low angle of the rays makes little difference. An absorption which is already nearly complete for a zenithal ray, is not much increased in passing at a lower angle. Meanwhile, in spite of the absorption, there are regions of the spectrum between the bands which are passing through the air freely. This was first demonstrated beyond a doubt by my observation, in October 1884, at the Allegheny Observatory, of lunar radiation in the spectrum from a rock-salt prism between deviations

38° and 33°, corresponding to the radiation from bodies at terrestrial temperatures. This observation was afterward repeated many times by Mr. Langley and myself, always with the same result.

Further direct comparisons, not of an ideal "black body," as a passage on p. 174 of the *Annals* implies, but of the lunar surface and "some of the materials likely to be prevalent on the moon's surface," gave me for the effective midday temperature of the equatorial regions of the moon 454° Abs. C.,[1] which is 61° higher than the value which Abbot and Fowle assign by estimation, using their value of the solar constant. As I can see no reason for abandoning the result of my lunar measurement, I prefer to take it as a basis for an estimate of the solar constant, getting about 3 calories.

On p. 15 of the *Annals* the categorical statement is made that "we know that in general the lower layers of the air have smaller transmission coefficients than the upper ones, owing to the generally low level of the larger quantities of dust and humidity." The assertion, which is made without any reservation, and concerning solar radiation, is true only for the unsifted rays, and does not apply to the actual residual radiation which has already experienced its greatest absorption by aqueous vapor in the upper air. Layers of saturated or nearly saturated vapor have an additional and especially strong absorption. Such layers may be found at any altitude. The dust, it is true, is most prevalent in the lower layers of air, but its coefficient has not been considered separately in the *Annals*.

The coefficient of transmission of terrestrial radiation which I have given is supported by the data of meteorology. Percival Lowell, in his equation for cooling by nocturnal radiation,[2] establishes his thesis upon the fact that nocturnal cooling goes on two and one-half times as fast under a clear sky as below clouds. Taking 50 per cent for the transmission of telluric radiation by clear air, he obtains a "relative emissivity," $e=0.47$. Direct measurements having given me 40 per cent for the transmission by clear air, substitution in Lowell's equation with the following data:

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[1] *Astrophysical Journal*, **8**, 284, 1898.

[2] "Temperature of Mars," *Proc. American Acad. of Arts and Sciences*, **42**, 661, March 1907.

cloud transmission $=2/5\times 0.40=0.16$, average for day sky $=0.28$, average day and clear night $=0.34$, average day and cloudy night $=0.22$, and using his data for absolute temperature and for cooling in centigrade degrees, gives

$$\frac{292-10}{292-\ 4}=0.9196=\sqrt[4]{\frac{y(1-0.34e)}{y(1-0.22e)}}\,,$$

$$e=0.585$$

This represents the radiated part of the loss, and if the true emissivity of the earth's surface is unity, 0.415 of the energy obtained from solar radiation is left to be dissipated by convection.

Lowell's figures are for temperate latitudes. In the very moist tropics, nocturnal cooling is only about half as great, while over the ocean the total diurnal change of temperature of the water is less than ½° C. Eleven cloudy days in the tropics gave me for the nocturnal cooling an average of 2°0, whereas sixteen clear days at the same station gave 5°4 during the same interval. Mean temperature $=+30^\circ$ C. Assume transmission $=0.20$,

$$\frac{303-5.4}{303-2.0}=0.9887=\sqrt[4]{\frac{y(1-0.255e)}{y(1-0.165e)}}\,,$$

$$e=0.185.$$

These examples show that whereas about 60 per cent of surface heat may be emitted as radiation from temperate regions, only one-third as much heat escapes in this way in the tropics. It is possible that the low value of 10 per cent, adopted in the *Annals* for the average transmission of telluric radiation, may apply to saturated air over the tropical oceans where the moisture is in an especially absorbent form; but this value cannot be admitted in the reduction of land observations in middle latitudes.

While the absorption of terrestrial radiation of great wave-length is overrated in the *Annals*, that of the solar radiation is undervalued. An allowance of about one-half of 1 per cent is made for all of the solar radiation of greater wave-length than 2.5μ, "determined by consideration of the form of the energy spectrum of the 'black body' at 6000° absolute temperature" (*op. cit.*, p. 56). But this allowance for unobserved infra-red radiation is inadequate, and the mode of extrapolation is inadmissible, because, as I have shown in my paper on "The Solar Constant," published by the United States Weather Bureau in 1901, the solar radiation is richer in long waves than the theory assumes, besides having suffered a large aqueous absorption which is relatively more effective here than for terrestrial radiation, since the solar curve is ascending rapidly throughout the region of great aqueous absorption where the terrestrial curve is descending to its limit.

The selective depletion of the shorter waves by the air may best be approached by considering the phenomena of sky light. The sky sends us one-fourth as much light as the sun, and for a middle altitude an equal amount is reflected by the air to space, or one-half of the sunlight received is dissipated in this way. This leaves a remnant of 0.67, since

$$0.67+(0.5\times 0.67)=1.005.$$

But the air-mass for altitude 45° is 1.42, which gives

$$(p)^{1.42}=0.67, \text{ and } p=0.75.$$

This coefficient of transmission corresponds to wave-length 0.57 μ in the lemon yellow; but the maximum luminosity for sky light is in the blue where the transmission-curve given in the *Annals* reads 0.67 instead of 0.75. Hence there must be further depletion of these rays in addition to the ordinary selective scattering which gives sky light.

The sources of sky light are complex. Though, in general, blue and violet are in excess in the light of the sky, there is a strong and wide absorption band in the violet whose limits I find from photometric measures are 0.405 μ and 0.460 μ, with the minimum at 0.433 μ. This band, which also appears in the spectrum of the direct rays, is attributed in the *Annals* to the abundance of solar lines near G. But though the wave-length of G is $\lambda=4308$ Å, and thus is not far from the minimum of the diffuse band, there is no extraordinary gathering of solar lines between the given limits sufficient to account for the band which represents an absorption of about 25 per cent. Consequently, the band is *not* of solar origin.

Now Edward L. Nichols has found (especially in high mountainous regions) that the sky spectrum has a variable *emission* band at wave-length 0.42 μ which, allowing for the rather vague identification of the maximum point in his curves, may possibly have some connection with this band. My own observations, however, unlike those of Nichols, show the band as an *absorption* band in the sky spectrum. Further investigation will be needed to clear up these discrepancies, but the band is undoubtedly telluric and much more extensive than the gathering of solar lines near G. That a band or line should appear sometimes bright and at other times dark, that is, either as emission or absorption, is nothing new, although the controlling conditions, which are usually those of temperature, are not easily recognized if this band be such a case.

Nichols, it is true, found that the ratios of the spectral curves of sky luminosity to his "typical dawn curve" showed a *minimum* at 0.42 μ, but diminishing in strength as the sun rose higher, and succeeded by the aforesaid maximum in the middle of the day. My observation of the violet depression was made on the night sky illuminated by moonlight. This suggests that the "dawn curve" may be produced by selective scattering of shorter waves, in which violet predominates, and that the scattering is caused by a special material in the upper air which is in relatively strong illumination at the hour of dawn as compared with the deeper layers of the atmosphere. The latter furnish the absorption band, temporarily overpowered by the light from the upper layer, but reappearing soon after sunrise, when the upper light loses its early predominance; while later in the day the emission band again asserts itself in the sky spectrum at high altitudes through the continual formation of the special material in the upper air during the daylight hours.

The diversity of the phenomenon is further enhanced in the variability of the process of production of the special material which must be formed by ultra-violet sunlight, and must also be unstable, returning quickly to a normal and inactive condition, so that the active material becomes conspicuous only during that part of the day when the rate of its formation exceeds its decay. At low levels the emission band is liable to be neutralized in the sky spectrum by the absorption band present both by day and by night in the spectrum of the lower air.

Another observation by Nichols may possibly contain the clue to the substance which produces this band. He found that the emission band "appears regularly as the day advances in the mountainous regions of Switzerland during fine summer weather when there is a gradual formation of mist in the upper atmosphere. . . . . The maximum is commonly well established before noon and frequently persists until toward sunset."[1]

I suggest that the growth of this band is possibly connected with the formation of the complex water molecules, described in my *Atmospheric Radiation* (p. 100), to which Sutherland has given the general name of hydrols. Increasing ionization of the upper air by the sun's ultra-violet rays leads to the formation of nulcei of condensation, and this, combined with the midday upward movement of aqueous vapor at considerable altitudes, conduces to the formation of an exceptional amount of peculiar diffracting particles in the upper air during the middle of the day. The slopes of the logarithmic transmission-curves obtained by combining observations at high sun with others for a lower altitude of the sun have too low a slope, because the quality of the air has gradually changed in the middle of the day, and the transmission is overrated in a part of the spectrum where a small error makes a large difference in the resulting solar constant. The continual fluctuation of the solar radiant intensity through a wide range, shown by Crova's recording actinograph in the middle of the day with the clearest sky, is an evidence of the irregular distribution of an invisible obstructing substance which is most potent to deplete the solar rays when the sun is highest. Instead of rejecting low-sun measures, as has been done in the *Annals*, it might be better to reject those made near meridian altitude as being under suspicion to say the least.

If the wave-length assigned to the maximum of sky emission by Nichols could be shown to be correct, we might have further reasons for supposing that his band at 0.42 μ may be a hydrol band, because: (1) This wave-length agrees with that of a hypothetical band "No. 14" in extension of Kayser's diffuse series for aqueous vapor as shown in the series:

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[1] Edward L. Nichols, "A Study of Overcast Skies," *Physical Review*, **28**, 126, 1909.

Band No. 6 (observed),	$1/\lambda=1684$	
Band No. 9 (observed)	1833,	difference = 149
Band No. 11 (observed)	1978,	difference = 145
Band No. 12 (computed)	2118,	difference = 140
Band No. 13 (computed)	2253,	difference = 135
Band No. 14 (computed)	2381,	difference = 128

(2) The character of the band, which is diffuse and apparently irresolvable, agrees with that of the other members of the series. (3) The band seems to have some connection with the formation of mist, and, in general, depletion by selective scattering increases with the moisture in the air. Thus I have found from observations with the spectrobolometer that depletion of the shorter waves is greatest in summer. But this is equally indicated by the obvious fact that the red tints of sunset and sunrise are deepest in summer and especially in the tropics. Now the absorption lines of aqueous vapor are strongest at the *red* end of the spectrum. Their exceptional development should therefore increase the blue rays by comparison. On the contrary, it is the blue rays which are lacking, hence they have been depleted by something which increases with the abundance of water-vapor, but which is not water-vapor; and this substance, which appears to be most extensively formed out of aqueous vapor in summer, has a depleting action on the shorter waves which increases at a more rapid rate than the line absorption in the red. Thus the aqueous vapor does more than double duty, for it strikes out its own peculiar absorption bands in the red and infra-red, and also supplies a still more potent substance, depleting the rays of short wave-length.

Considering the nature of the depleting process, we find that the selective scattering of light by the atmosphere is a complex of diffraction and reflection from gaseous molecules, from condensation nuclei, and from dust. Incipient mist, apart from its own peculiar band absorption, increases the reflection of longer waves. The air of the tropical regions is especially loaded with material which produces scattering reflection, particularly that of the general, or nonselective sort. Ward says:[1] "The intensity of the light from tropical skies by day is trying, even almost unbearable." The

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[1] *Climate Considered Especially in Relation to Man*, 88.

"pale whitish appearance" of the tropical skies is especially noted. The reflection from fine dust exceeds that from cloud. *Saturn's* rings are an example. They are composed entirely of discrete dust particles, and far outshine the clouded body of the planet (area for area). In consequence, the albedo of *Saturn* (0.78) is greater than that of the other major planets (0.65 to 0.75). The value which Abbot and Fowle give for the reflection of total radiation by the earth is 0.37. I have no hesitation in saying that this is too small. In my paper on "The Greenhouse Theory and Planetary Temperatures" in the *Philosophical Magazine* for September 1908 I have given 0.44 as a minimum value. The albedo, or light reflection, is larger. *Mars* has 0.27 and *Venus* 0.92. A mean of these, 0.60, is more likely than not to be exceeded for the earth, which resembles *Venus* more than it does *Mars*. A larger albedo means, of course, a smaller transmission of solar rays, and a larger value of the solar constant.

I wish now to pass to a meteorological argument which shows the necessity of a smaller coefficient of transmission for solar rays than Abbot and Fowle are willing to admit. If q is a quantity of heat, and dq the variation of this quantity on the unit of horizontal surface in the time dt, A being the solar constant, r the sun's distance from the earth, ζ the zenith distance of the sun, p the coefficient of atmospheric transmission, and ϵ the air-mass corresponding to a particular solar altitude, the general expression for the insolation is

$$dq=\frac{A}{r^2}\,p^\epsilon \cos\zeta\, dt\,.$$

Angot has summed this equation for all latitudes with various values of the average transmission, p_a, and Ward, in his work on climate just cited, gives curves of insolation for June 21, founded on these data, to which I have added a lower curve for $p_a=0.25$ (Fig. 1). Now actually the distribution of temperature on the earth at the summer solstice agrees better with the lower curve, or even with one a little lower yet, as may be seen in the next figure (Fig. 2), where the curves should be congruous. The insolation curve is already a little too low on the left-hand side of the maximum, and yet it requires still further diminution on that side to allow for the greater band absorption between latitudes 20° N. and 20° S. where the aqueous absorption reaches a maximum. I consider that the best value for the average transmission of solar rays is 0.18, as far as this can be inferred from the form of the curves. It must be remembered that this is an average for the entire sunlit hemisphere, because we have taken average tempera-

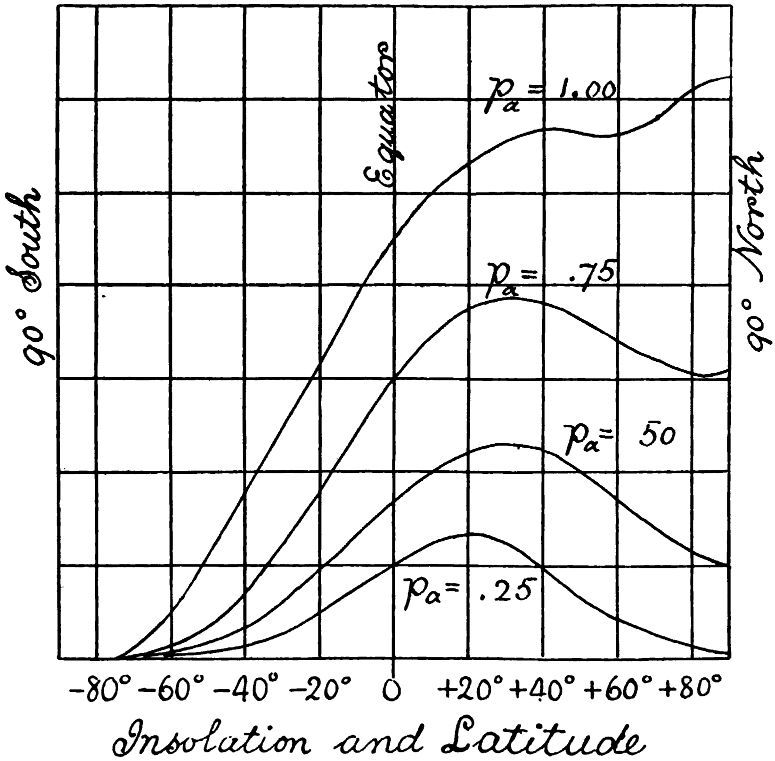

Fig. 1

tures. Such a transmission is entirely incompatible with a value of the solar constant as low as 2.1 calories. It requires a value greater than 3 calories, but perhaps not much greater.

Let us suppose, for illustration, that A, the solar constant, is 3 cal. per sq. cm per min., p, the coefficient of transmission obtained by considering the depletion by selective scattering alone, is a factor=0.6, and B, the lost radiation, is 0.6 cal., equivalent to a coefficient of absorption of 0.2. Then

$$Ap-B=(0.6\times 3)-0.6=1.2=R,^{1}$$

and

$$(R+B)\div p=A.$$

Now suppose that A is unknown, and that there are many fine absorption lines which escape measurement by the spectrobolometer, or that there are dust particles so large and opaque that they absorb a part of the radiation, so that B is estimated to be 0.2 cal., or $6\frac{2}{3}$ per cent of the original radiation, instead of 0.6 cal., or 20 per cent. Then

$$\frac{R+B}{p}=\frac{1.2+0.2}{0.6}=2.33 \text{ cal.}$$

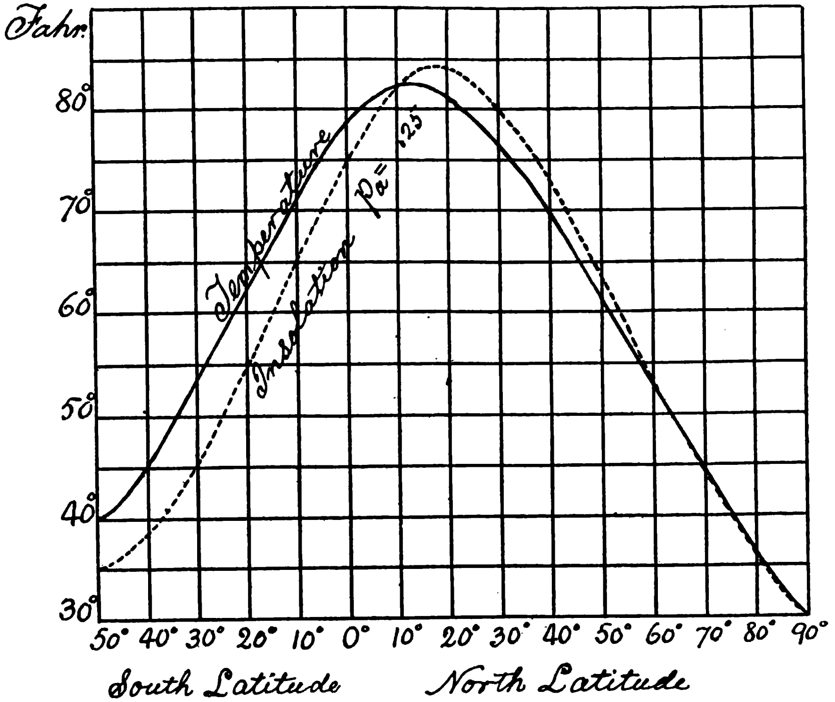

FIG. 2

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[1] The bands do not much affect the region of the spectrum in which selective scattering occurs. Hence, although the processes are simultaneous, they are best treated as if they were separate in such an approximate computation as this.

If p is also overestimated and presumed to be 70 per cent, instead of 60 per cent,

$$\frac{R+B}{p}=\frac{1.2+0.2}{0.7}=2.00 \text{ cal.}$$

It seems to me that this is exactly what has been done in these deductions of a solar constant by inadequate methods. The depletion of the incoming radiation has been underestimated. Too low an effective temperature has been assigned to the earth's radiating layer in consequence, and it has been necessary to assign too small a value to the atmospheric transmission of terrestrial radiation in order to reconcile data which are incompatible.

Too little account is taken in the *Annals* of a significant feature of the diurnal variation, namely, the apparent increase of transmission for large air-masses with a low sun. The frequent departure of the logarithmic curves of atmospheric transmission from straight lines is dismissed as being without special significance. The air layers are presumed to differ from each other in transparency, but only by a gradual progression. But the actual atmosphere contains ingredients which are sharply confined to definite layers with distinct limits. Of two stations, one at a low level and the other near the upper limit of the dust-bearing air, the more elevated experiences but a small fraction of the depletion of radiation by the dust which is suffered by the lower station, even though the air-masses may not differ much. The dust factor must be treated separately from the others. It has usually been completely ignored.

The flat-topped diurnal actinometer-curves, recorded by Crova's registering actinometer, are a record of the disappearance of a large amount of radiation in the middle of the day. The aqueous origin (whether direct or indirect) of this feature is shown by its partial disappearance in cold dry weather. There is a lack of symmetry in the curves obtained on mountains, the morning values of the insolation being higher than those obtained in the afternoon with the same altitude of the sun. This also is to be connected with the increase of the tension of aqueous vapor in the upper air in the afternoon. These details show that in every way the absorption of the incoming solar rays has been underrated.

It has been repeatedly asserted that the temperature of *Mars* must be far below the freezing-point, and this in spite of the obvious melting of polar snows, and the determination of the amount of water-vapor in the Martian atmosphere, which gives a dew-point not far from freezing temperature,[1] and of course a dry-bulb temperature considerably above the freezing-point. These facts point to a solar constant of 3 calories or more.

Even our earth, according to Abbot and Fowle, has an effective temperature of only −10° C., and hence must be assumed to radiate from an upper layer of frosty air, since the surface is so much warmer. I present a revision of this estimate, continuing my illustration of a hypothetical constant of 3 calories. After subtracting the lost radiation, 0.6 cal., I take the mean radiation received by a half-clouded earth = 2.4 cal. per sq. cm per min. before application of the transmission coefficient. The average transmission of the sifted remnant corresponds very nearly to that for a solar altitude of 27°, for which, assuming $p=0.56$,

$$p^{\epsilon}=(0.56)^{2\,2}=0.279.$$

Radiation directly received from the sun,

$$R=2.4\times0.279=0.67 \text{ cal.}$$

to which 10 per cent is to be added for sky reflection, making 0.74 cal. In emission this energy is distributed over a doubled area, becoming

$$\text{Emission}=1/2\times0.74=0.37 \text{ cal./sq. cm min.}$$

In considering the radiant temperature we shall do well enough to confine the computation to what goes on with a clear sky. The mean temperature of the earth is about 288° Abs., and its emissivity is not far from unity. Unobstructed radiation to space should give by the fourth-power law

$$0.523 \text{ cal./sq. cm min.}$$

Only 0.4 of this radiation can penetrate through the clear air, as I have shown (*ante*, p. 375). This gives for radiation directly escaping:

$$0.4\times0.523=0.209 \text{ cal./sq. cm min.}$$

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[1] *Lowell Observatory Bulletin*, No. 36, 1, 211.

The surface receives 0.370 cal., and the difference

$$0.370-0.209=0.161 \text{ cal./sq. cm min.}$$

is transferred to the air and emitted at a temperature of 215° Abs. $= -58°$ C. But this is almost precisely the temperature of the upper static layer which is truly the radiant layer of the atmosphere, since convection has ceased, and the heat received can be dissipated only as radiation.

The abandonment of low-sun observations because low-lying mists render such measures uncertain, and the selection of midday observations on the plea that the sky is then clearer, is at first sight plausible; but an examination of the diurnal curves of insolation obtained by the actinograph shows that the midday observations are the most irregular ones. If any observations are to be rejected, it should be these, subject as they are to continual variations and to the suspicion of a large constant error which is reduced to a practical certainty by the arguments which have been presented as well as by others which are to follow. The midday observations need not be absolutely thrown away; but we can recognize that, taken by themselves, they are misleading, that they require large corrections, and that the constant fluctuations of solar radiation in the middle of the day with the clearest sky, point to the presence of invisible "clouds" of absorbent vapor, which are more dangerous than low-lying mist at the time of low-sun observations. The mist, if it exists at all, is visible; but the diurnal variation in the transmissive quality of the upper air has to be inferred from the observations themselves. The mists of the morning may compensate for the midday diminution of transparency of the upper air, and comparisons of actinometer readings taken at morning and at noon may sometimes yield transmissions near the truth. Apparent transmissions founded on afternoon series of actinometer readings are usually too large, owing to the change of atmospheric quality. Such measures should be tested by carefully timed observations at the lowest observable altitude of the sun, which will at least show whether the measurements are capable of reduction. They usually demonstrate that the absorbent quality of the medium has changed.

If spectrum measures are used, it is found that the blue and violet are subject to extraordinary variations which are not yet entirely understood. The yellow and red are thickly strewn with atmospheric lines. In the intermediate green, however, there is a region sufficiently free from these disturbances to yield an approximate transmission coefficient if widely different air-masses are compared.

Observations covering a limited range of air-mass may be in apparent agreement. The real test is to see whether the agreement continues when the range is extended. The almost total absence of low-altitude records in the *Annals* is to be regretted; and still more unfortunate is the dependence which the authors have placed on summer observations almost to the exclusion of winter work, although the absence of moisture is of the greatest importance in securing a reliable measurement of the solar constant.

Langley's *bon mot:* "That is the *best* value of the solar constant which is nearest to three calories," may be more than a bit of humor; but, on the whole, the indications point to a value greater than three. A value of the solar constant not less than three, nor more than four, calories, accords with the data of meteorology. It also satisfies the demands of the best measures of nocturnal radiation and of the transmission of telluric radiation, agrees well with the temperature of the sun inferred from Wien's formula with Paschen's constant, as modified by Lummer and Pringsheim, and from the actual temperature at the focus of a large mirror, requires no modifications of the methods of evaluating the transmission of direct solar rays which are not abundantly demonstrable as requisite, and in every way harmonizes a great body of facts which would otherwise be found to be in conflict.

WESTWOOD, MASS.
July 1911

THE PRODUCTION OF LIGHT BY CATHODE RAYS

BY GORDON S. FULCHER

In a previous paper[1] it was shown that light is emitted as a result of the collision of canal rays with gas molecules and that its intensity is proportional to the mean energy transmitted times the number of collisions. An analysis of the Doppler effect seemed to indicate that the neutral rays are far less effective than the charged rays in producing this luminescence. Since then an attempt has been made to separate the charged from the uncharged rays by means of a magnetic field and to examine the light-producing effect of each; but, in agreement with recent results obtained by Wien and Koenigsberger, this was unsuccessful. Ionization and neutralization produced by the collisions with gas molecules and corpuscles, introduce charged and uncharged rays into the deflected and undeflected beams, so that, with the apparatus used, a very strong magnetic field reduced the number of charged rays in the undeflected beam by only about 10 per cent.

The phenomena occurring in the region traversed by canal rays are fewer than in the main discharge, but nevertheless are quite numerous. The beam itself consists of both atoms and molecules; some are positively charged, some are neutral, and some negatively charged. Ionization, neutralization, dissociation, and association are continually taking place; secondary cathode rays are being shot out of the ionized atoms producing more ionization in their turn; and in connection with each one of these transformations light may be emitted. As the attempt to separate neutral and charged rays had failed, I turned my attention to the secondary cathode rays, since these are identical with cathode rays of the same velocity and can be most easily separated from other kinds of rays.

Only two papers dealing definitely with the subject have been found. Wüllner,[2] in a brief note, says that he carefully separated cathode from canal rays, though he gives no further description of his apparatus, and reports the following results: In hydrogen, the cathode rays produce the compound line spectrum, wrongly termed the "band spectrum." In oxygen, they produce the cathode bands at λλ 5980, 5600, 5260 and the triplet 5330, but no spark lines. In air, the results were inconclusive. Lewis[1] concluded from a comparison of the spectra of the light from various parts of the discharge in air that the cathode rays cause the emission of the negative bands of nitrogen. He also investigated the luminescence produced by cathode rays in certain metallic vapors.[2] Because of the importance of the matter, an attempt has been made to verify these conclusions. Some crude preliminary results obtained with the apparatus described below may be of interest.

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[1] *Astrophysical Journal*, **33**, 28, 1911.

[2] *Physikalische Zeitschrift*, **1**, 132, 1899.

APPARATUS

The anode *D* (Fig. 1) was made of soft iron 1 mm thick in the central portion, and through it a hole (*H*) 0.020 cm in diameter was drilled. The cathode (*C*) was arranged so that its distance from the anode could be varied by turning the key *S* which fitted into a conical ground greased joint. The space behind the anode was inclosed by a brass cylinder *E*, except for the opening *W* which allowed light from the beam emerging from *H* to reach the single-prism spectrograph. The fine beam itself was adjusted so as to be at the focus of the collimating lens *L*, and thus took the place of the slit. It was found—what is doubtless well known—that when the prism was turned from the position of minimum deviation so as to decrease the angle of incidence, the width of the image of a wide slit increased more rapidly than the dispersion; but that when the prism was turned so as to increase the angle of incidence, the width of the image decreased more rapidly than the dispersion, so that lines could be separated in this position which would overlap with the prism placed for minimum deviation. By turning a large 45° prism, having a length of 10 cm at the base of its faces, so as to increase the angle of incidence as much as possible, the width of the image was reduced to but half that of the slit, while the dispersion was not much less than with the prism placed for minimum deviation.

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[1] *Astrophysical Journal*, 17, 258, 1903.

[2] *Ibid.*, 16, 31, 1902.

A soft iron pole-piece *P* was fastened inside *E* and a weak electromagnet *M* placed just outside the tube. The discharge chamber (*I*) and observation chamber (*II*) could each or both be connected to a reservoir of gas through a fine capillary tube and also to a Gaede pump. Because of the smallness of the hole connecting the two chambers, the pressure in either chamber could be kept several times greater than that in the other by connecting it to a capillary and the other to the pump; or the pressure could be maintained the same in both. Usually a continuous current was secured by the arrangement described in the previous paper (*op. cit.*)—a condenser kept charged by an induction coil and allowed to discharge continuously through a high resistance in series with the discharge tube.

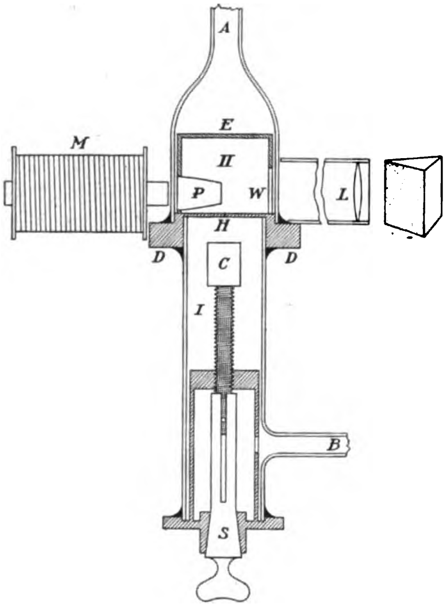


Fig. 1

RESULTS

Air.—Fig. 2 (Plate XVII) shows reproductions, enlarged 5 times, of a number of spectrograms of the nitrogen bands, made under various circumstances. Nos. 55, 56, 81, and 82 were taken with the apparatus described above. The spectrum of the deflected beam is seen to consist solely of the negative bands, in agreement with Lewis' conclusion. No. 82 is badly overexposed (12 hours), yet shows only a trace of the strongest positive bands. Compare also Nos. 65, 76, and 78. In the case of 56, the arrangement was such that retrograde canal rays as well as cathode rays passed through

PLATE XVII

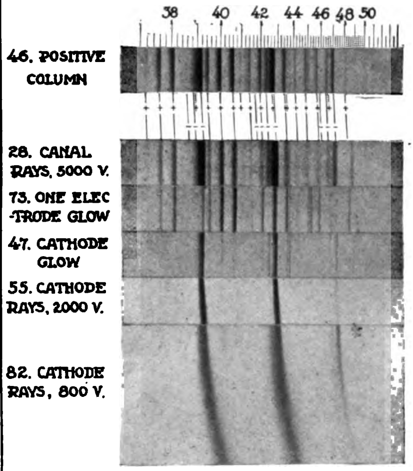


Fig. 2 Air.

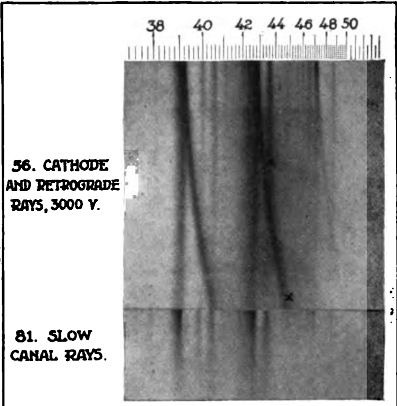


Fig. 3 Air.

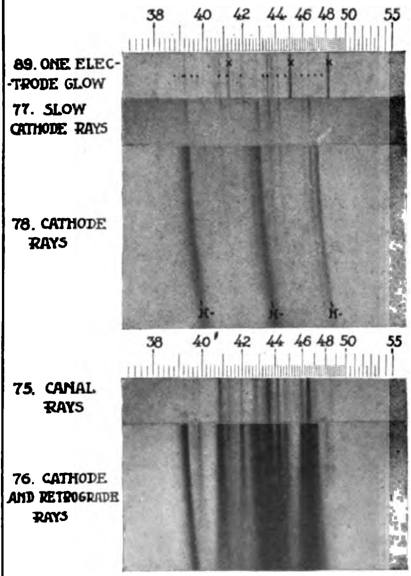


Fig. 4 Oxygen

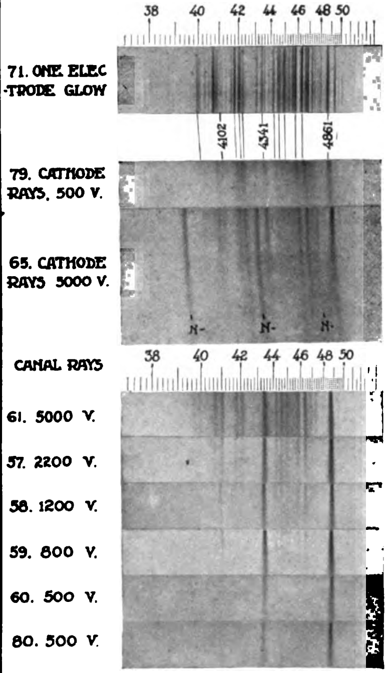


Fig. 5 Hydrogen.

The Production of Light by Cathode Rays

the hole in the anode. The former produced the undeflected lines, the latter the deflected lines. The line marked with two crosses is a scratch on the negative. The difference between the spectra due to the canal and cathode rays is thus strikingly shown.

It was thought that the spectrum due to very slow canal rays might also be simpler. However, very slow canal rays are quickly dispersed, have a short range, and also produce very little light. No. 81 was taken with an ordinary discontinuous discharge, 1000 volts, and shows both positive and negative bands. I hope to try much slower rays with a better apparatus.

It was noticed that when the vacuum was such that the discharge passed with difficulty between the two electrodes, a luminescence appeared in the tubes leading from the discharge tube to the gas reservoir and pump. The spectrum was photographed and showed the positive bands much more strongly developed than the negative, No. 73. When one electrode of a Geissler tube was connected to the high-potential electrode of a tube through which a high-potential discharge was passing, a glow appeared in the Geissler tube whose spectrum was identical with that of the luminescence in the connecting tubes. This will be referred to as the one-electrode glow discharge.

Hydrogen.—In hydrogen, the cathode rays produce both the compound line and the main series spectrum, Nos. 79, 65; but the latter is relatively much less intense than in the light due to canal rays, and less intense in the spectrum produced by slow cathode rays, No. 79, than in that caused by the faster rays, No. 65. In the spectrum of the one-electrode glow discharge, No. 71, the lines of the main series are no stronger than neighboring strong lines of the compound spectrum.

From the series of spectrograms of light produced by canal rays, Nos. 57 to 61, it is clear that the compound spectrum is relatively less intense for the slower rays. The intensity of the light in the case of the slowest rays (500 volts) was so small that both plates 60 and 80 are underexposed in spite of long exposures (20 hours), so that they do not prove the entire absence of the compound spectrum under those circumstances; but it seems probable that in the case of sufficiently slow canal rays the main series line-spectrum of hydrogen is the only one excited.

Oxygen.—All the spectrograms obtained show the elementary or spark spectrum lines of oxygen, even that produced by slow cathode rays. The main spark lines are marked by dots on No. 89. The crosses indicate *CO* bands. Also the main series lines λλ 3947, 4368 seem present on all the spectrograms. The first subordinate series line λ 4773 appears on Nos. 75 and 89, the line λ 4968 on the other three, Nos. 76, 77, 78. The second subordinate series lines λλ 4802, 5020, 5435 are faintly visible on No. 76. The cathode bands λλ 5260 and 5580 are present in Nos. 76 and 78; they show no undeflected streak in No. 76. All the lines appearing in No. 76 seem to have a deflected branch, certainly the spark lines, and probably also the series lines. Properly to study the behavior of the various spectra of oxygen, an apparatus sensitive to λ 5600 and having greater dispersion is necessary. The spectrograms Nos. 76, 77, 78 represent exposures of about 20 hours each.

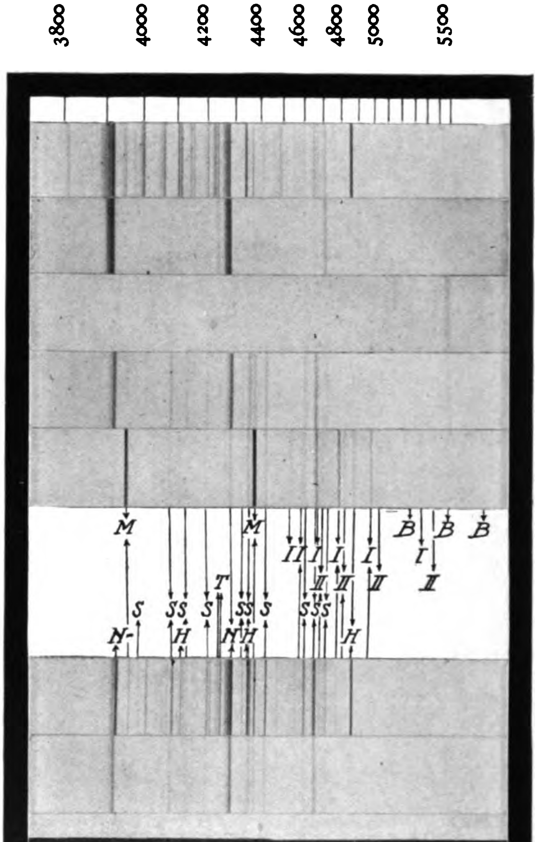
Summary.—The results described above show that the spectrum of the light produced by cathode rays (1) in nitrogen consists solely of the negative bands; (2) in hydrogen consists chiefly of the compound spectrum together with the main series lines, relatively weak; (3) in oxygen consists of the negative bands together with the spark lines and series of triplets. They agree in general with the results of Wüllner and Lewis summarized above.

Also it seems probable that slow canal rays in hydrogen produce only the main series lines.

Discussion.—In this preliminary note, I shall not attempt to explain the numerous observations regarding the variations in the relative intensity of the different spectra of the above gases under varying circumstances which have been reported, but shall merely indicate how this line of approach promises to afford definite information as to which atomic system emits each spectrum. Consider the case of cathode rays passing through a diatomic gas. They will collide with the molecules and may conceivably ionize them either (1) by shooting out an electron from one of the atoms of the molecule without causing dissociation or (2) by hitting between the two atoms so as to render the molecule unstable, causing dissociation as well as ionization. If we add neutralization, recombination of the ionized molecules, and negative ionization by the attachment of a stray corpuscle to a molecule, we have probably listed all the pos-

PLATE XVIII

FIG. 7



136. Air-Canal Rays

117. Air-Cathode Rays (3000 v.)
λλ 3914, 4278, 4709

120. *O*-Cathode Rays (3000 v.)
Trichromatic Plate

121. *O*-Cathode Rays (3000 v.)
Seed 27 Plate

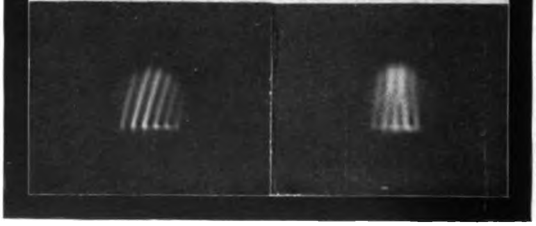
133. *O*-Diffuse Canal Rays (1500 v.)

B, Negative Bands: λλ 5260, 5600, 5860
M, Main Series: λλ 3947, 4368
I, 1st Sub. Series
II, 2d Sub. Series
S, Spark Lines *T*, Triplet, λλ 4223, 4233
N, Nitrogen *H*, Hydrogen

131. *O*-Canal Rays (5000 v.)

121. *O*-Cathode Rays

FIG. 8



(*a*) Cathode Rays (*b*) Diffuse Canal Rays sible transformations taking place in the space traversed by the cathode rays. Because of the relatively long mean free path of molecules at that pressure, the last three transformations will not be confined to the path of the rays, but will be distributed throughout a considerable space around it; whereas the direct ionization by the cathode rays must take place largely in the path of the beam (since the scattered rays will be slower, less concentrated, and less effective). Hence it is reasonable to assign to the two ionizing transformations most of the light photographed with the above apparatus. In the case of very slow cathode rays it seems probable that ionization without dissociation alone may be possible or may at least largely predominate. If, then, the light produced by very slow cathode rays consists solely of a band spectrum, that spectrum is probably emitted by the positively charged molecule of the gas in question.

In the case of nitrogen, slow canal rays produce only a band spectrum; that is, no dissociation is produced. The difference between the band spectrum produced by canal and cathode rays may be due to the difference in the method of ionization because of the difference in size and in the electric field around the two kinds of rays. Hence the vibrations aroused, though not unrelated, are different.

September 4, 1911

AN ADDITIONAL EXPERIMENT

Since the above was written, a *new apparatus* has been made which gives more light, permitting the use of a focusing lens and a slit. The cathode is the same as before, but the anode is changed as shown in Fig. 6. There are several parallel slits cut in it, 1 mm wide and 1.5 mm apart. A comb containing a similar set of slits is placed 5 mm back of the anode so as to intercept any rays which pass through the anode in a direction normal to its front surface. Two magnetic pole-pieces, 3 mm apart, are so placed that when the field is adjusted to a suitable strength, cathode rays passing through the slits in the anode are deflected so as to pass through those in the comb, while the less deflectable atomic rays are still intercepted.

A photograph of the deflected bundles of cathode rays, viewed edge-on, is shown in Fig. 8 (*a*), Plate XVIII. The apparatus is not as successful as had been hoped in eliminating atomic rays from the chamber back of the anode for a wide range of cathode-ray velocities. It was found that when the terminals are reversed so that the anode becomes the cathode and canal rays pass through the slits, the rays are not entirely intercepted in the case of low velocities; or rather, diffuse secondary canal rays, created by collision with gas molecules, produce luminous undeflectable bundles, which are shown photographed in Fig. 8 (*b*). However, with a cathode-fall of 2000 volts or more, quite pure bundles of cathode rays can be obtained, as is shown by the fact that, when the magnetic field is zero, the chamber back of the anode is practically dark. Yet the range of velocities which can be tested with this apparatus is unfortunately very limited. This disadvantage is partially offset by the opportunity, unexpectedly afforded, to obtain the spectrum of the light produced by the slow diffuse canal rays, which turns out to be of some interest. But with this apparatus, it must be kept in mind that we analyze the light from the region traversed by the rays

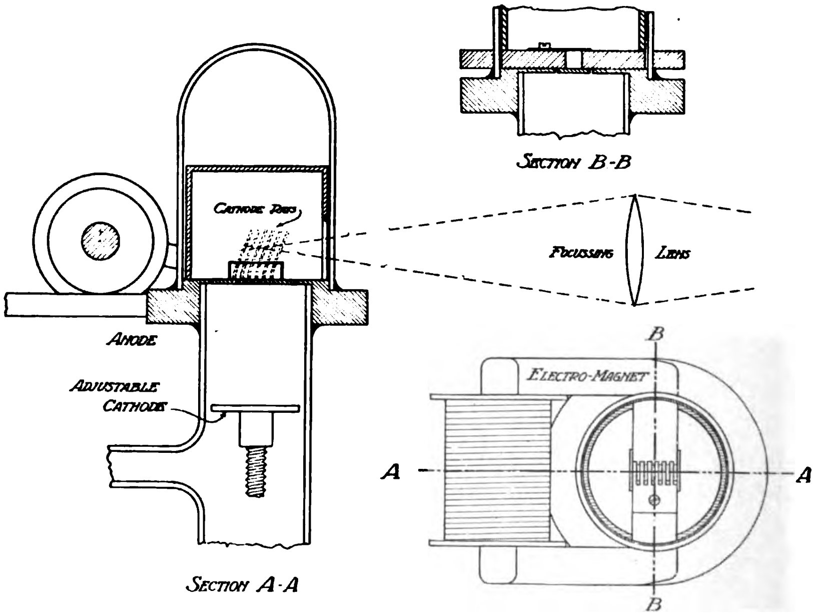


Fig. 6

and not merely the light due to the primary action of the rays as with the previous apparatus.

The *results* (Plate XVIII) confirm and supplement those reported above.

Air.—Cathode rays cause the emission of the negative nitrogen bands, $\lambda\lambda$ 3914, 4278, 4709. No. 117 shows merely a trace of the nitrogen positive bands and no oxygen lines. An exposure twelve times as long (100 hours) would be necessary to bring out the oxygen cathode-ray spectrum as shown in No. 121, which required an exposure of 24 hours. *Diffuse canal rays* produce (primarily and secondarily) the same spectrum as slow canal rays—positive and negative bands both strong.

Hydrogen.—Cathode rays produce both the compound line and the series spectrum, but the latter is relatively very much weaker than in the light produced by canal rays. *Diffuse canal rays*, like slow canal rays, cause the emission of both spectra (directly and indirectly), but the series spectrum is much the stronger. As Stark and Hermann[1] have found, the spectrum of the light from the canal ray region shows the series spectrum relatively more intense the greater the velocity of the rays. This seems a direct contradiction to the statement made above (p. 391) that slow canal rays produce only the series spectrum in hydrogen; but when we consider that we analyze in one case light from the whole region, including that due to secondary ionization, and in the other case only that due to the primary action of the canal rays in the path of the beam, the contradiction is seen to be easily explicable and very significant. The primary luminescence depends directly on the energy of the rays, while the secondary luminescence, due to secondary cathode rays, is, within certain limits, independent of the velocity of the rays. Thus the greater the energy of the rays, the more the primary will predominate over the secondary luminescence. When, however, the beam itself is used as a line source of light so that the secondary luminescence scattered throughout the surrounding region is relatively ineffective, the spectrum of the light due to the primary action of the rays is obtained. It seems probable, therefore, that the series line spectrum of hydrogen is emitted as a result of the collision of canal rays with molecules;

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[1] *Physikalische Zeitschrift*, 7, 95, 1905.

the compound line spectrum as a result of the ionization of hydrogen molecules by cathode rays.

Oxygen.—The spectrum due to *cathode rays* consists chiefly of the negative bands, λλ 526, 560, 586, but spark lines and series triplets are also present, though faint. No. 120 was made on a Seed No. 27 extra-rapid plate, 24 hours' exposure; in the case of No. 121, a Cramer Trichromatic plate was used, 30 hours' exposure. In the spectrum due to *diffuse canal rays* (No. 123), the main series lines, λλ 4368 and 3947 (see Runge and Paschen, *Wied. Annalen*, **61**, 641, 1897), are remarkably strong; the two subordinate series are well developed, while the spark lines are relatively weak. With fast *canal rays* (No. 131), the spark lines predominate, while slower canal rays give rise to spectra intermediate between No. 131 and No. 123. The variation in the relative intensity of the various groups of lines when produced in various ways, is shown roughly in the following table. The intensities of the various lines on the plates were estimated, and the average intensity was computed for the spark lines (S), λλ 3974, 4072, 4186, 4318, 4348, 4415, 4591, 4642; for the first subordinate series triplets (I), λλ 4773, 4968; for the second subordinate series triplets (II), λλ 4673, 4802. Then the scales were adjusted so that the average for the spark lines came out the same for all. No trace of the line λ 3955 nor of the series of doublets was found (cf. Runge and Paschen, *op. cit.*).

Oxygen	Diffuse Canal Rays (1500 v.)	Canal Rays (2000 v.)	Canal Rays (5000 v.)	Cathode Rays (3000 v.)
Negative Bands (B) (No. 121)	—	—	—	20
Spark Lines (S)	4	4	4	4
λ 4368	40	12	4	4
λ 3947	25	8	2.5	1
First Sub. (I)	6	3	0.5	0.4
Second Sub. (II)	2.5	1.3	0.3	0.2

As a result of the discussion above on p. 393, it was concluded that when cathode rays produce a band spectrum it may reasonably be assigned to the positively ionized molecule of the gas in question. The negative bands of nitrogen and the negative bands of oxygen, then, are probably emitted by the respective molecules as a result of ionization by cathode rays.

UNIVERSITY OF WISCONSIN
November 12, 1911

EFFECTS OF VARIATIONS OF VAPOR-DENSITY ON THE CALCIUM LINES H, K, AND *g* (λ 4227)

BY OLIVER J. LEE

The element calcium has from the beginning played an important part in astrophysics. Two of its principal spectrum lines, H, λ 3968, and K, λ 3934, have been chiefly used by solar physicists in the study of the sun with the spectroheliograph. The importance of these lines is being further enhanced by their growing significance in stellar spectroscopy. There arises the question of the interpretation of strong, sharp lines of calcium in a stellar spectrum when accompanied by broad lines of hydrogen, helium, and other elements. Radial velocity observations of stars of the spectroscopic binary type have in various cases shown an apparently anomalous behavior for these lines.

Reference to Kayser's *Handbuch der Spectroscopie*, V, 235, shows that no less than 130 articles have been published which are concerned wholly or in part with the spectroscopic treatment of calcium.

The well-known experiment of Sir William and Lady Huggins on vapor-density effects in the spectrum of calcium, as well as the recent researches of King, St. John, and others, have suggested the desirability of trying to isolate one of the variables in the production of the calcium spectrum and to govern its variation, while keeping the others approximately constant. The present paper gives the results of an attempt to get quantitative data in the case of varying vapor-density, and limits itself to determining the relative vapor-densities at which self-reversal occurs in the lines H, K, and *g*.

Methods used to determine vapor-density constants for elements that volatilize at low temperatures, such as sodium and sulphur, are obviously unsuitable for calcium. Owing to the high temperature required, about 2500° C., it was out of the question to confine any given amount of vapor and still be able to get its light. Instead, the rate of volatilization of the element under standard conditions was assumed to be constant and was made the basis of the experiment. Thus it was assumed that if twice as great a surface area of calcium was exposed in one case as in a second case, the density of the vapor after a given time of heating would be twice as great in the former instance as in the latter. The conditions for each observation therefore became:

1) Constant temperature sufficiently high to volatilize calcium freely;

2) Volatilizing areas of calcium which should be proportional to given predetermined ratios;

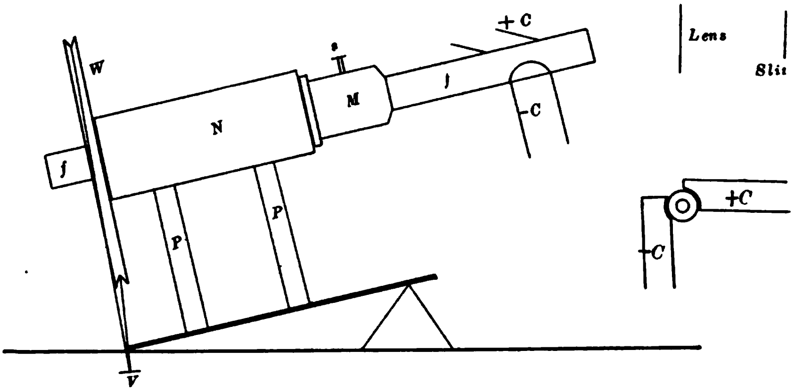
3) Exposure-times short enough to eliminate as much as possible inevitable changes of conditions.

APPARATUS

The apparatus employed consists of a rotating carbon heating tube in which calcium is vaporized and a concave grating spectrograph.

The heater is a carbon rod, *ff*, 1 cm in diameter, which has a hole in one end 5 mm in width and 15 mm deep. The carbon is mounted in the hollow spindle *M*, to which it is fastened by means of the set-screw *s*. At the other end of the spindle is fastened a light grooved wheel *W*, 8 inches in diameter, which is connected to the small pulley of a motor, *V*, by means of a stout cord. Spindle and wheel are mounted in the brass bearing *N*, and this is rigidly bolted to the cast-iron base of the arc apparatus by means of the steel rods *PP*. The carbon-tube was made to rotate smoothly about 100 times a minute between the poles of an arc, each a carbon rod 13 mm in diameter. This arrangement practically gave the heating effect of two arcs, the rotating heater being a negative pole on the upper side and a positive pole on the lower. A stationary heater similarly mounted was first tried, but in that case the negative pole, *C*, burned its way through the thin wall of the tube so quickly that no reliance could be placed upon the temperature within. With the rotating heater this wear took place so slowly that the temperature could be kept comparatively constant for the 2 to 4 minutes during which each was in use.

The current used throughout was 20 amperes at 40 volts. This produced a temperature of about 2500° C. After finding a radiation pyrometer impracticable for such a small heated surface, and an electric thermo-couple inadequate, the heat within the tube being actually greater than was expected, the temperature given above was determined indirectly. Several substances whose melting points are established, such as salt, 800° C., iron, 1503°, platinum, 1753°, and iridium, 2350°±, were introduced into fresh heating tubes and the current was slowly increased until they melted. The currents were plotted against melting temperatures and the required temperature was read from the resultant curve.



By means of a 3-inch lens the light from the center of the tube was projected upon the slit of the spectrograph, the dispersion piece of which is a Michelson concave grating of 2 meters radius. The ruled surface is 50 mm×82 mm and has 590 lines to the mm. In practice, with a fairly wide slit, two good lines 0.3 Å units apart were well resolved. The plate was held in the camera, curved in an arc of 1 m radius, and 7 exposures could be made on each plate without opening the camera.

METHOD OF OBSERVING

Two distinct methods have been used in this experiment to govern the relative amounts of calcium introduced into the various tubes.

1. A supersaturated solution of calcium chloride was made by taking equal parts by weight of distilled water and the salt. Call this standard solution A. By means of glass droppers, solutions of calcium strength A, A/2, A/4, A/8 . . . . A/128 were introduced into tubes of the adopted standard size, 5×15 mm. Series of such tubes were filled and left in an upright position to evaporate part of the water and to let the salt crystallize on the walls and bottom of the carbon bore. When the tubes were dry, they were placed in the rotating spindle and slowly heated until the water of crystallization was given up without explosive scattering of the material. From 3 to 7 exposures of 30 seconds each were made with every tube so that its photographic record might be complete. The normal plate first shows the calcium lines weakly, then as the heater gets its proper temperature and vaporization of the calcium is rapid the lines increase in strength to a maximum for that tube.

It was found that this method gave consistent results for the lower densities, the ratio of which could be determined with considerable accuracy. For the greater densities required to reverse H and K, however, the large amount of material in the tube made the vaporization more or less uncertain and the resulting reversals were in consequence erratic. The second method was found to give more satisfactory results for these higher densities.

2. Metallic calcium was sawed into very thin rectangular pieces ½ mm thick and 3 to 4 mm wide. These were clipped across into slivers fairly uniform in size. The slivers, weighing on an average 2 milligrams each, were weighed out into charges of from 5 to 100 milligrams. After tipping the apparatus as shown in the diagram the tube with its charge was fastened in the spindle and slowly heated while revolving. As the chamber walls became heated the metallic particles would spread themselves uniformly over them and adhere while vaporizing. Multiple exposures were made as in the first case. With neither method was a tube used for more than one charge. About 800 photographs were made with 225 tubes.

The following tables give typical data for reversals with change of vapor-density. The first gives results obtained by method (1) and covers the low order of vapor-density required to reverse *g*.

Calcium Charge	Number of Exposures	Results
A/4	20	*g* reversed on all, K on 2, H on 1
A/8	13	*g* reversed on 10
A/16	14	*g* reversed on 8
A/32	20	One doubtful reversal of *g*

The following table gives results obtained by method (2) covering the range for H and K reversals:

CALCIUM CHARGE	NUMBER OF EXPOSURES	TIMES REVERSED		
		g	K	H
0.100 gram	33	33	27	16
0.050 gram	33	33	22	10
0.025 gram	31	28	7	0
0.010 gram	32	22	1	0
0.005 gram	29	9	0	0

Comparing the reversal times with calcium charges A/16, 0.010 gram and 0.005 gram, it is seen that an A/16 charge of calcium chloride corresponds in this experiment to one of about 0.0085 gram of metallic calcium.

CALCIUM IMPURITIES IN CARBONS AND IN AIR

An attempt was made to purify the carbons used for heating tubes by repeatedly raising them to a temperature of several hundred degrees in a strong atmosphere of chlorine and then boiling them in distilled water. The desired effect was not obtained, for there seemed to be no diminution in the amount of calcium present. The reason for this failure is explained by an experiment by W. N. Hartley, reported to the Royal Society, May 11, 1911, in which he shows that there are copper and calcium impurities in the air in sufficient quantities to give the characteristic spectrum lines of these elements. As all of my exposures were made with the arc in air, even as radical a treatment of the tubes as that described would of course not eliminate the calcium lines. The order of magnitude of the impurities in carbons and air was found by noting at what strength of calcium solution the lines should begin to show an increase over their intensity when the carbon only was used. No increase was visible until a charge of A/100 was reached. At A/50 the lines were estimated to be twice as strong as those derived from the carbon alone in air. From this it was concluded that the calcium impurity in the carbon heaters and the air together did not exceed 2 per cent of an A charge, and that its effect on the ratio A to A/8, for example, was negligible.

APPEARANCE OF THE LINES

All the tubes were exposed at atmospheric pressure. To what extent electrical and chemical action took place in the vaporizing process is not known. It was assumed to be constant, and nothing in the appearance of the vapor while being generated would lead the observer to another opinion.

The H and K lines have always been observed together, K being always a trifle the wider, at least, if not the more intense. With faint H and K lines present, *g* was frequently missing. Whenever the background of continuous spectrum appeared strong, photographic reversal took place.

It is important to notice the changes in the lines upon approaching reversal. The line *g* is never as hard as the other two lines, even when these are very faint. As the vapor-density increases, it becomes more diffuse until reversal occurs. At this point K begins to lose its sharpness, and when it reverses H is almost ready to reverse.[1] In each case there seems to be a more or less gradual breaking up of the structure of the line before reversal takes place. When the three lines are reversed on the same plate, the wings of *g* flare out about a wide core which has clean sharp edges, while H and K flare very little, preserving their hard form tenaciously. The accompanying plate XIX shows the changes in these lines as the vapor-density is increased, the lowest spectrum corresponding to the greatest density.

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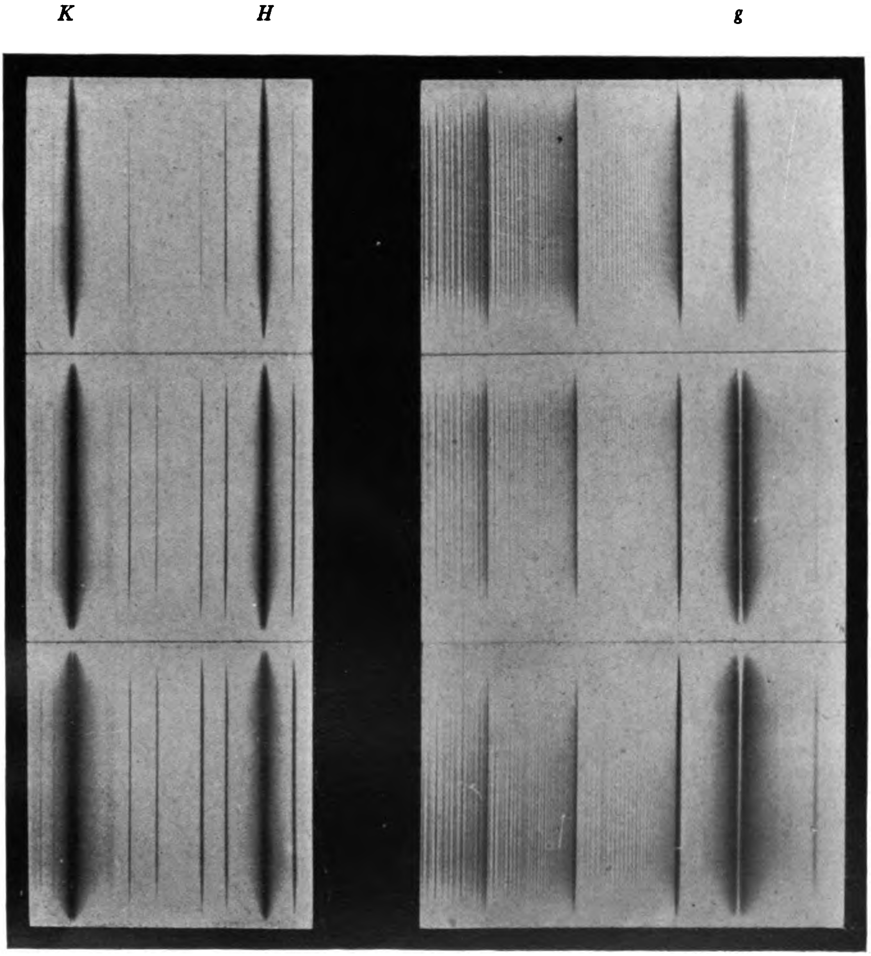
[1] Concerning Dr. A. S. King's observations, in the *Astrophysical Journal*, **28**, 395, 1908, that H and K refused to show "self-reversal" in the furnace, the present results would seem to indicate that in his case the vapor of calcium was too rare to reverse these lines.

PLATE XIX



Successive Reversal of Calcium Lines with Increasing Vapor-Density

CONCLUSION

The lines *g*, λ 4227, and H, λ 3968, require for reversal, vapor-densities that are 1/7 and 3/2 as great respectively as that necessary for a reversal of K, λ 3934, when the vapor is observed at a temperature of 2500° C. and at atmospheric pressure.

It will be understood that the figures given above are approximations only. The difficulty of the problem and the limitations of the method alike render quite useless any claim to greater accuracy.

RYERSON PHYSICAL LABORATORY
THE UNIVERSITY OF CHICAGO
May 1911

NICKELED GLASS REFLECTORS FOR CELESTIAL PHOTOGRAPHY

By R. W. WOOD

The failure of silver to reflect well in the ultra-violet region, places a limit to the extent of the spectrum of a star or other celestial object obtained with a reflector of the ordinary type. As is well known, the spectrum of light which has penetrated the earth's atmosphere extends to about wave-length 2900. In the region around wave-length 3160 silver reflects only 4 per cent of the incident light; in other words, the silvered mirror is no more efficient in this region than unsilvered glass. For extending the work which I commenced two years ago on the ultra-violet photography of the moon, it became necessary to construct a long-focus mirror of large aperture capable of reflecting copiously the light transmitted through the silvered-quartz ray-filter. The great weight of a large mirror of speculum metal, and difficulties in its construction, with its liability to tarnish, led me to try to work out some method of depositing a film of some suitable metal upon a glass mirror.

Consulting Rubens' tables of the reflecting power of metals, it appeared that nickel would prove most suitable, its reflecting power throughout the entire spectrum being only a little less than that of speculum metal. So far as I was able to find, no method is known of depositing nickel chemically, except the process in which the gas nickel carbonyl is employed. This gas is extremely poisonous, and I understand that the metal films are bright only on the surface which is in contact with the glass. I accordingly determined to use this method only as a last resort, and commenced experiments at my East Hampton laboratory this summer on the electrolytic deposition. It was found that solutions with an acid reaction either frilled or completely removed the silver film, which was deposited chemically to serve as a substratum for the electrolytic film. I commenced work with copper, and found that if a dilute solution of the sulphate was converted into cuprammonium by the addition

PLATE XX

Fig. 1
Cd and *Sn* Spark

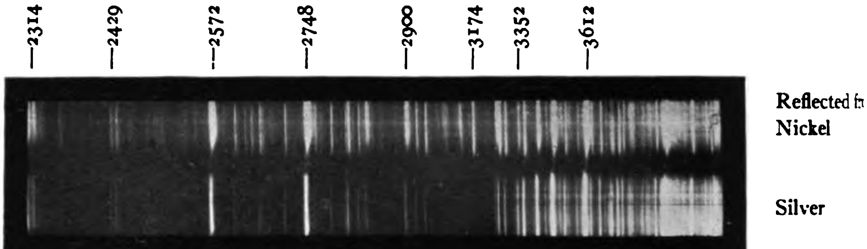


Fig. 2
Fe Spark

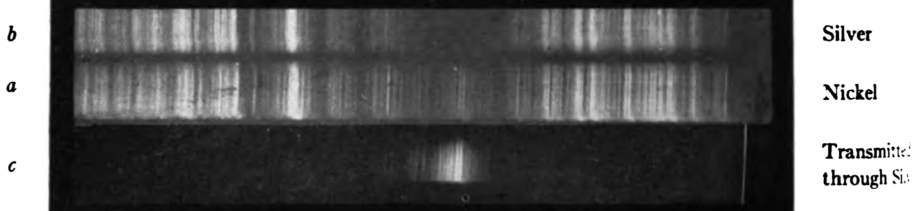
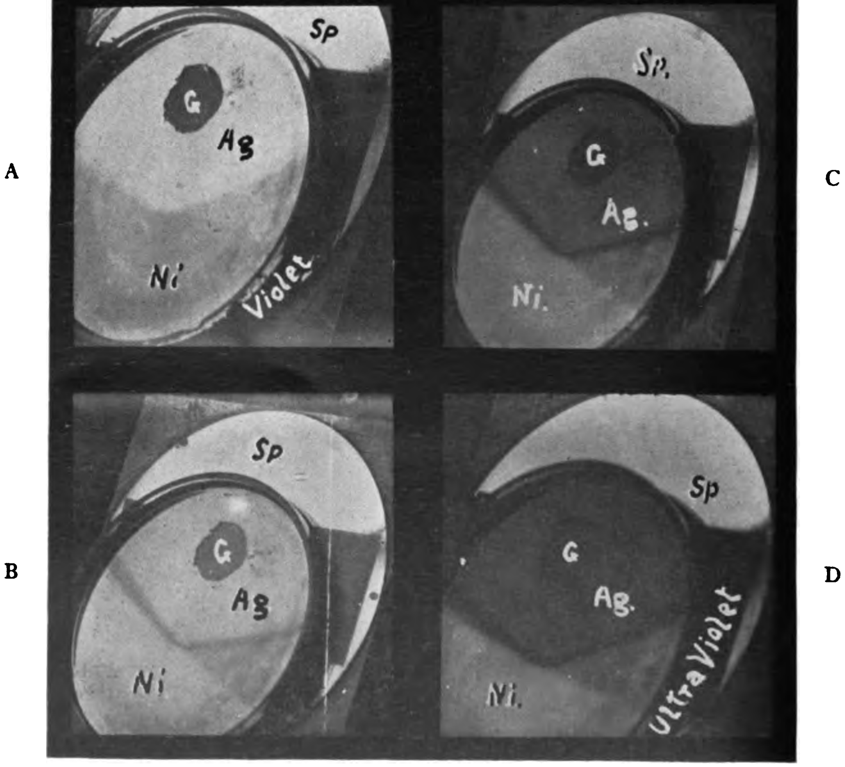


Fig. 3



Reflection from Nickel and Silver Films

of ammonia, a beautiful deposit could be thrown down in about 30 seconds with two dry cells. I then tried the double sulphate of nickel and ammonia. This gave some trouble at first, but it finally yielded good deposits. The solution must be very dilute, however, say 10 grams to the liter, and it is a good plan to add a little ammonia to it, sufficient to give it a very slight odor. From one to three dry cells can be used, according to their condition and the size of the mirror to be coated. The silver substratum should be thin enough to show a deep blue color when examined by a strong light, and the nickel should be thrown down until it becomes opaque. Some experience will be necessary before good deposits are obtained. If the silver coating is too thin, the fall of potential along the surface from the point where the battery wire joins it is too great, and the deposit is not uniform. It is a good plan to practice with small glass plates until one becomes familiar with the difficulties, and learns how to avoid them.

A nickel anode of about the same area as the plate to be coated is joined to the carbon pole of the battery, though in the case of my 16-inch long-focus mirror, I used a 16-inch disk of copper; it makes little or no difference which is used, as very little metal is thrown out of solution. Before depositing the nickel I at first dried the silvered mirror, and polished off the light deposit of dust which usually clouds the surface. This is a bad plan, for it is almost impossible to avoid long scratches, which cut through and destroy the continuity of the conducting film and result in an erratic distribution of the current; if this is the case, the nickel deposits along one side of the scratch only, and the deposit is very streaky. If the current is too weak or if the current is applied for an insufficient time, one obtains a very thin, yellowish deposit which has a very low reflecting power in the ultra-violet, not much better than that of the silver. Continuing the process a little longer usually remedies this defect.

To study the reflecting power of the various films I reflected the light from the cadmium or iron spark from them into a quartz spectrograph, and obtained in this way spectrograms which enabled their behavior to be judged at once. On Plate XX, Fig. 1, we have two spectra of the spark of cadmium and tin, the upper, *a*, taken with the light reflected from a nickel-on-glass film, the lower, *b*, with a silver film. As is at once apparent, one region of the spectrum in the latter photograph is wholly absent, and the entire ultra-violet below this region is relatively weak. Fig. 2 shows the same thing with the iron spark. Spectrogram *c* is made with direct light of the spark transmitted through the quartz-silver ray-filter, and shows that the region which the silver fails to reflect is copiously transmitted. Fig. 3 is extremely interesting and shows the reflecting power of glass, silver, speculum metal, and nickel for various regions of the spectrum.

A shallow glass dish was silvered on the inside. The lower portion of this deposit was then nickel plated, and a portion of the silver wiped from the upper portion with a bit of cotton moistened with very dilute nitric acid. The clear glass is marked "*G*" on the pictures, the nickel "*Ni*," the silver "*Ag*," and the plate of polished speculum metal, which stands behind the dish, "*Sp*."

The mirrors were mounted near an open window at such an angle as to reflect the light of the sky to the camera, which was provided with a quartz lens. Photograph *A* was made with the quartz lens unsilvered, that is, chiefly through the action of the blue and violet rays. The silver and the speculum metal come out very bright, while the nickel is much darker, and the unsilvered glass spot *G* is practically black. *B* was made with the quartz lens coated with a film of silver of such thickness that a brightly lighted window appeared very deep blue through it. In this case we have a mixture of violet and ultra-violet in such proportion, that the silver, nickel, and speculum all reflect about equally well. The dark V-shaped band which separates the nickel area from the silver is the thin yellow deposit above referred to, which diminishes the reflecting power in the ultra-violet. No trace of this appears in the first picture. In the small triangular area below the "*Ag*" the nickel deposit is thinner than over the rest of the lower portion of the plate, and the reflecting power is less. Photograph *C* was made through a silver film of such thickness that a brightly lighted window was barely visible through it, and the sun appeared as a disk of a bluish-white color easily supported by the eye. This picture is made chiefly by the ultra-violet region between wave-lengths 3000 and 3200. The silver reflects only very little better than the spot of clear glass, and the speculum somewhat more strongly than the nickel. In the case of *D* the silver film was so thick that the sun could be seen only as a feebly luminous ball of a deep violet color, resembling the lighted window as seen with the thinner film. A tungsten lamp was absolutely invisible through it. This is the best thickness to employ for ultra-violet photography. The glass and silver both come out almost black, the nickel fairly bright, and the speculum a little brighter.

It appears to me that there is a wide field of investigation open in various branches of science, by means of photography with these obscure rays. I have found that many white substances are absolutely black in this ultra-violet light, zinc-oxide being a conspicuous example. White garden flowers have very different reflecting powers in this region of the spectrum. Common phlox comes out coal black, while white geraniums are very much lighter. The earlier photographs which I made were taken with a silver film which was much too thin, as the investigations of the past summer have shown, and it is impossible to predict what will be found in the pictures of the moon and planets which are soon to be made with a very thick silver film and the long-focus nickel mirror.

We will now take up the method of coating a large mirror with a deposit of nickel.

The mirror which I have nickeled for the ultra-violet photography of the moon and planets is 16 inches in diameter and has a focal length of 26 feet. It was made by Mr. John E. Mellish of Cottage Grove, Wis., and is of very good figure. The silver substratum was thrown down by the formaldehyde process[1] and the plate transferred to the nickel bath without drying, the surface, however, being first well washed under a rather heavy stream of water to remove as much of the sediment as possible. The deposition of the nickel was carried on in a cylindrical pan of galvanized iron, which I found in the local hardware store, the inner surface being coated with paraffine to prevent the solution from attacking the metal. The silver surface of the mirror was about an inch below the surface of the nickel solution, the anode consisting of a circular disk of sheet copper of the same diameter as the mirror. A short piece of heavy brass tubing was soldered to the center of the copper disk to serve as a handle, and the wire from the battery was twisted around this. A V-shaped notch about an inch in depth was cut in the margin of the disk to facilitate making contact with the silver film. The other wire from the battery was attached to a slender rod of brass with a scrap of platinum-foil soldered to its lower end. This piece of foil was crumpled up so as to make contact at many points with the silver. The copper disk was carefully immersed in the solution and held so as to be everywhere nearly at the surface. This insured a uniform distance at every point between the anode and the silver film. The platinum electrode was now brought in contact with the silver film by passing the brass rod down through the V-shaped notch, and the current was allowed to flow for thirty seconds, after which the mirror was removed and held up to the light. The film was much darker than before but not quite opaque. Thirty seconds more finished the operation and gave a beautiful coat, which was washed, dried, and rubbed lightly with cotton to remove the slight film of sediment.

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[1] Wood, *Physical Optics*, revised edition.

If the current-strength and density of solution have been right, the surface of the nickel, as soon as it is removed from the plating bath, should appear bright, though distinctly duller than the silver. It is a good plan to prepare a small glass plate with half of its surface silvered and the other half nickeled. In this way it is possible to judge of the quality of the nickel film, which should appear of about the same brightness as the silver film viewed through a nicol prism, which reduces the intensity of the reflected light by one-half. If the film appears yellow, or brownish, like smoked glass, something has gone wrong. If the silver peels from the glass or frills, the current has been too strong or it has been applied for too long a time. The coating which I have applied to my mirror is very nearly as bright as the surface of my 13-inch short-focus speculum. It may happen that the film appears to be all right until an attempt is made to clean it after it is dry, when it is found that the slightest rubbing with soft cotton strips the film from the glass. This is probably due to the fact that too much metal has been deposited.

As is well known, nickel deposited by electrolysis comes down under a tremendous tension. Even in technical work, where it is deposited on solid metal, it is apt to scale off if deposited too thickly. Under the circumstances, it is rather surprising that the silver film is able to stand up under the strain.

In addition to the advantage to be derived from the higher reflecting power in the ultra-violet, it is possible that the nickel film may prove more permanent in atmospheres contaminated with sulphur, though I have not had my mirror under observation for a sufficient length of time to be able to speak of its lasting qualities. One mirror which I prepared early in the summer appears to have changed, the surface having a crystalline appearance in spots, as if the two metals had alloyed. This mirror was made, however, before I had worked out the process to my satisfaction, the film being of the brownish color referred to above. The electrical process does not interfere with the figure, as the thickness of the layer deposited is but a very small fraction of the wave-length of light.

These experiments are a continuation of work commenced two years ago in which I was aided by a grant from the Elizabeth Thomson Science Fund.

East Hampton, Long Island
September 28, 1911

REVIEWS

An Introduction to the Theory of Optics. By ARTHUR SCHUSTER. Second Edition, revised. London and New York: Longmans, Green & Co., 1910. $4.00.

This is the revised edition of Professor Schuster's well-known text-book on light, which first appeared seven years ago. The corrections called for by the latter were few in number; but the alterations have been numerous and are all helpful.

This book is different in principle and in detail from all others on the same subject, and on this account—apart from all other reasons—it makes delightful reading. It forms, in a way, the complement of an equally valuable book on Light, that by Professor R. W. Wood. One feels in reading each of these that one is in the hands of a master of the subject; each author gives both theory and experiment, but in Schuster's book theory is emphasized and made clear, while in Wood's it is the description of the experiments which one most enjoys.

It may not be amiss to call attention to the qualities which seem to the reviewer to distinguish this book from others bearing similar titles.

1. All theories are developed in such forms as to lend themselves to numerical calculations and verification.

This is an extraordinarily valuable feature; for students can with such ease lose sight of arithmetic when enjoying the calculus, and so often defer the searching test of a theory by the actual measurements of experiment.

In this connection reference may be made to the sections dealing with diffraction, optical instruments (especially the spectroscope), damping of vibrations of electrons, molecular scattering.

2. The various phenomena discussed, either on a large scale or in detail, are explained in terms both of quasi-homogeneous radiation and of ether impulses.

The homogeneous train of waves, as ordinarily written, is, as Schuster says, "a mathematical abstraction"; and so he introduces the idea of what he calls quasi-homogeneous radiations, such as are emitted by gases when made luminous by electrical excitation. To this idea he gives mathematical expression, by considering the summation of a number of simple waves, whose amplitudes and wave-lengths vary slightly. White light may then be defined by making the range of wave-length sufficient.

But white light may be considered as an irregular succession of impulses; and as Schuster says: "We take the disturbance as it is and try to represent it analytically, and just as there are many ways of resolving a system of forces, so are there many ways of resolving the motion of light into elements with which we can deal analytically. The resolution by homogeneous waves is one, the resolution by impulses, another. Whenever the two methods seem to yield different results, a mistake has been made in their application."

The many uses Schuster makes of the impulse-treatment are most instructive; e.g., in dispersion, interference, Talbot's bands, etc.

3. Many questions, often unimportant ones, are discussed, in which every student is interested and yet which are passed over by many writers. (The difficulty of the explanation may be the reason for this omission.) A great part of the enjoyment one has in reading the book comes from these remarks which are often of the nature of *obiter dicta.*

Again, the consideration given the great features of the subject is always ample and always original. New points of view are emphasized, and the basis of all theories in mechanical principles is made clear. A few sections in which this is most noteworthy are the ones on emission of energy, the nature of light, absorption, etc.

In reading a book which one enjoys and finds inspiring, it is not one's duty to look for errors of commission or omission; this is certainly true of this book, in spite of the fact that Professor Schuster by a remark in the preface to this new edition tempts one to make a painstaking search for mistakes. Yet a reviewer must note what he sees. The name of Lewis Rutherfurd is given wrongly on p. 142; and it is doubtful if, by using metallic surfaces on which to rule gratings the life of the diamond point is prolonged. The hardness of speculum metal is much greater than is supposed. The great advantage in its use as a grating surface comes from the fact that in all probability grooves ruled in it reproduce most exactly the plane faces of the ruling-diamond; so that, from a knowledge of the angles of the latter, the shape of the groove may be known. This is contrary to the belief of Rowland and to the statement by Schuster on p. 111; but its justification may be found in a recent article in this journal (May 1911) by Anderson and Sparrow. In this same article there is a critical discussion of gratings, which points out essential errors in that portion of Schuster's treatment given on pp. 122 and 123.

The reviewer is forced to confess that certain sections of the book have been understood with great difficulty: e.g., the discussion of absorption on pp. 270 and 271; the explanation of "the nature of light" on pp. 336–338; etc. One would not complain, if one did not believe that Professor Schuster might have made his treatment much simpler.

The order used in the arrangement of the subject-matter is not one to which we are accustomed; and the manner in which the student is referred to subsequent sections for vital discussions, are points of presentation or of pedagogy on which any teacher is privileged to differ with Professor Schuster. It may be, of course, that the training in English schools would make his scheme the natural one.

In conclusion, it should be stated clearly that no book on optics known to the reviewer is so eminently satisfactory, especially owing to the fact that it answers so many questions and reduces such a large part of the subject to the principles of mechanics.

JOSEPH S. AMES

JOHNS HOPKINS UNIVERSITY
June 1, 1911

The Principles and Methods of Geometrical Optics, Especially as Applied to the Theory of Optical Instruments. By JAMES P. C. SOUTHALL. New York: Macmillan, 1910. Pp. xxiv+626. $5.50.

This work aims to fill a recognized gap among scientific treatises in English. In extent and detail of treatment and in the scientific importance of the topics dealt with, it goes far beyond most other works in English; in fact, can be compared easily only with the recent book by Taylor, which is more restricted in scope and devoted primarily to the mathematical methods devised or used by its author, while the present volume is planned as a general treatise.

In the main the treatment is in the spirit of the recent German literature on the subject, from which indeed some portions appear rather as careful compilations than as independent presentation. Especial indebtedness is naturally owed and frankly acknowledged to the works of Czapski and of von Rohr and his colleagues. But the material from various sources has been thoughtfully fused into a coherent whole, and is supplemented by abundant and well-chosen bibliographic references. The notation in the formulas has evidently received unusual care, and the reader is further aided by a complete list of the algebraic symbols, given in an appendix which accompanies the good index.

The early chapters discuss the scope and limitations of geometric optics as a physical theory, presenting its basic concepts and laws by an eclectic use of the alternative methods of rays, wave-surfaces, and optical paths. A brief study of refraction at a plane surface prepares for a long chapter on prisms, which deals chiefly, however, with rays in a meridian plane; the treatment of oblique rays seems unduly meager, and the problem of the curvature of spectrum lines is dismissed with a list of references.

The elementary or paraxial theory of spherical surfaces and of lenses is given first in the older form of an approximation based on the law of refraction for rays of small obliquity and is then exhibited as the ideal implied in the purely geometric theory of optical imagery as developed principally by Abbe. To this modern fundamental theory of the general properties of optical collineations is devoted a full and excellent chapter, which will doubtless be especially welcome to readers to whom the German publications are not accessible.

Following this are given various sets of exact trigonometric formulas for rays following the physical law of refraction, together with a study of astigmatic pencils of arbitrary inclination. The remainder of the volume, except for the closing chapter on aperture, field, and illumination, presents the theory of the various spherical and chromatic aberrations of the first order, mainly in the form due to Petzval and Seidel, aided by some of the concepts and notations introduced by Abbe. For the pure theory the treatment here would be hard to improve.

Some shortcomings might be noted, partly in matters of physical optics, but more particularly in matters touching on the practical side, the detailed work of the maker or user of optical instruments. The formulas are not always adapted for computation and the numerical examples are meager and rather trivial. The list of Jena glasses is unfortunate in its selections, since half of the types given are perishable and no longer made, while the remainder are hardly representative of the optical materials available at the present time. But the book excludes from its plan a systematic discussion of special types of optical constructions and is therefore judged best simply as a repository of the general basic theory. As such it can hardly be said to have a superior in any language, and must surely be ranked as a standard work.

A. C. L.

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ERRATA

Vol. **34**, September 1911, in Professor Stebbins' article on "A New Bright Variable Star, β *Aurigae*":

Page 119, in (2), *for* m_1+etc., *read* m_1−etc.

Page 125, twelfth line from bottom, in coefficient of δi, *for* sin 2η, *read* sin η. This equation should be numbered (5).

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